

Improving the demultiplexing performances of the multiple Bessel Gaussian beams (mBGBs)

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ARTICLE INFO

Keywords:

Bessel Gaussian (BG) beams
Free-space optical (FSO) communication
Multiple Bessel Gaussian beams (mBGBs)
Multiplexing phase hologram (MPH)
Orbital angular momentum (OAM)
Helical phase

ABSTRACT

As approximate non-diffracted beams, Bessel Gaussian (BG) beams are more suitable to be used in free-space optical (FSO) communication system than Laguerre Gaussian (LG) vortex beams because of its self-recovering property. However, due to the limitation of its detection method, the multiplexing communication based on multiple Bessel Gaussian beams (mBGBs) has not been investigated deeply. In this paper, we propose a method for simultaneous demodulation of 36-mBGBs using 6×6 multiplexing phase hologram (MPH). To solve the problem that the energy of some BG beams are too low at the demodulation end, a method is proposed to optimize the pinhole size of the pinhole plate at the demodulation end. Compared with the traditional decomposition method, the proposed method not only improves the decoding efficiency greatly, but also has better robustness. In addition, the communication performance of 32-mBGBs encoded with four ASCII codes has been constructed and investigated numerically. The results show that the transmitted information can be clearly decoded by setting a reasonable threshold value, which is beneficial to the practical application of mBGBs in FSO communications.

Introduction

In the traditional free-space optical (FSO) communication systems, the modulation and multiplexing of information carrier is mainly based on plane wave, which revolves around the dimension resources of optical wave, such as amplitude, wavelength, time and polarization [1–5]. However, the capacity crisis of optical communication technology still exists and needs to be solved urgently because the development of these existing dimensions has almost reached their limits. In 1992, Allen et al. demonstrated that the Laguerre-Gaussian (LG) beams with helical wavefront carried the orbital angular momentum (OAM) [6], which contains phase factor of $\exp(il\varphi)$ and carries $l\hbar$ OAM for each photon, where l called the OAM mode value or topological charge (theoretically infinite values), and φ is the azimuthal angle, $\hbar$ is reduced Planck constant [7]. Theoretically, vortex beams carrying different OAM modes are orthogonal to each other, so they can be used in the FSO communication to increase the capacity of information [8–12].

Similar to LG beams, high-order Bessel beams also have helical phase structure. However, in practical application, Gaussian truncation

modulation is often used to obtain approximate Bessel beams, and these new beams are called Bessel Gaussian (BG) beams [13]. As an approximate non-diffracted beam, BG beams has the recovery characteristics in a specific propagation range, which impel researchers to use BG beams in OAM-FSO communication systems [14–17]. Current researches on BG beams mainly focus on various generation methods [18–21] and their intensity distribution and mode crosstalk in turbulent environment [22,23] or inhomogeneous medium [24–26]. At present, geometrical transformation [27,28] and mode decomposition [29,30] are still used in BG beams demodulation. The study of BG beams communication involved only considers the case of single receive [15–17], and the research on the information transmission performance of multiple Bessel Gaussian beams (mBGBs) in the FSO communication system is still relatively lacking [14]. At present, scholars have proposed a variety of different OAM demodulation schemes based on the characteristics of OAM beams. Common traditional schemes mainly include schemes based on helical phase plate [31], diffracted optical element [32–35], phase hologram [36,37], interference or diffraction [38–40], optical geometric transformation [41], mode conversion method [42], OAM

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<https://doi.org/10.1016/j.rinp.2021.104829>

Received 28 July 2021; Received in revised form 6 September 2021; Accepted 15 September 2021

Available online 21 September 2021

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spectrum analysis [43], machine learning [44–47], etc. These schemes can be also used in BG beams to increase its demodulation speed and accuracy.

