

International Conference on Computational Science 2013

Shape-Preserving Response Prediction for Engineering Design Optimization

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Abstract

Contemporary engineering design relies heavily on computer simulations. The structures and systems considered in many engineering disciplines are far too complex to be described accurately using simple theoretical models; using simulations is often the only way to adequately assess the performance of the design. Unfortunately, high-fidelity simulations are computationally expensive so that their use in an automated design process based on conventional optimization algorithms is often prohibitive. Despite the availability of faster computers and more efficient simulation software: growing demand for improved accuracy and the need to evaluate larger and larger systems effectively diminishes the benefits of this increase in computing power. One of the most promising ways of alleviating these problems are surrogate-based optimization (SBO) techniques which are capable of reducing the number of expensive objective function evaluations in a simulation-driven design process. In SBO, the direct optimization of the expensive model is replaced by iterative updating and re-optimization of its cheap surrogate model. Among proven SBO techniques, the methods exploiting physics-based low-fidelity models are probably the most efficient. This is because the knowledge about the system of interest embedded in the low-fidelity model allows constructing the surrogate model that has good generalization capability at a cost of just a few evaluations of the original model. In this paper, we review one of the most recent techniques of this kind, the so-called shape-preserving response prediction (SPRP). We discuss the formulation of SPRP, its limitations and generalizations, and, most importantly, demonstrate its applications to solve design problems in various engineering areas, including microwave engineering, antenna design, and aerodynamic shape optimization.

Keywords: Microwave engineering, aerodynamic optimization, surrogate modeling, space mapping, shape-preserving response prediction.

1. Introduction

The computer simulation is one of the most important tools in contemporary engineering and science. Starting from small devices such as electronic devices and circuits, through complete systems such as aircraft, to large-scale phenomena (e.g., climate models), simulation is used to evaluate the performance and validate the designs. Commercial simulation packages have matured and the computing resources are cheaper and in abundance. Yet, in many cases, accurate, high-fidelity simulation is computationally expensive, to the extent that its use in the design process, e.g., by employing it in an automated design optimization loop, may be prohibitive. The primary reason is that conventional optimization algorithms, both gradient-based [1] and derivative-free [2] require large number of objective function

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evaluations. In some cases, the use of adjoint sensitivity [3] can alleviate this difficulty; however, this technique is not always available through commercial simulation packages. On the other hand, design automation becomes a necessity in all situations where simple theoretical models are no longer capable to adequately account for complex interactions between the system components and, therefore, yield nothing but an initial approximation of the optimum design that has to be tuned further in order to meet the given performance requirements. In practice, design “tuning” is often based on parameter studies guided by engineering experience. This combination is often sufficient to obtain satisfactory designs in reasonable time; however, it is far from being an automated process.

Surrogate-based optimization (SBO) [4], [5] is probably the most promising approach to alleviate the difficulties discussed in the previous paragraph. In SBO, direct optimization of the expensive high-fidelity simulation model is replaced by iterative updating and re-optimization of its computationally cheap representation, the surrogate. Because of shifting the optimization burden into the surrogate, the overall design cost can be greatly reduced. The high-fidelity model is referenced rarely, to verify the prediction produced by the surrogate and to improve the latter. Various SBO methods differ mostly in the way the surrogate is created. There is a large group of function approximation modeling techniques, where the surrogate is created by approximating sampled high-fidelity model data. The most popular methods include polynomial approximation [5], radial basis function interpolation [6], kriging [7], support vector regression [8], and neural networks [9]. Once created, approximation models are very fast, however, a large number of training samples—and a high CPU cost of gathering the simulation data—are necessary to ensure reasonable accuracy. Furthermore, the number of samples grows exponentially with the dimensionality of the design space (the *curse of dimensionality*). Depending on the model purpose, this initial computational overhead may (e.g., multiple-use library models) or may not (e.g., one-time optimization) be justified.

