

Multi-Objective Design Optimization of Planar Yagi-Uda Antenna Using Physics-Based Surrogates and Rotational Design Space Reduction

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Abstract

A procedure for low-cost multi-objective design optimization of antenna structures is discussed. The major stages of the optimization process include: (i) an initial reduction of the search space aimed at identifying its relevant subset containing the Pareto-optimal design space, (ii) construction—using sampled coarse-discretization electromagnetic (EM) simulation data—of the response surface approximation surrogate, (iii) surrogate optimization using a multi-objective evolutionary algorithm, and (iv) the Pareto front refinement. Our optimization procedure is demonstrated through the design of a planar quasi Yagi-Uda antenna. The final set of designs representing the best available trade-offs between conflicting objectives is obtained at a computational cost corresponding to about 172 evaluations of the high-fidelity EM antenna model.

Keywords: Multi-objective optimization, antenna design, simulation-driven design, surrogate modeling, design space reduction

1 Introduction

Design of antenna structures is a challenging task that requires taking into account a number of objectives, concerning electrical performance parameters (such as reflection response, gain, radiation pattern) as well as the antenna geometry, in particular, minimization of the antenna size or footprint in case of planar structures. In practice, typically only one of the objectives is optimized directly, whereas others are handled through suitably defined constraints or penalty functions. In many

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situations, however, it is advantageous to identify certain set of alternative designs representing the best possible trade-offs between various figures of interest (also referred to as a Pareto front). In this case, multi-objective optimization is necessary. The most popular solution approaches for this type of problem include population-based metaheuristics, such as genetic algorithms (Kuwahara, 2005; Yang, *et al.* 2008), particle swarm optimizers (Afshinmanesh, *et al.* 2008; Chamaani, *et al.* 2011), or differential evolution (Goudos and Sahalos, 2010; Rocca, *et al.* 2011). While metaheuristics allow for generating the entire Pareto set in a single algorithm run, their drawback is a high computational cost, typically measured in thousands of objective evaluations (Bekasiewicz *et al.* 2014a; Koziel *et al.* 2014a; Koziel *et al.* 2014b). This is a fundamental limitation when high-fidelity full-wave electromagnetic (EM) analysis is utilized for antenna evaluation.

The difficulties related to the high computational cost of the optimization process can be alleviated by using surrogate models (Koziel and Yang, 2011; El Zooghby *et al.* 2000). Surrogate-based optimization (SBO) replaces direct handling of expensive EM antenna models by iterative construction and optimization of fast replacement models – the so-called surrogates (Bandler *et al.* 2004; Koziel and Bandler, 2008; Queipo *et al.* 2005). In (Koziel and Ogurtsov, 2013), an efficient SBO procedure for multi-objective antenna design was introduced, utilizing a multi-objective evolutionary algorithm (MOGA) and auxiliary response surface approximation (RSA) model. The method was further improved in (Koziel, *et al.* 2014a) by endowing it with a procedure for design space (DS) reduction that allows for constructing the RSA model with a reasonable number of samples even in higher-dimensional spaces. Design cases with more than a dozen of adjustable parameters have been successfully solved using this approach (Bekasiewicz *et al.* 2014a; Koziel *et al.* 2014a). Nevertheless, even with the reduced design space, the overhead related to data acquisition for constructing of the RSA model may still be considerable and can become a dominant contributor to the overall optimization cost.

In this paper, an enhanced design space reduction algorithm is proposed. Our approach extends the reduction scheme (Bekasiewicz *et al.* 2014a)—based on the extreme points of the Pareto set obtained through single-objective optimization runs—by means of coordinate system rotation aimed at more precise location of the Pareto front. It allows for further reduction of the cost related to construction of the RSA model, utilized by the algorithm (Koziel *et al.* 2014a). The technique is demonstrated using a planar Yagi-Uda antenna example with 24 designable parameters, and a Pareto set representation found at the cost corresponding to only 172 evaluations of the high-fidelity EM antenna model.

2 Formulation of the Multi-Objective Antenna Design Problem

Accurate evaluation of antenna performance requires full-wave electromagnetic (EM) analysis. Here, a high-fidelity EM model of the antenna structure under design will be denoted as $\mathbf{R}_f(\mathbf{x})$, where $\mathbf{x}$ stands for a vector of adjustable geometry parameters. The model response represents antenna characteristics such as reflection versus frequency, antenna gain, or radiation pattern.