In this letter, by designing a novel multiplexing phase hologram (MPH), we present a high-efficiency mBGBs demodulation for FSO communication system. Just make the designed mBGBs incident at the MPH, from which the propagation directions of OAM states can be manipulated. Then, use the CCD camera to capture light intensity of the diffraction array, which is selected by a mask with a pinhole array for obtaining the demultiplexing energies of every OAM states. In this way, we can judge whether there are incident OAM modes in transmitted mBGBs. In addition, we have also found that the size of diffraction pattern at the Fourier diffraction plane is related to the radial wave vector of the incident BG beams. By adjusting the intensity pattern of the diffraction surface, more OAM states can be demodulated at one time. However, for smaller OAM states, the conversion energy in the center region of the diffraction order is too small. Therefore, the size of the pinhole in the mask can be optimized for making sure the transmittance intensities corresponding to different OAM states close to each other. Finally, 32-OAM states from ± 18 to ± 3 were selected to encode the four ASCII codes. By selecting an appropriate threshold, the demodulation terminal could easily demodulate the corresponding 32-bits information. This proves the superiority of the demodulation mBGBs method in the actual FSO communication system.

Methods

First of all, the light field distribution of the mBGBs with a set of target OAM states of $\{l_1, l_2, \dots, l_m\}$ in cylindrical coordinates (r, φ, z) can be expressed as follows [29]:

$$E_M(r, \varphi, z) = \sum_{l=l_1}^{l_m} \sqrt{\frac{2}{\pi}} J_l \left(\frac{z_R k_r r}{z_R - iz} \right) \exp \left(\frac{i w_g^2 k_r^2 z - 2kr^2}{4(z_R - iz)} \right) \exp(il\varphi - ik_z z). \quad (1)$$

where J_l is the l th order first-kind Bessel function, l is the topological charge, w_g is the waist radius of the Gaussian beam that confines the BG beam, $z_R = \pi w_g^2 / \lambda$ is the Rayleigh range, $k_r = k \sin(a/k)$ is the radial wave vector, $k_z = k \cos(a/k)$ is the longitudinal wave vector, k is the wave number, and “ a ” is the axicon parameter [48,49].

Previous mBGBs demodulation techniques often lead to severe pattern crosstalk [27–30]. In 2019, J. Pinnell et al. proposed and demonstrated that an inner product algorithm could be used to calculate the mode proportion of a vortex beam, as long as the radial amplitude

and directional phase of the vortex beam can be separated [30]. This method not only can detect only one OAM state with one anti-phase plate, but also the central light intensity expression at the Fourier plane is related to the order l of the first kind of Bessel function when the BG beams pass through the corresponding anti-phase plate directly. Therefore, if we want to make full use of the advantages of BG beams channel multiplexing, we need a new optimization method to solve the problem of low-detection efficiency and uneven energy spectrum distribution.

As a method to demodulate multiple topological charge states at the same time, the MPH has been widely used in demodulation of LG beams [37]. However, no one has yet attempted it to demodulate mBGBs. In fact, our research shows that MPH is actually more suitable for mBGBs. The phase of MPH can be described as:

$$\phi(x, y) = \sum_{n=1}^N \exp(i(k \sin \alpha_n x + k \sin \beta_n y + l_n \varphi)). \quad (2)$$

where N is the number of OAM beams, x and y indicate the horizontal and vertical coordinates respectively, α and β represent the horizontal and vertical angles respectively.

As shown in Fig. 1, the MPH is designed by combining the helical phase mask and the grating phase to form the fork grating. The mBGBs is incident on the inverse helical phase mask $\exp(il\varphi)$, and go through the Fourier transform, from which the BG mode can be converted to a bright spot in the concrete center [48]. The grating phases have the parameters of α and β , which allow to easily control the propagating directions vertically and horizontally of the desired OAM beams respectively. By conveniently choosing α and β in the grating phase, we are able to manipulate the relative diffraction positions of the OAM beam in x-y detection plane. By adding all types of fork-shaped grating phase, the multiplexing phase hologram can be obtained for the generation or detection of multiple OAM states, in which their propagating can also be manipulated simultaneously.

When the incident mBGBs illuminate the MPH and is transformed by a Fourier lens, the light field at the focal plane of the lens satisfies:

$$\begin{aligned} U(x, y) &= F\{\phi(x, y) \times E(x, y)\} \\ &= \sum_{n=1}^N F\{\exp(i(k \sin \alpha_n x + k \sin \beta_n y) \times \exp(il_n \varphi) \times E(x, y)\} \\ &= F\{\exp(i(k \sin \alpha_n x + k \sin \beta_n y)\} \otimes \sum_{n=1}^N F\{\exp(il_n \varphi) \times E(x, y)\} \end{aligned} \quad (3)$$