Another approach to surrogate-based optimization is by correcting an auxiliary low-fidelity (or coarse) model. Low-fidelity model is a reduced-accuracy but faster representation of the system of interest. This can be obtained in various ways, such as by using simplified-physics, leaving out certain second-order effects, or describing the system on a different physical level (e.g., equivalent circuit versus full-wave electromagnetic simulation in case of microwave components). In many areas, engineers have been using such simplified models for decades: before the computer era they were the only way to conduct the design process. Because of the fact that the low-fidelity model contains certain knowledge about the system of interest, physics-based surrogates offer good generalization capability and can be set up using a limited number of training points, which is their biggest advantage over purely approximation models. Several techniques have been proposed to exploit physics-based surrogate models in the SBO process, including the approximation model management optimization (AMMO) framework [10], space mapping (SM) [11], manifold mapping [12], and simulation-based tuning [13]. Many of these methods are based on correcting the low-fidelity model output (response). If the correction is realized to ensure both zero- and first-order consistency [10] between the surrogate and the high-fidelity model, the SBO process is provably convergent to the high-fidelity model optimum [13] when embedded in the trust-region framework [14]. In some cases (with a notable example of SM), the correction can be done by introducing a shift or an even more complex mapping between the parameter space of the low- and high-fidelity models.

One of the recent techniques exploiting physics-based low-fidelity models is the shape-preserving response prediction (SPRP) technique [15]. This method, originally developed in microwave engineering area [15], has been applied to problems in antenna design [16] and aerodynamic design [17]. SPRP is a parameter-less method where the surrogate model response is constructed by tracking the changes of the low-fidelity model response when moving from a certain reference design to another one, and applying these changes (represented by translation vectors) to a reference response of the high-fidelity model. The SPRP surrogate exploits the knowledge embedded in the low-fidelity model to a larger extent than, e.g., SM, and, therefore, its generalization capability is usually better than that of SM [15]. In this paper, we review the SPRP technique, its basic and generalized formulations, and attempt to give an intuitive explanation of its efficiency. We also illustrate its operation and performance using several design examples from various engineering disciplines.

2. Surrogate-Based Optimization and Shape-Preserving Response Prediction

In this section, we formulate the engineering design optimization problem, recall the concept of surrogate-based optimization and discuss the shape-preserving response prediction (SPRP) methodology [15]. Examples illustrating application of SPRP in various engineering fields are provided in Sections 3 to 5.

2.1. Engineering Design Task as an Optimization Problem

The engineering design optimization problem can be defined as

$$\mathbf{x}_f^* = \arg \min_{\mathbf{x}} U(f(\mathbf{x})) \quad (1)$$

where $f: X_f \rightarrow R^m$, $X_f \subseteq R^n$, denotes the response vector of a high-fidelity (or fine) model of the device or system of interest; $U: R^m \rightarrow R$ is a given objective function, e.g., minimax [18]. In microwave engineering, the response vector may contain, for example, the values of transmission coefficient $|S_{21}|$ evaluated over certain frequency band.

2.2. Surrogate-Based Optimization

Because of the high computational cost of evaluating f , its direct optimization is replaced by an iterative procedure [5]

$$\mathbf{x}^{(i+1)} = \arg \min_{\mathbf{x}} U(s^{(i)}(\mathbf{x})) \quad (2)$$

that generates a sequence of points (designs) $\mathbf{x}^{(i)} \in X_f$, $i = 0, 1, \dots$. Each $\mathbf{x}^{(i+1)}$ is the optimal design of the surrogate model $s^{(i)}: X_s^{(i)} \rightarrow R^m$, $X_s^{(i)} \subseteq R^n$, $i = 0, 1, \dots$. $s^{(i)}$ is assumed to be a computationally cheap and sufficiently reliable representation of the fine model f , particularly in the neighborhood of the current design $\mathbf{x}^{(i)}$. Under these assumptions, the algorithm (2) is likely to produce a sequence of designs that quickly approach $\mathbf{x}_f^*$. Because f is evaluated rarely (usually once per iteration), the surrogate model is supposedly fast, and the number of iterations for a well-performing algorithm is substantially smaller than for most direct optimization methods, the process (2) may lead to substantial reduction of the computational cost of solving (1). If the surrogate model satisfies zero- and first-order consistency conditions with the fine model, i.e., $s^{(i)}(\mathbf{x}^{(i)}) = f(\mathbf{x}^{(i)})$ and $(\partial s^{(i)} / \partial \mathbf{x})(\mathbf{x}^{(i)}) = (\partial f / \partial \mathbf{x})(\mathbf{x}^{(i)})$ (verification of the latter requires f sensitivity data), and the algorithm (2) is enhanced by the trust region method [19], then it is provably convergent to a local optimum of the fine model [10]. Convergence can also be guaranteed if the algorithm (2) is enhanced by properly selected local search methods [20].