Let $F_k(\mathbf{x})$, $k = 1, \dots, N_{obj}$, be a k th design objective. A typical performance objective would be to minimize the antenna reflection over a certain frequency band of interest, and to ensure that $|S_{11}| < -10\text{dB}$ over that band. There might be also some geometrical objectives such as to minimize $F_k(\mathbf{x}) = A(\mathbf{x})$ – the antenna size defined in a convenient way (e.g., maximal lateral size, height, the maximal dimension, area of the footprint, antenna volume). Similar objectives can be formulated with respect to the antenna gain, radiation pattern, efficiency, etc.

If $N_{obj} > 1$, then any two designs $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ for which $F_k(\mathbf{x}^{(1)}) < F_k(\mathbf{x}^{(2)})$ and $F_l(\mathbf{x}^{(2)}) < F_l(\mathbf{x}^{(1)})$ for at least one pair $k \neq l$, are not commensurable, i.e., none is better than the other in the multi-objective sense. For the sake of multi-objective optimization, we utilize a Pareto dominance relation $\prec$ (Deb, 2001) defined as follows: for the two designs $\mathbf{x}$ and $\mathbf{y}$, we have $\mathbf{x} \prec \mathbf{y}$ ($\mathbf{x}$ dominates over $\mathbf{y}$) if $F_k(\mathbf{x}) < F_k(\mathbf{y})$ for all $k = 1, \dots, N_{obj}$. The goal of the multi-objective optimization is to find a representation of a

so-called Pareto front (or a Pareto-optimal set) X_P of the design space X , such that for any $\mathbf{x} \in X_P$, there is no $\mathbf{y} \in X$ for which $\mathbf{y} \prec \mathbf{x}$ (Deb, 2001).

3 Surrogate-Based Multi-Objective Optimization

Due to a high computational cost of EM simulations, a direct multi-objective optimization of the high-fidelity antenna model $\mathbf{R}_f$ is not practical. Instead, we use a surrogate-based approach of (Koziel and Ogurtsov, 2013) which exploits two types of lower-fidelity models. The first one is a coarse-mesh EM model $\mathbf{R}_{cd}$, which is evaluated using the same electromagnetic solver as the one used for $\mathbf{R}_f$, however, with a coarser discretization of the structure. As a result, the simulation time can be lowered considerably (typically, $\mathbf{R}_{cd}$ is 10 to 50 times faster than $\mathbf{R}_f$), of course, at the expense of limited accuracy. Unfortunately, $\mathbf{R}_{cd}$ is still too expensive for direct multi-objective optimization. Therefore, we utilize another auxiliary model, which is a fast response surface approximation (RSA) surrogate $\mathbf{R}_s$ obtained from sampled $\mathbf{R}_{cd}$ data. Here, we use kriging interpolation (Queipo *et al.* 2005; Siah *et al.* 2004). Latin Hypercube Sampling (Giunta *et al.* 2003) serves as the design of experiments technique. The sampling process and RSA model construction are carried out in the reduced design space (see Section 4 for a detailed description of the design space reduction algorithm).

An initial approximation of the Pareto set is subsequently obtained by optimizing the RSA surrogate using a multi-objective evolutionary algorithm (MOEA) (Koziel and Ogurtsov, 2013). We utilize a rather standard implementation of MOEA with Pareto-ranking-based tournament selection, fitness sharing and mating restrictions (Deb, 2001). The Pareto set obtained at this stage corresponds to the low-fidelity model $\mathbf{R}_{cd}$ so that its further refinement is necessary in order to obtain a representation at the high-fidelity model level. Thus, the final optimization stage consists of the refinement of K designs $\mathbf{x}_s^{(k)}$, $k = 1, \dots, K$, selected from the Pareto front found by the MOEA. To simplify the description, we make the assumption of having two objectives; generalization for larger number of objectives is straightforward. The refinement process is realized as follows (Bandler *et al.* 2004):

$$\mathbf{x}_f^{(k,i+1)} = \arg \min_{\mathbf{x}, F_2(\mathbf{x}) \leq F_2(\mathbf{x}_s^{(k,i)})} F_1(\mathbf{R}_s(\mathbf{x}) + [\mathbf{R}_f(\mathbf{x}_s^{(k,i)}) - \mathbf{R}_s(\mathbf{x}_s^{(k,i)})]) \quad (1)$$

i.e., the refined design is obtained by optimizing the corrected RSA surrogate. Here, we use output space mapping (Bandler *et al.* 2004) as the correction method. More specifically, we aim at improving the first objective without degrading the second one. In practice, 2 or 3 iterations of (1) are sufficient to find a refined high-fidelity model design $\mathbf{x}_f^{(k)}$ (Bekasiewicz *et al.* 2014a; Koziel *et al.* 2014a). The final outcome of the multi-objective optimization process is a set of refined Pareto-optimal high-fidelity model designs $\mathbf{x}_f^{(k)}$, $k = 1, \dots, K$.