where $E(x, y)$ is the mBGBs equation at the MPH, F indicate Fourier

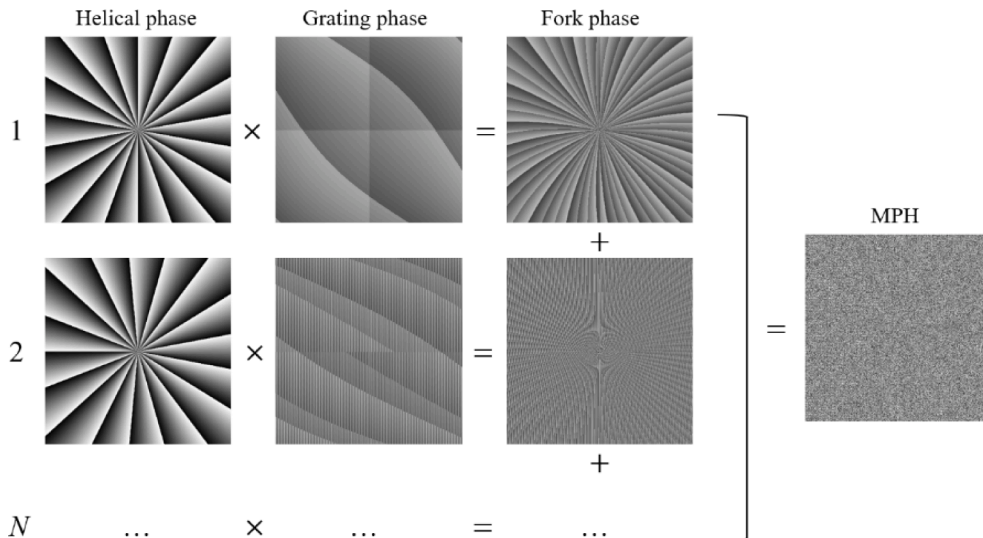


Fig. 1. Method of generating the MPH.

transform, and $\otimes$ means convolving two functions. In fact, based on the shifting theory of Fourier transform, $F\{exp(i(ksina_nx + ksin\beta_ny))\}$ only changes the center position of the light field on the Fourier plane and has no effect on the shape of the light field. When the BG beam passes through the MPH and then goes through a Fourier lens (corresponds to the far field diffraction result), the shapes of the regions corresponding to different diffraction orders of the field in the focal plane are consistent with the image shapes obtained by mode decomposition method [30].

The schematic diagram of the transmission system is shown in Fig. 2. At the transmitter, a Gaussian beam with the wavelength of $\lambda = 1550$ nm and beam waist $w_g = 1$ cm (as shown in Fig. 2(a)) is sent onto a spatial light modulator (SLM) encoded with a transmission function of an axicon [29,47]. Phase masks corresponding to different OAM modes are loaded on the SLM1 to modulate Gaussian beam into the mBGBs (as shown in Fig. 2(b)). After the 1000 m transmission, at the receiver, the 6×6 MPH loaded onto the SLM2 will control the propagating directions and also multiplies mBGBs with different inverse phases for detection. The processed BG beams are then passed through a Fourier lens (Lens 1, focal length: f_1), while a pinhole plate is placed at the focal plane (the Fourier plane) to filter out diffraction pattern except for the central intensity (as shown in Fig. 2(d)). Finally, Lens 2 (focal length: f_2) will focus the light beam through the pinhole plate to the CCD camera, which is used to record the light intensity transmitted by each aperture at the focal plane, and the computer is used to calculate the OAM states carried in the channel.

Results

As shown in Fig. 3(a), in order to explore the cause of uneven energy distribution in the detection of different topological charges, we incident the mBGBs with $l = 14, 9, 4, -2, -7$ and -12 at MPH to observe the energy distribution at different diffraction positions. Fig. 3(b) shows the intensity distribution of the mBGBs' diffraction pattern in the Fourier plane after passing through MPH. The green dot circles indicate the diffraction region corresponding to the OAM states contained in the incident mBGBs. Consistent with the theoretical analysis, bright spots appear in the center of the diffraction pattern only when the contained OAM spectrum exists, and there is almost no energy distribution in the center of the other positions. Fig. 3(c) magnifies the energy distribution of the corresponding diffraction pattern in the six circles in Fig. 3(b).