2.3. Shape-Preserving Response Prediction: Concept [15]

Shape-preserving response prediction (SPRP) [15] has been initially introduced in microwave engineering to reduce the cost of optimizing electromagnetic (EM)-simulated structures such as filters [15]. In SPRP, the surrogate model is constructed assuming that the change of the fine model response due to the adjustment of the design variables from can be predicted using the actual response changes of the auxiliary low-fidelity (or coarse) model $c: X_c \rightarrow R^m$, $X_c \subseteq R^n$, that describes the same object as the high-fidelity model; c is less accurate but much faster to evaluate than f . The choice of the coarse model very much depends on the engineering discipline. In microwave engineering, the coarse model might be an equivalent circuit of the considered microwave structure, that describes the structure using circuit theory methods rather than through solution of the Maxwell equations. It is critically important for SPRP that the coarse model is physically based, which ensures that the effect of the design parameter variations on the model response is similar for both the fine and coarse models. The change of the coarse model response is described by the translation vectors corresponding to certain (finite) number of characteristic points of the model's response. These translation vectors are subsequently used to predict the change of the fine model response with the actual response of f at the current iteration point, $f(\mathbf{x}^{(i)})$, treated as a reference.

Here, we explain the concept of SPRP using the specific case of a microwave filter. Figure 1(a) shows the example of the coarse model response, $|S_{21}|$ in the frequency range 8 GHz to 18 GHz, at the design $\mathbf{x}^{(i)}$, as well as the coarse model response at some other design $\mathbf{x}$. The responses come from the double folded stub bandstop filter [15]. Circles denote five characteristic points of $c(\mathbf{x}^{(i)})$, here, selected to represent $|S_{21}| = -3$ dB, $|S_{21}| = -20$ dB, and the local $|S_{21}|$ maximum (at about 13 GHz). Squares denote corresponding characteristic points for $c(\mathbf{x})$, while small line segments represent the translation vectors that determine the “shift” of the characteristic points of c when changing the design variables from $\mathbf{x}^{(i)}$ to $\mathbf{x}$. Because the coarse model is physics-based, the fine model response at the given design, here, $\mathbf{x}$, can be predicted using the same translation vectors applied to the corresponding characteristic points of the fine model response at $\mathbf{x}^{(i)}$, $f(\mathbf{x}^{(i)})$. This is illustrated in Fig. 1(b). Figure 2 shows the predicted fine model response at $\mathbf{x}$ as well as the actual response, $f(\mathbf{x})$, with a good agreement between both curves.

2.4. Shape-Preserving Response Prediction: Formulation [15]

SPRP can be rigorously formulated as follows. Let $f(\mathbf{x}) = [f(\mathbf{x}, \omega_1) \dots f(\mathbf{x}, \omega_m)]^T$ and $c(\mathbf{x}) = [c(\mathbf{x}, \omega_1) \dots c(\mathbf{x}, \omega_m)]^T$, where $\omega_j, j = 1, \dots, m$, is the frequency sweep (it can be assumed without loss of generality that the model responses are parameterized by frequency). Let $p_j^f = [\omega_j^f \ r_j^f]^T$, $p_j^{c^0} = [\omega_j^{c^0} \ r_j^{c^0}]^T$, and $p_j^c = [\omega_j^c \ r_j^c]^T$, $j = 1, \dots, K$, denote the sets of characteristic points of $f(\mathbf{x}^{(i)})$, $c(\mathbf{x}^{(i)})$ and $c(\mathbf{x})$, respectively. Here, ω and r denote the frequency and magnitude components of the respective point. The translation vectors of the coarse model response are defined as $t_j = [\omega_j^t \ r_j^t]^T, j = 1, \dots, K$, where $\omega_j^t = \omega_j^c - \omega_j^{c^0}$ and $r_j^t = r_j^c - r_j^{c^0}$. The shape-preserving response prediction surrogate model is defined as follows

$$s^{(i)}(\mathbf{x}) = [s^{(i)}(\mathbf{x}, \omega_1) \dots s^{(i)}(\mathbf{x}, \omega_m)]^T \quad (3)$$

where

$$s^{(i)}(\mathbf{x}, \omega_j) = \bar{f}(\mathbf{x}^{(i)}, F(\omega_j, \{-\omega_k^t\}_{k=1}^K)) + R(\omega_j, \{r_k^t\}_{k=1}^K) \quad (4)$$