The design optimization procedure can be summarized as follows:

1. Perform design space reduction (cf. Section 4);
2. Sample the design space and acquire the $\mathbf{R}_{cd}$ data;
3. Construct the RSA surrogate model $\mathbf{R}_s$;
4. Obtain the Pareto front by optimizing $\mathbf{R}_s$ using the MOEA;
5. Refine selected elements of the Pareto front, $\mathbf{x}_s^{(k)}$, to obtain the corresponding high-fidelity model designs $\mathbf{x}_f^{(k)}$.

Steps 1, 2, and 5 of the above procedure are the most significant contributors to the overall optimization cost. Step 1 is normally realized by means of an SBO algorithm (with coarse-discretization EM simulations used as the underlying low-fidelity model) so that its cost may be kept reasonably low (about a dozen or so high-fidelity antenna simulations per extreme Pareto set point).

Constructing the RSA model requires a few hundred to over one thousand $\mathbf{R}_{cd}$ samples. It should be emphasized, however, that this would not be possible—within a reasonable timeframe—without the space reduction step, if the number of design variables exceeds a few (in practice, 5 or 6). The cost of the refinement stage depends on the number of designs to be obtained as the algorithm outcome (typically around 10). The combination of the design space reduction and the surrogate model allows for lowering the overall optimization cost from thousands or tens of thousands of EM simulations required in direct MOEA-based optimization to dozens (or at most a few hundred for very complex cases).

4 Reduction of the Design Space

Constructing a reliable approximation model is one of the most important steps of the multi-objective optimization flow of Section 3. Although the RSA model is obtained from coarse-discretization EM simulation data (computationally much cheaper than the high-fidelity ones), the data acquisition is still a considerable challenge in computational terms. In practical problems, the number of the degrees of freedom in antenna structures tends to be large (cases with 10 to 20 parameters are not uncommon). Moreover, the ranges of parameters are wide because initial guesses about optimal dimensions are very difficult. Additionally, the solutions of interest from the multi-objective-optimization point of view normally occupy a small region of the design space. This is visualized in Fig. 1 using an example of a three-parameter UWB monocone antenna. Consequently, the required number of training samples may be impractically large even for relatively low-dimensional design spaces. Moreover, a vast majority of these samples are not relevant for antenna optimization since they are allocated outside the subset of the space containing Pareto-optimal solutions. To alleviate this difficulty, we utilize a three-step scheme for the design space reduction. The first step is a technique introduced in (Koziel *et al.* 2014a) that allocates the extreme points of the Pareto set (cf. Section 4.1). The second and the third step are based on an appropriate rotation of the coordinate system and are described in Section 4.2.

4.1 Initial Space Reduction through Extreme Pareto Set Points

The initial design space reduction relies on identifying the extreme points of the Pareto front. Let $\mathbf{x}^{*(k)} = \operatorname{argmin}\{\mathbf{x} : F_k(\mathbf{R}_{cd}(\mathbf{x}))\}$, $k = 1, \dots, N_{obj}$, be the optimum design with respect to the k th objective. The vectors $\mathbf{x}^{*(k)}$ determine the extreme points of the Pareto set. The reduced design space is then defined through the following lower and upper bounds, $\mathbf{l}$ and $\mathbf{u}$, respectively: $\mathbf{l} = \min\{\mathbf{x}^{*(1)}, \mathbf{x}^{*(2)}, \dots, \mathbf{x}^{*(N_{obj})}\}$, and $\mathbf{u} = \max\{\mathbf{x}^{*(1)}, \mathbf{x}^{*(2)}, \dots, \mathbf{x}^{*(N_{obj})}\}$ (cf. Fig. 2(a) for conceptual explanation). In practice, the reduced space is a very small fraction of the initial one, which makes the creation of the RSA model computationally feasible. While there is no guarantee that the design space reduced as above will contain the entire Pareto front, it will typically contain the majority of it (including, of course, the extreme points).

4.2 Design Space Reduction through Coordinate System Rotation

The second stage of the design space reduction procedure aims at more precise identification of the region containing the Pareto set, which includes determining its orientation within the design space. This stage allows for a further reduction of the number of samples required for creating the RSA model, which might be essential for highly-dimensional search spaces. The main assumption here is that majority of the Pareto-optimal solutions are—in practical cases—allocated close to the diagonal of the reduced space connecting the extreme points $\mathbf{x}^{*(1)}$ and $\mathbf{x}^{*(2)}$ (the current version of the method only works for $N_{obj} = 2$), cf. Fig. 2(b).