The fact that we chose 6×6 MPH for demodulating the mBGBs, does not maximize the performance of our designed MPH. Because after the BG beam multiplied by the additional phase and the Fourier transform, the order of Bessel function in the BG beam equation is different from the topological charge of vortex phase. Therefore, it is not possible to simplify the beam equation by the following equation:

$$\int_0^\infty J_l(up)J_l(vp)\rho dp = \frac{1}{u}\delta(u-v) \quad (4)$$

In other words, the BG beam equation cannot be reduced to the Dirac function, and the BG beam cannot be transformed into a perfect vortex where the radius is independent of the topological charge and the energy is confined to the ring [48]. But we find that the radius of the beam is still related to “ a ”, the radial wave vector of the incident BG beams. Fig. 3(b) shows the mBGBs incident to MPH with an axicon parameter of $a = 20$. Fig. 4(a) and 4(b) respectively show the diffraction patterns of the mBGBs with an axial parameter $a = 30$ and 10 respectively. The maximum quasi-non-diffracting transmission distance of mBGBs corresponding to these axicon parameters are 11.5, 7.7 and 23 km [49], respectively, which meet the requirements of long-distance transmission. Therefore, we can manipulate the axicon parameter to reduce the radius of diffractive vortices at the Fourier diffraction plane, so that when more vortices are demodulated simultaneously, the mutual crosstalk can be minimized. Theoretically, the number of OAM states of BG beams simultaneously demodulated by MPH can reach $7 \times 7, 8 \times 8$, or even more. Nevertheless, it is worth to note that when “ a ” is a large value, the diffraction pattern is large, and the diffraction pattern in adjacent areas may overlap with each other, which will have a great impact on the detection results. When the value of “ a ” is small, the size of the center bright spot will also decrease, which requires higher machining accuracy for the pinhole plate, and its robustness is weak. Therefore, for different required MPH, we need to choose appropriate axial parameters.

Although our proposed 6×6 MPH can simultaneously demodulate 36 different OAM states (-18 to $+18$) from left to right and top to bottom in different diffraction regions (as shown in Fig. 2(c), Fig. 3(b) and Fig. 4 (a, b)), which greatly improves the channel capacity. However, it also faces the problem that the central strength of different OAM states demodulation has a big difference, just like the mode decomposition method [30]. To solve this problem, they propose two methods to average the spectrum distribution of each OAM channel in Ref. 30. One is to modify the incident spectrum distribution, and the other is to divide by the designed OAM states-dependent factor at the receiver. However, in practical application, the former is inconvenient to adjust at the transmitter, and the optimal energy ratio needs to be calculated in advance for different channel combinations. Meanwhile, when the incident energy ratios of different OAM states are different, the channel interference will also have different influences on the detecting performances. Moreover, too much energy will aggravate the influence on the energy distribution in the surrounding area for the demodulation with diffraction plate; the latter modification of the spectral distribution by several orders of magnitude would greatly weaken the robustness of the system.

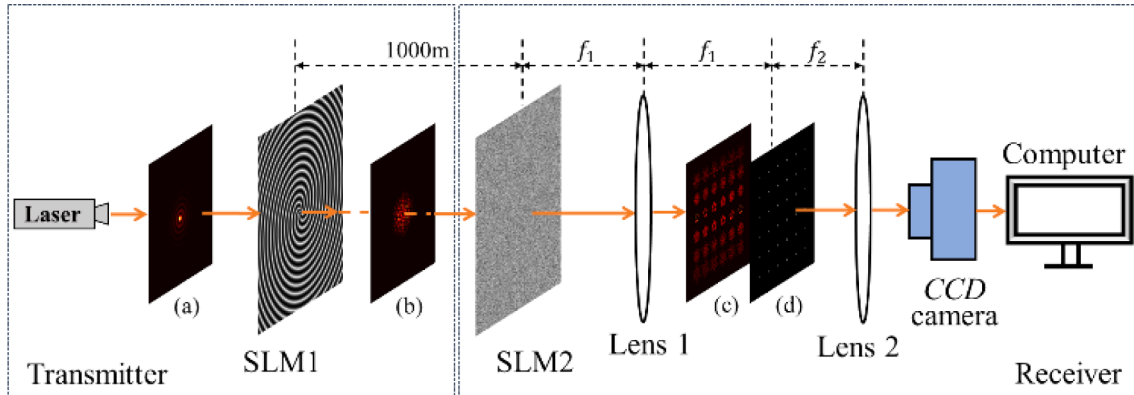


Fig. 2. Schematic diagram of the transmission system. (a), (b), (c) and (d) respectively correspond to Gaussian beam, mBGBs, far-field diffraction pattern after MPH and the mask with pinhole array.