for $j = 1, \dots, m$. $\bar{f}(\mathbf{x}, \omega)$ is an interpolation of $\{f(\mathbf{x}, \omega_1), \dots, f(\mathbf{x}, \omega_m)\}$ onto the frequency interval $[\omega_1, \omega_m]$. The scaling function F interpolates the data pairs $\{\omega_1, \omega_1\}$, $\{\omega_1^f, \omega_1^f - \omega_1^t\}$, $\dots$, $\{\omega_K^f, \omega_K^f - \omega_K^t\}$, $\{\omega_m, \omega_m\}$, onto the frequency interval $[\omega_1, \omega_m]$. The function R does a similar interpolation for data pairs $\{\omega_1, r_1\}$, $\{\omega_1^f, r_1^f - r_1^t\}$, $\dots$, $\{\omega_K^f, r_K^f - r_K^t\}$, $\{\omega_m, r_m\}$; here $r_1 = R_c(\mathbf{x}, \omega_1) - R_c(\mathbf{x}^f, \omega_1)$ and $r_m = c(\mathbf{x}, \omega_m) - c(\mathbf{x}^f, \omega_m)$. In other words, the function F translates the frequency components of the characteristic points of $f(\mathbf{x}^{(i)})$ to the frequencies at which they should be located according to the translation vectors t_j , while the function R adds the necessary magnitude component. The interpolation onto $[\omega_1, \omega_m]$ is necessary because the original frequency sweep is a discrete set.

Formally, both the translation vectors t_j and their components should have an additional index (i) indicating that they are determined at iteration i of the optimization algorithm (2), however, this was omitted for the sake of simplicity.

Figure 4 shows the plots of the functions F and R corresponding to the fine/coarse model response plots of Fig. 1. The interpolation of $\{f(\mathbf{x}, \omega_1), \dots, f(\mathbf{x}, \omega_m)\}$, F and R is implemented using cubic splines.

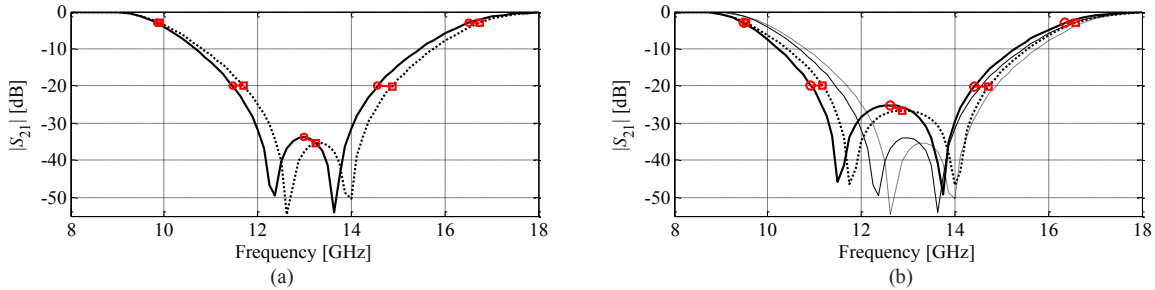


Fig. 1. The SPRP concept [15]: (a) Example coarse model response at the design $\mathbf{x}^{(0)}$, $c(\mathbf{x}^{(0)})$ (—), the coarse model response at $\mathbf{x}$, $c(\mathbf{x})$ (....), characteristic points of $c(\mathbf{x}^{(0)})$ (o) and $c(\mathbf{x})$ (□), and the translation vectors (short lines); (b) Fine model response at $\mathbf{x}^{(0)}$, $f(\mathbf{x}^{(0)})$ (—) and the predicted fine model response at $\mathbf{x}$ (....) obtained using SPRP based on characteristic points of Fig. 1(a); characteristic points of $f(\mathbf{x}^{(0)})$ (o) and the translation vectors (short lines) were used to find the characteristic points (□) of the predicted fine model response; coarse model responses $c(\mathbf{x}^{(0)})$ and $c(\mathbf{x})$ are plotted using thin solid and dotted lines, respectively [9].