An outline of the entire reduction procedure is as follows:

Step 1: Execute the reduction procedure (Koziel *et al.* 2014a) to obtain the initially reduced space (the “box”);

Step 2: Perform a rotation of the box (cf. Fig. 2(b));

Step 3: Perform a reduction of the box size (cf. Fig. 2(c));

Step 2 is realized by an appropriate rotation of the coordinate system. Let $\mathbf{x}_0 = (\mathbf{x}^{*(1)} + \mathbf{x}^{*(2)})/2$ be the center of the box. Let $\mathbf{v}$ be a unit vector $\mathbf{v} = (\mathbf{x}^{*(1)} - \mathbf{x}^{*(2)})/\|\mathbf{x}^{*(1)} - \mathbf{x}^{*(2)}\|$, and $\mathbf{e}_1$ be a unit vector of the standard basis associated with the largest box dimension. The box is rotated around $\mathbf{x}_0$ with respect to a two-dimensional subspace spanned by vectors $\mathbf{v}$ and $\mathbf{e}_1$, so that the points $\mathbf{x}^{*(1)}$ and $\mathbf{x}^{*(2)}$ become the centers of its two faces as shown in Fig. 2(b). A rigorous description of the rotation process is omitted here for the sake of brevity. The dimensions of the rotated box are the same as the corresponding (through the rotated standard basis vectors) dimensions of the original box, except the one corresponding to the main diagonal (now equal to $\|\mathbf{x}^{*(1)} - \mathbf{x}^{*(2)}\|$).

In the last step of the process (Step 3), the rotated box is linearly reduced in all dimensions except the “diagonal” one. The appropriate reduction rate can be found iteratively (by identifying the Pareto sets for increasing reduction rates, and looking for front “saturation”). Here, we reduce a rotated box by a factor of 3, which is a rule of thumb found by considering multiple practical cases. Note, that this reduction rate results in approximately (assuming a relatively small rotation angle θ) 3^{n-1} reduction of the design space volume compared to that obtained through (Koziel *et al.* 2014a). For $n = 10$ dimensions, this additional reduction is over 10^4 (volume-wise).

5 Design Case: Yagi-Uda Antenna

Consider a planar quasi Yagi-Uda antenna (Wu *et al.* 2014) as shown in Fig. 3. The structure is designated to operate on a Taconic RF-35 dielectric substrate ($\epsilon_r = 3.5$, $\tan\delta = 0.0018$ and $h = 0.762$). The antenna is composed of a driven element in the form of a dipole excited through a coplanar stripline (CPS) and two directors. The structure is fed by a microstrip-to-CPS transition which provides wideband operation for 50-Ohm input impedance through a microstrip matching transformer. The antenna is miniaturized using two symmetrical ground plane stubs that increase the current path within a ground plane.

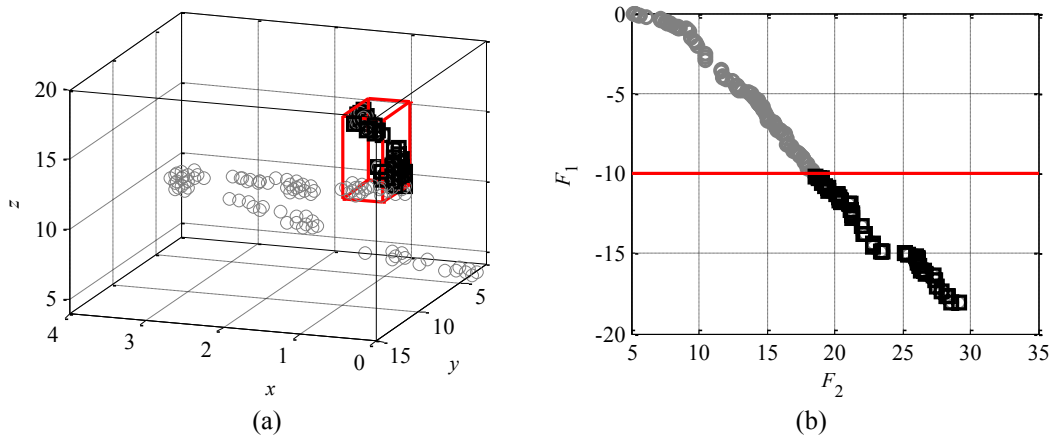


Figure 1: Visualization of the design space, the feature space and the Pareto set of an exemplary antenna (Bekasiewicz *et al.* 2014b): (a) Pareto optimal solutions ($\circ$) within the 3-dimensional solution space. The relevant Pareto optimal designs ($F_1 \leq -10$) are located in a subset, whose boundaries are determined by a red cuboid. (b) Pareto optimal set of interest ($\square$) versus the entire one mapped to the feature space ($\circ$).