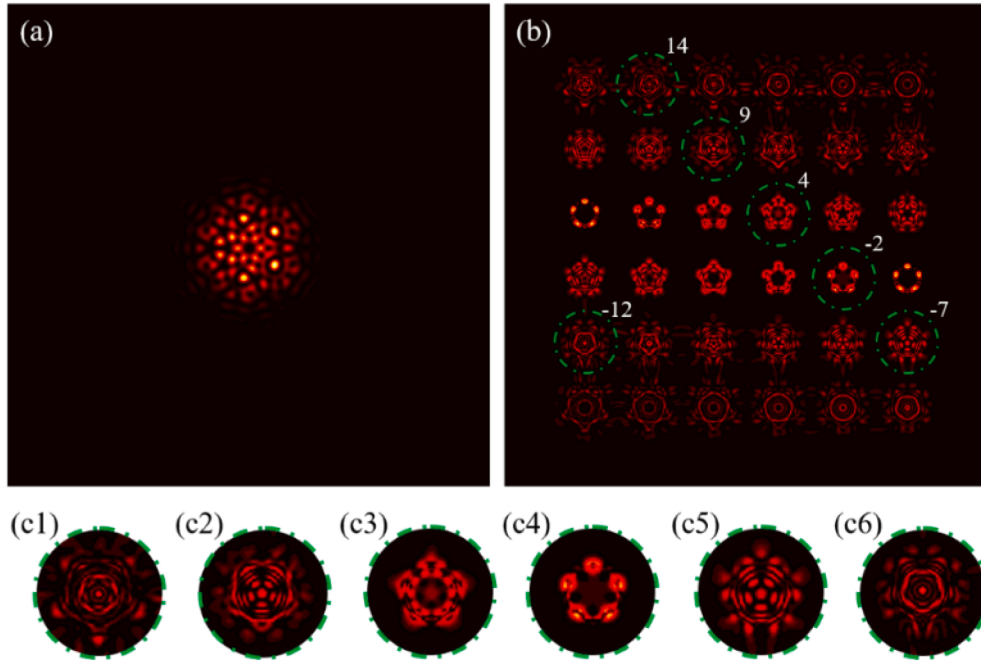


Fig. 3. The generated mBGBs (a) and the corresponding diffraction pattern by MPH (b) with $l = 14, 9, 4, -2, -7$ and -12 . (c1, c2, c3, c4, c5, c6) The zoom-in intensities profiles for $l = 14, 9, 4, -2, -7$ and -12 respectively with a bright spot in their centers.