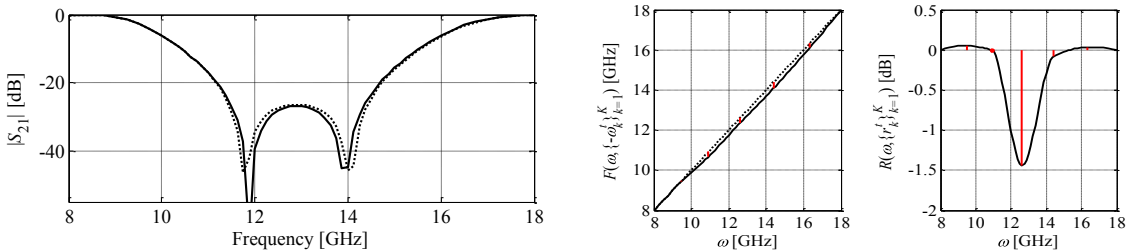


Fig. 2. (a) Fine model response at $\mathbf{x}$, $f(\mathbf{x})$ (solid line), and the fine model response at $\mathbf{x}$ obtained using the shape-preserving prediction (dotted line). Good agreement between both curves is observed, particularly in the areas corresponding to the characteristic points of the response; (b) Interpolating function F (—) corresponding to the fine/coarse model plots in Fig. 1; the identity function is denoted using the dotted line, the frequency components of the translation vectors are denoted as short solid lines; (c) Interpolating function R (—); the magnitude components of the translation vectors are denoted using short solid lines.

As follows from its formulation, SPRP is developed assuming that the frequency components of the translation vectors are zero at the edges of the frequency spectrum (i.e., at ω_1 and ω_m). This limitation can be easily overcome either by extending the frequency range of the coarse model and applying extrapolation (cf. [15]). Also, it is assumed that the overall shape of both the fine and coarse model response is similar. This means, in particular, that the characteristic points of responses of both the coarse model c and the fine model f are in one-to-one correspondence. If this assumption is not satisfied, the surrogate model (3), (4) cannot be evaluated because the translation vectors t_i are not well defined. Generalizations of SPRP that allow alleviating this difficulty in some cases can be found in [15].

3. SPRP for Microwave Design Optimization

In this section, we demonstrate the use of SPRP for the design optimization of microwave components. Consider the dual-band bandpass filter [21] (Fig. 3(a)). The design parameters are $\mathbf{x} = [L_1 \ L_2 \ S_1 \ S_2 \ S_3 \ d \ g \ W]^T$ mm. The fine model is simulated in Sonnet *em* [22]. The design specifications are $|S_{21}| \geq -3$ dB for $0.85 \text{ GHz} \leq \omega \leq 0.95 \text{ GHz}$ and $1.75 \text{ GHz} \leq \omega \leq 1.85 \text{ GHz}$, and $|S_{21}| \leq -20$ dB for $0.5 \text{ GHz} \leq \omega \leq 0.7 \text{ GHz}$, $1.1 \text{ GHz} \leq \omega \leq 1.6 \text{ GHz}$ and $2.0 \text{ GHz} \leq \omega \leq 2.2 \text{ GHz}$. The coarse model is implemented in Agilent ADS [23] (Fig. 3(b)). The initial design is $\mathbf{x}^{(0)} = [16.14 \ 17.28 \ 1.16 \ 0.38 \ 1.18 \ 0.98 \ 0.98 \ 0.20]^T$ mm (the optimal solution of c). The following characteristic points are selected to set up functions F and R : four points for which $|S_{21}| = -20$ dB, four points with $|S_{21}| = -5$ dB, as well as 6 additional points located between -5 dB points. For the purpose of optimization, the coarse model was enhanced by tuning the dielectric constants and the substrate heights of the microstrip models corresponding to the design variables L_1, L_2, d and g (original values of ϵ_r and H were 10.2 and 0.635 mm, respectively) [15]. The filter was optimized using two versions of SPRP, a regular one and SPRP enhanced by input SM (cf. Table 1). Figure 4 shows the initial fine model response as well as the fine model response at the design obtained using the shape-preserving response prediction method.

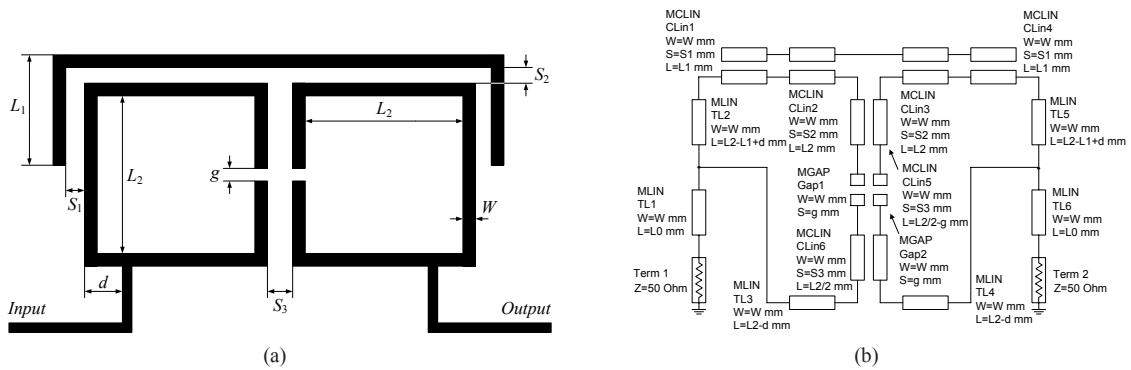


Fig. 3. Dual-band bandpass filter: (a) geometry [21], (b) coarse model (Agilent ADS).