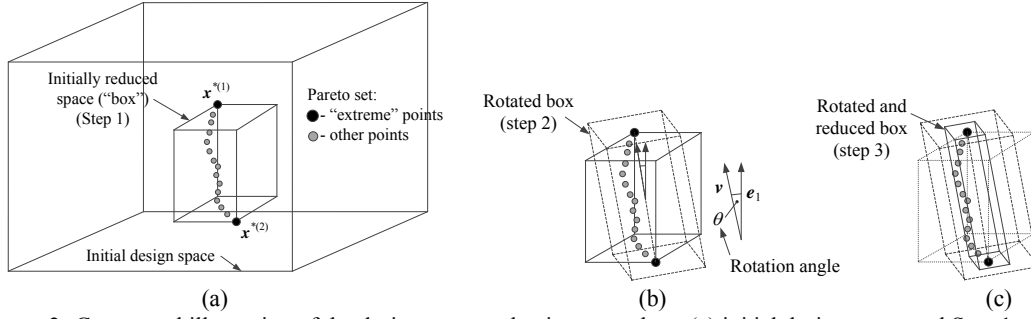


Figure 2: Conceptual illustration of the design space reduction procedure: (a) initial design space and Step 1 of the reduction process (finding the hypercube encompassing extreme Pareto front points $x^{*(1)}$ and $x^{*(2)}$), (b) Step 2: box rotation (here, v is a unit vector defining the box diagonal, and e_1 is a unit vector along the largest box dimension), (c) Step 3: reduction (linear scaling) of the rotated box.

The high-fidelity model ($\sim 2,100,000$ mesh cells and average simulation time of 13 min) and the low-fidelity model ($\sim 80,000$ mesh cells, average simulation time of 40 s) of the antenna are implemented in CST Microwave Studio (CST, 2013) and evaluated using its time domain solver. The design objectives are: F_1 – minimization of reflection, and F_2 – maximization of mean gain. Both are considered within 4 GHz to 10 GHz frequency range of interest. Additionally, the in-band reflection responses of the Pareto optimal design solutions are supposed to be kept below an acceptable level of -10 dB.

5.1 Results

The antenna structure is initially described by a vector of 24 independent design parameters $x^{(0)} = [w_0 \ d_1 \ d_2 \ g_1 \ l_1 \ l_2 \ l_3 \ l_4 \ l_6 \ l_7 \ l_8 \ l_f \ o_1 \ o_2 \ s_1 \ s_2 \ s_3 \ w_1 \ w_2 \ w_3 \ w_4 \ w_5 \ w_7 \ w_8]^T$ (see Fig. 3(b) for visualization). Additionally, we assume $l_5 = 3l_3$, whereas $d_3 = 3$, $d_4 = 6.5$, $w_f = 1.7$, $w_6 = 1$ remain fixed to reduce the design space dimensionality and provide 50-Ohm input impedance (all dimensions in mm). The initial design is as $x^{(0)} = [30 \ 0.65 \ 3 \ 2.9 \ 1.7 \ 2.15 \ 0.6 \ 3 \ 9 \ 7 \ 9 \ 0 \ 0 \ 2 \ 2 \ 0.7 \ 1.7 \ 2.5 \ 3.4 \ 1.6 \ 1.4 \ 1.6]^T$. The initial design space is defined by the following l/u bounds (chose arbitrarily): $l = [20 \ 0 \ 1 \ 1 \ 0.1 \ 1 \ 0.2 \ 0.2 \ 1 \ 5 \ 5 \ 0 \ 0 \ 1 \ 1 \ 1 \ 0.2 \ 1.2 \ 2 \ 2.9 \ 0.5 \ 0.5 \ 0.5]^T$ and $u = [40 \ 1 \ 5 \ 5 \ 4 \ 4 \ 1.2 \ 1.2 \ 5 \ 13 \ 13 \ 13 \ 2 \ 2 \ 4 \ 4 \ 4 \ 1.2 \ 2.2 \ 3 \ 3.9 \ 2.5 \ 2.5 \ 2.5]^T$. Such a configuration of parameters is utilized for the reduction of a design space (DS) by means of sequential optimizations with respect to one design objective at a time (cf. Section 4).