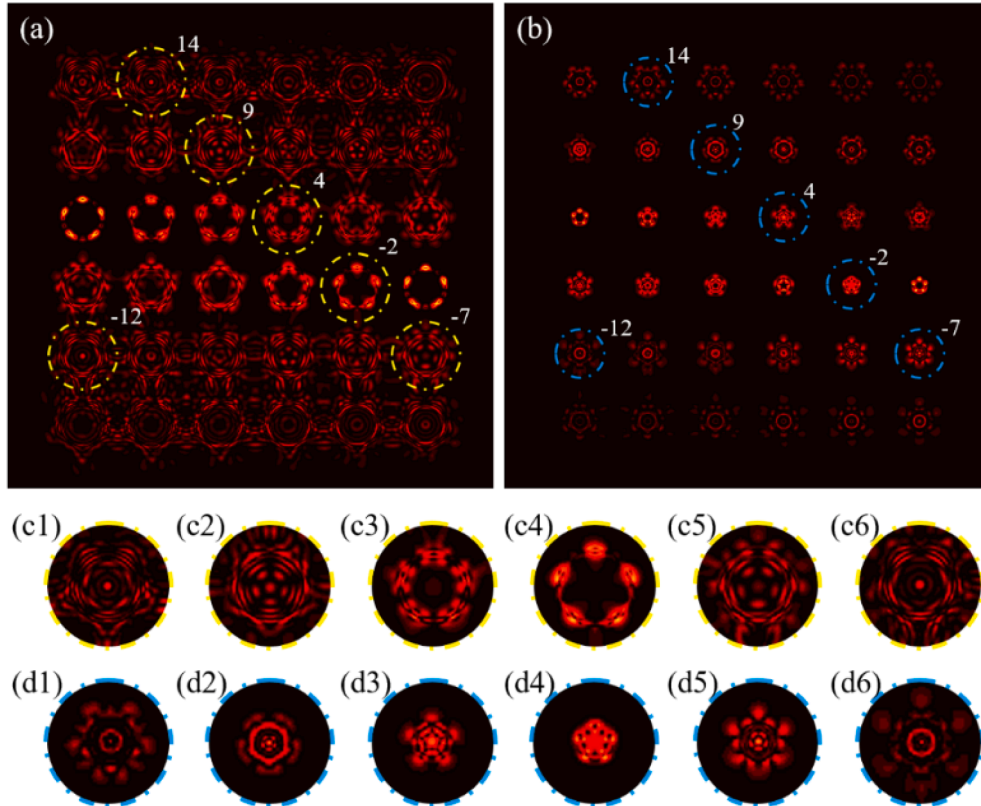


Fig. 4. Far-field diffraction pattern of mBGBs of (a) $a = 30$, (b) $a = 10$ and $l = 14, 9, 4, -2, -7$ and -12 (c1, c2, c3, c4, c5, c6 and d1, d2, d3, d4, d5, d6) after incident MPH.

Fig. 3 (c1-c6), Fig. 4 (c1-c6), Fig. 4 (d1-d6) respectively correspond to the cases of $a = 20, 30, 10$ and topological charges from 14 to -12 . It can be seen that when the value of OAM state is smaller, the range of bright spot in the center is larger, and the intensity at the most center is lower (independent of symbol). It can be reasonably speculated that the

reason why the demodulating center energy of small-OAM BG beam is small, is that the center highlight spot would be diffused into a relative larger-area center bright spot, and the smaller the OAM state, the more serious the diffusion. In view of this phenomenon of energy diffusion, the best solution is undoubtedly to change the pinhole sizes in the mask

at different positions (for different OAM states). But at first, we need to determine the possible effects of other factors on the center energy (especially the energy crosstalk between different diffraction orders). To qualify the proportion of detection energy of OAM channel l , we define the relative power η_l as:

$$\eta_l = \frac{I_l}{\sum_{n=1}^N I_n} \quad (5)$$

where N is the number of OAM states, I_n represent the intensity of the n th OAM state in the OAM demultiplexing array.

Fig. 5(a) shows the relative power distributions on the central axis after demodulation with a single anti-phase plate in the case of all OAM states from -18 to 18 . Fig. 5(b) shows the relative powers distribution after using pinhole plate filtering after simultaneously demodulating these OAM states in different diffraction regions using our designed MPH (pinhole size in each position is the same). Through comparison, the relative power distributions in Fig. 5(a) and Fig. 5(b) are basically the same. Although the powers of some specific OAM states are inconsistent due to diffraction, it is within the acceptable range compared with the influence of the size of OAM states on the energy proportion. We then modified the pinhole size in the mask, so that the region with relatively small OAM state can pass through a larger-area center bright spot light for obtaining more energy. After the correction, the relative power distributions transmitted by the demodulation terminals in each OAM state has been shown in Fig. 5(c), which are not exactly the same, because the size of each pinhole plate cannot be arbitrarily set due to the limitation of resolution in the simulation calculation (the simulation pattern is discrete into 1000×1000 pixels, and the aperture area of the

pinhole plate is $1-9$ pixels, the shape of the aperture is irregular circle). But in the actual applications, through precision machining, the size and shape of each pinhole in the pinhole mask can be set to what we want, so that the demodulation terminal can receive the same relative power for every OAM states.