Subsequently, a number of the optimized parameters of an antenna, namely: $l_f = 7.5$, $w_1 = 0.7$, $w_0 = 28.86$, $l_4 = 0.58$, $o_2 = 0.13$, $d_2 = 3$, $l_7 = 9.47$, and $l_8 = 6.21$ have been fixed since they remain unchanged for the extreme designs. As a result, the solution space dimensionality has been confined to 16 independent design variables: $x = [d_1 \ g_1 \ l_1 \ l_2 \ l_3 \ l_6 \ o_1 \ s_1 \ s_2 \ s_3 \ w_2 \ w_3 \ w_4 \ w_5 \ w_7 \ w_8]^T$ and the refined lower/upper bounds are as follows: $l^* = [0.65 \ 1.63 \ 1.9 \ 2.02 \ 0.72 \ 2.6 \ 0.09 \ 2 \ 2 \ 2 \ 1.7 \ 2.3 \ 2.9 \ 1.6 \ 1.68 \ 2.24]^T$ and $u^* = [0.68 \ 3.09 \ 2 \ 2.07 \ 0.96 \ 2.83 \ 1.09 \ 2.2 \ 2.13 \ 3.73 \ 2.04 \ 2.92 \ 3.4 \ 2.08 \ 2.43 \ 2.45]^T$. One should emphasize that after a first step of DS reduction, the refined space is 18 orders of magnitude smaller (volume-wise) in comparison to the initial one.

Next, Steps 2 and 3 of the procedure of Section 4.2 have been applied to rotate the cuboid defined by the l^*/u^* bounds; the size of the rotated box is the reduced by a factor of 3 with respect to all dimensions except the one corresponding to the main diagonal of the original box. The final DS is defined by a set of 16 independent design variables. Moreover, it is 7 orders smaller (volume-wise) in comparison to the one obtained after first step of the introduced reduction procedure and about 10^{25} smaller than the one defined initially for 24 design variables.

After the DS reduction procedure, a kriging interpolation model R_s is constructed within the reduced space using 502 samples of the R_{cd} model (here, Latin Hypercube Sampling (LHS) is utilized

as the design of experiments procedure). The average relative RMS error of the model $\mathbf{R}_s$ is 2%. Subsequently, a multi-objective evolutionary algorithm is executed to find a Pareto set. The algorithm is run for 50 iterations with the population of the size of 500 individuals. A set of 10 design solutions selected across the Pareto set are then used for design refinement using the output space mapping algorithm (Bandler *et al.* 2004). A comparison of the $\mathbf{R}_f$ and $\mathbf{R}_s$ model representations of the Pareto front is shown in Fig. 4.

The design featuring the highest average gain of 6.8 dB simultaneously offers the worst acceptable in-band reflection (the gain values vary by 6% over the entire Pareto set), whereas the design with the best reflection of -16.7 dB exhibits the smallest gain (reflection variability is 34%). The obtained results clearly indicate conflicting nature of design objectives. The details for selected Pareto-optimal designs are gathered in Table 1, whereas their frequency and gain characteristics are shown in Fig. 5.

For the sake of comparison a kriging interpolation model $\mathbf{R}_s^*$ has been constructed within initially reduced solution space using a set of 2002 LHS-based samples of the $\mathbf{R}_{cd}$ model. Despite of using four times more training samples, the average relative RMS error for the $\mathbf{R}_s^*$ model is 3% (50% higher than for $\mathbf{R}_s$). The model $\mathbf{R}_s^*$ has also been optimized using a multi-objective evolutionary algorithm with 50 iterations and a population size of 500 individuals. A comparison of the obtained Pareto-optimal sets is given in Fig. 6. It should be emphasized that the Pareto set obtained within the rotated solution space is characterized by greater diversity of solutions than the set obtained with the model $\mathbf{R}_s^*$. The results also indicate correctness of the proposed rule of thumb, which considerably decreased the number of samples required for a construction of a reliable antenna model.

The total aggregated cost of quasi Yagi-Uda antenna optimization corresponds to about 172 $\mathbf{R}_f$ simulations (~ 38 hours of the CPU time) and includes: 2,917 $\mathbf{R}_{cd}$ model simulations for the initial design space reduction (1,076 $\mathbf{R}_{cd}$ and 1,841 $\mathbf{R}_{cd}$ simulations for optimization required for minimization of S_{11} and maximization of gain, respectively), 502 $\mathbf{R}_{cd}$ evaluations for a construction of the $\mathbf{R}_s$ model, and 30 $\mathbf{R}_f$ simulations for a refinement of 10 points selected across the Pareto-optimal set. One should emphasize that the proposed methodology allows for obtaining comprehensive information about the antenna without involving considerable computational resources. The estimated cost of direct optimization of such a structure using only high-fidelity model simulations is 25,000 $\mathbf{R}_f$ model evaluations (over 260 days of the CPU time), whereas the cost of antenna optimization using only $\mathbf{R}_{cd}$ representation would correspond to about 1,111 $\mathbf{R}_f$ evaluations (~ 278 hours). Therefore, the proposed design and optimization methodology outperforms direct optimization schemes driven by multi-objective evolutionary algorithms (being 145 and 6.5 times faster comparing to the former and the latter, respectively).

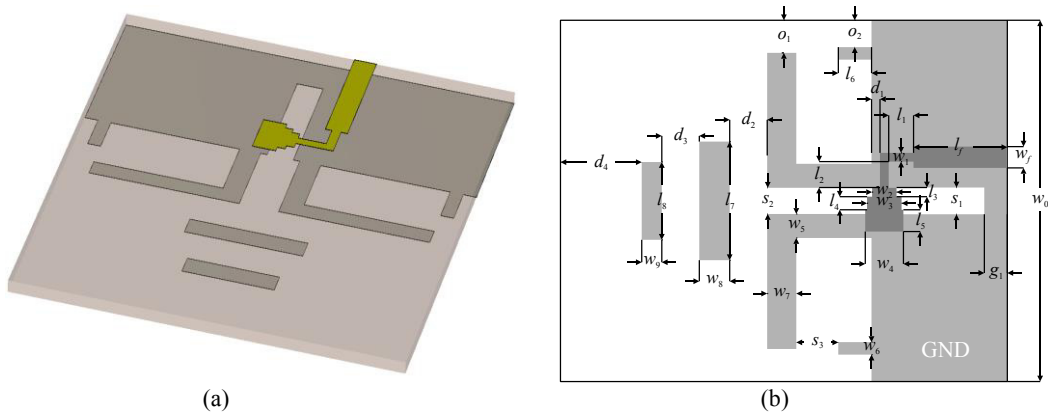


Figure 3: Planar quasi Yagi-Uda antenna (Wu *et al.* 2014): (a) three-dimensional visualization of a top and bottom layers of a structure; (b) detailed antenna dimensions.

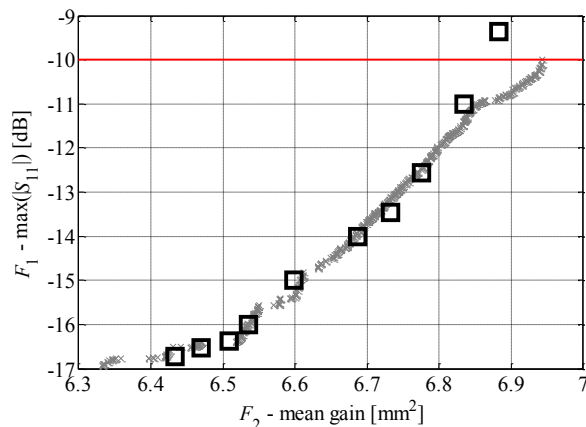


Figure 4: Pareto sets obtained using the proposed multi-objective optimization procedure with rotational design space reduction: responses of the surrogate model $\mathbf{R}_s$ ($\times$) and the high-fidelity model $\mathbf{R}_f$ ($\square$).