Discussion

To further evaluate the feasibility of using our proposed “MPH + improved pinhole mask” demodulation mBGBs communication system, we have designed four kinds of 8-bit ASCII codes for simultaneous transmission, 32 coaxial single-OAM BG beams with $l = -18$ to -11 , -10 to -3 , 3 to 10 and 11 to 18 are used to form the mBGBs with the OAM modes determined by the value “1” of the binary code. Taking the word “HFUT” as example, the 8-bit ASCII codes are 01001000B, 01000110B, 01010101B, 01010100B. The mBGBs used to transmit the letter “HFUT” contains 12 OAM states of $l = -17, -14, -9, -5, -4, 4, 6, 8, 10, 12, 14$ and 16 . For the mBGBs transmitting the word “HFUT”, using our designed MPH and optimized pinhole mask demodulation, the relative powers of the OAM modes are shown in Table 1. The maximum and the minimum relative powers of the OAM states contained in the multiplexed BG beams are 0.1148 and 0.0568 , respectively. And the maximum relative powers of non-target OAM states is 0.0159 , which is much smaller than 0.0568 . The results show that the relative powers of binary codes “1” corresponding to the OAM states are high, while the relative powers of binary codes “0” corresponding to the OAM states are close to zero. The difference between the energies for the OAM states encoded as “1”, is mainly due to the design of the pinhole mask which