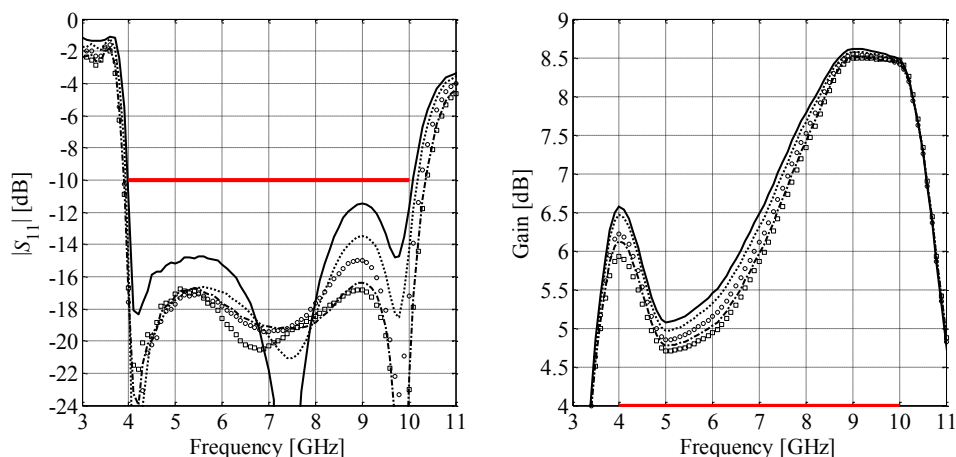


Figure 5: Frequency responses of optimized quasi Yagi-Uda antenna for selected designs: $\mathbf{x}_f^{(2)}$ — (—), $\mathbf{x}_f^{(4)}$ — ($\cdots$), $\mathbf{x}_f^{(6)}$ — ($\circ \circ \circ$), $\mathbf{x}_f^{(8)}$ — ($\cdot - \cdot$), $\mathbf{x}_f^{(10)}$ — ($\square \square \square$).

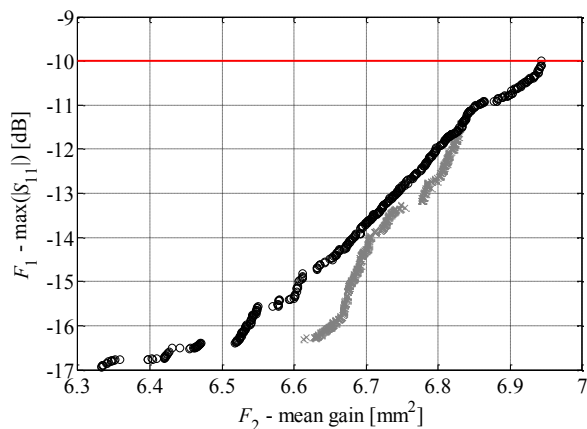


Figure 6: Comparison of Pareto fronts obtained using kriging model generated within initially reduced solution space $\mathbf{R}_s^*$ ($\times$) and generated within rotated solution space $\mathbf{R}_s$ ($\circ$).

Design		$\mathbf{x}_f^{(2)}$	$\mathbf{x}_f^{(4)}$	$\mathbf{x}_f^{(6)}$	$\mathbf{x}_f^{(8)}$	$\mathbf{x}_f^{(10)}$
F_1 [dB]		-11.0	-13.5	-15.0	-16.4	-16.7
F_2 [dB]		6.8	6.7	6.6	6.5	6.4
Design variables	d_1	0.67	0.67	0.66	0.66	0.65
	g_1	1.84	2.31	2.62	2.76	2.95
	l_1	1.99	1.96	1.93	1.91	1.91
	l_2	2.07	2.06	2.03	2.04	2.03
	l_3	0.73	0.81	0.88	0.91	0.93
	l_6	2.66	2.69	2.78	2.79	2.85
	o_1	1.01	0.64	0.31	0.21	0.29
	s_1	2.14	2.10	2.05	2.03	2.02
	s_2	2.09	2.07	2.03	2.02	2.02
	s_3	3.35	3.05	2.60	2.32	2.08
	w_2	1.95	1.90	1.83	1.79	1.76
	w_3	2.76	2.64	2.49	2.50	2.43
	w_4	3.05	3.13	3.21	3.28	3.42
	w_5	1.92	1.82	1.74	1.71	1.59
	w_7	2.33	2.16	1.99	1.87	1.84
	w_8	2.43	2.37	2.33	2.30	2.28

Table 1: Optimization results of quasi Yagi-Uda antenna

6 Conclusion

A methodology for computationally efficient multi-objective design optimization of antenna structures has been presented. The key to handling optimization problems in highly-dimensional design spaces is an appropriate design space reduction by identifying the search space region containing the set of Pareto optimal solutions. In this work, it is realized as a two-step process: (i) an initial reduction by means of finding the extreme points of the Pareto set (obtained through single-objective optimization w.r.t. one objective at a time), and (ii) a coordinate system rotation and further reduction of the rotated box (hypercube) that confines the Pareto set. This, together with exploitation of auxiliary models (coarse-discretization EM simulations and fast response surface approximation one) allows for finding the Pareto set at a cost corresponding to a few dozen of high-fidelity antenna simulations, even for relative large number of designable parameters, as demonstrated using a planar quasi Yagi-Uda antenna example. Future work will be directed towards reduction of the computational cost required to determine the extreme Pareto-optimal designs, which exceeds the cost of the kriging model construction for multi-dimensional designs. Moreover, extending the proposed rotational design space reduction technique for optimization problems with more than two design objectives will be conducted.

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