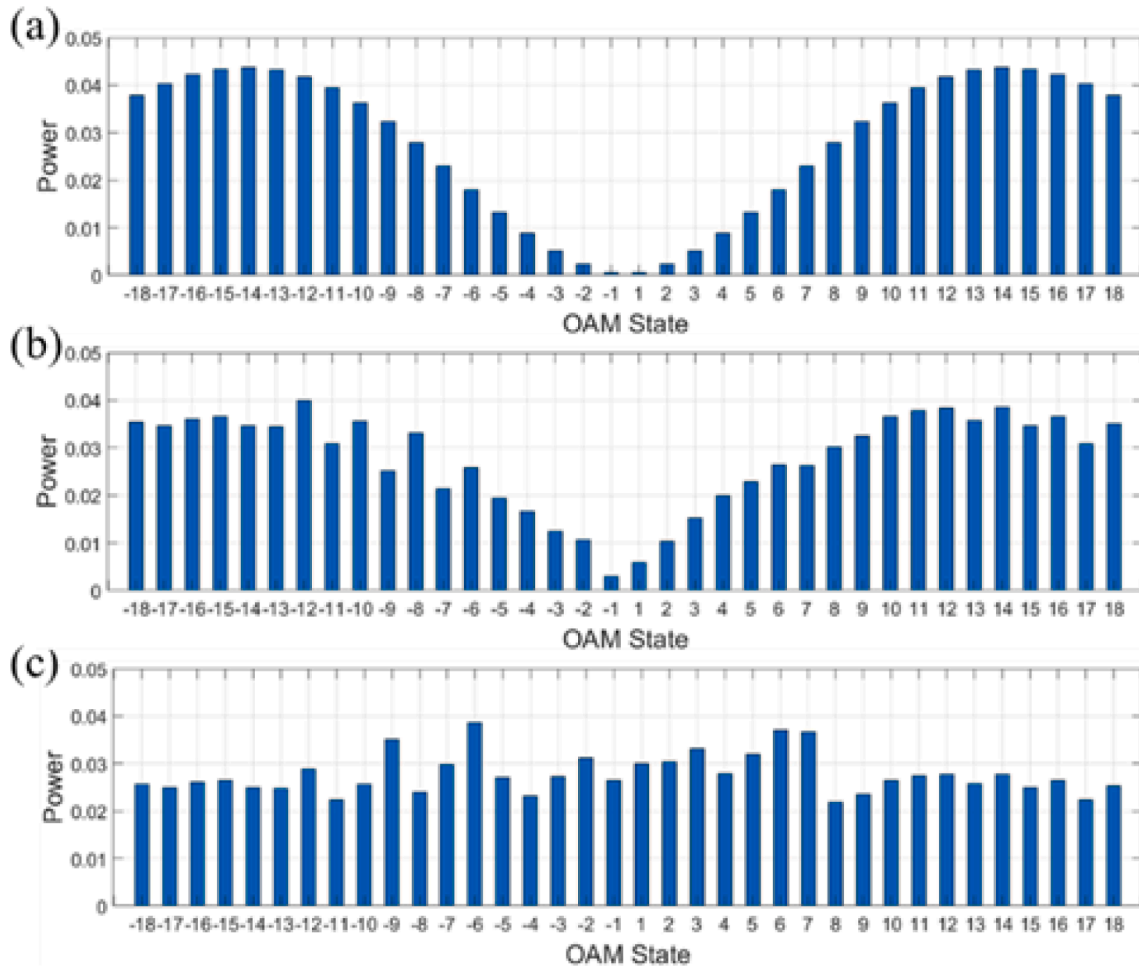


Fig. 5. The relative powers of each channel of BG beam with $l = 18$ to -18 is demodulated by (a) anti-phase plate, (b) MPH and (c) MPH + improved pinhole plate.

Table 1

Results of the decoded relative powers and the decoded binary codes for the mBGBs transmitting the word “HFUT”.

l	-18	-17	-16	-15	-14	-13	-12	-11	H
powers	0.0028	0.0682	0.0125	0.0107	0.0706	0.0018	0.0031	0.0026	
code	0	1	0	0	1	0	0	0	
l	-10	-9	-8	-7	-6	-5	-4	-3	F
powers	0.0055	0.1148	0.0027	0.0016	0.0046	0.0759	0.0652	0.008	
code	0	1	0	0	0	1	1	0	
l	3	4	5	6	7	8	9	10	U
powers	0.0067	0.064	0.0048	0.0918	0.0045	0.0568	0.0091	0.0631	
code	0	1	0	1	0	1	0	1	
l	11	12	13	14	15	16	17	18	T
powers	0.0084	0.0697	0.0038	0.0701	0.0159	0.0669	0.0048	0.0027	
code	0	1	0	1	0	1	0	0	

still has some defects. The slight energy for the OAM states encoded as “0”, is due to the energy leakage of other diffraction levels. Nonetheless, we can take 1/3 of the maximum relative powers of all OAM states as a threshold, and all OAM states with relative powers above this threshold are considered to exist (“1”) in the channel, while others are not (“0”). The decoding results are in good agreement with the expectation, which indicates that the demodulation speed of this method can reach 32 times of that of the mode decomposition, and meanwhile the communication system has higher reliability.

It’s worth mentioning that, after demodulating by the MPH, the size of diffraction patterns for each OAM state (BG beams) in diffractive region can be manipulated by the value of “ α ” effectively. Therefore, the number of detectable OAM states in BG beams demodulated by the MPH can be more than that of LG beams at one time (the current maximum MPH size for demodulating the multiplexing LG beams is 5x5). In addition to mBGBs’ self-recovery property, our proposed method provides a new scheme for FSO communications with high channel capacity and high accuracy.

Conclusion

In conclusion, we present a new method for improving the demultiplexing performances of the mBGBs. Compared with the traditional method using mode decomposition to demodulate BG beam, our method can not only significantly reduce the demodulation times and improve the demodulation speed, but also make the demodulation results more robust by introducing an improved pinhole mask. In additions, we have also proposed that the light intensity distribution on the diffracted plane could be manipulated by changing the axicon parameters (“ α ”) of the incident BG beam, so as to further increase the number of detectable OAM states. Finally, we use that proposed method to demodulate 36-OAM-mBGBs with $l = -18$ to 18 simultaneously, and apply it to a communication system that can transmit four ASCII codes simultaneously. We demonstrate that the information carried by all four ASCII codes can be accurately demodulated simultaneously by setting the threshold to 1/3 of the maximum relative energy. This mBGBs demodulation method based on MPH combined with an improved pinhole mask will provide a new and efficient option for high-speed self-recovery FSO communication system.

Funding

National Natural Science Foundation of China (NSFC) (61775050); Fundamental Research Funds for the Central Universities (PA2019GDZC0098).

CRediT authorship contribution statement

Chaofan Gong: Conceptualization, Formal analysis, Methodology, Software, Validation, Visualization, Writing - original draft. **Zhenzhen Pan:** Formal analysis, Software, Validation, Visualization. **Maxime Irene Dedo:** Formal analysis, Data curation, Methodology. **Jinghua Sun:** Investigation, Writing – review & editing. **Lulu Wang:** Investigation, Writing – review & editing. **Zhongyi Guo:** Conceptualization, Formal analysis, Funding acquisition, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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