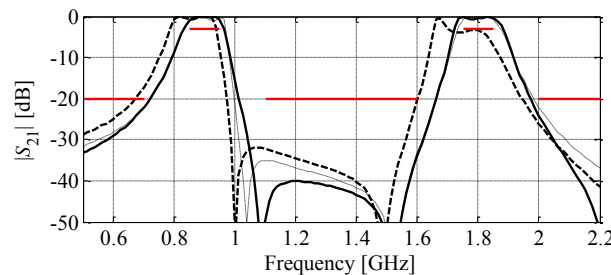


Fig. 4. Dual-band bandpass filter: fine model (dashed line) and coarse model (thin dashed line) response at $\mathbf{x}^{(0)}$, and the optimized fine model response (solid line) at the design obtained using shape-preserving response prediction.

Table 1. Optimization Results for Dual-Band Bandpass Filter

Algorithm	Final Specification Error [dB]	Number of Fine Model Evaluations ¹
Shape-preserving response prediction	-2.0 ³	3
Shape-preserving response prediction+ ISM ²	-1.9 ⁴	2

¹ Excludes the fine model evaluation at the starting point.

² The surrogate model is of the form $s^{(0)}(\mathbf{x}) = c(\mathbf{x} + \mathbf{d}^{(0)})$; $\mathbf{d}^{(0)}$ is found using parameter extraction [9].

³ Design specifications satisfied after the first iteration (spec. error -1.2 dB).

⁴ Design specifications satisfied after the first iteration (spec. error -1.0 dB).

4. SPRP for Antenna Design

In this section, we illustrate the use of SPRP for the design of antenna structures. As an example, consider an antenna shown in Fig. 5 [24] where $\mathbf{x} = [l_1 \ l_2 \ l_3 \ l_4 \ w_2 \ w_3 \ d_1 \ s]^T$ are the design variables. Multilayer substrate is $l_s \times l_s$ ($l_s = 30$ mm). The antenna stack (bottom-to-top) comprises: metal ground, 0.813 mm thick RO4003, microstrip trace ($w_1 = 1.1$ mm), 1.905 mm thick RO3006 and a trace-to-patch via ($r_0 = 0.25$ mm), driven patch, 3.048 mm thick RO4003, and four patches at the top. The antenna stack is fixed with four M1.6 bolts at the corners ($u = 3$ mm). Metallization is with thick 50 μm copper. Feeding is through an edge mount 50 ohm SMA connector with the 10 mm $\times$ 10 mm $\times$ 2 mm flange.

The design objective is $|S_{11}| \leq -10$ dB for 3.1 GHz to 4.8 GHz. Realized gain not less than 5 dB for the zero zenith angle is an optimization constrain over the frequency band. The initial design is $\mathbf{x}^{init} = [-4 \ 15 \ 15 \ 2 \ 15 \ 15 \ 20 \ 2]^T$ mm.

Both the high-fidelity model f (2,334,312 mesh cells at the initial design, 160 minutes of the evaluation time) and the low-fidelity model c (122,713 mesh cells, 3 min of the evaluation time) are simulated using the CST MWS transient solver [25]. Here, the first step is to find the rough optimum of c , $\mathbf{x}^{(0)} = [-4.91 \ 15.15 \ 15.07 \ 2.56 \ 14.21 \ 14.23 \ 21.07 \ 2.67]^T$ mm. The computational cost of this step is 82 evaluations of c (which corresponds to about 1.5 evaluations of the high-fidelity model). Figure 6(a) shows the responses of f at $\mathbf{x}^{init}$ and $\mathbf{x}^{(0)}$, as well as the response of c at $\mathbf{x}^{(0)}$. The final design $\mathbf{x}^{(4)} = [-5.21 \ 15.38 \ 15.57 \ 2.58 \ 14.41 \ 13.73 \ 21.07 \ 2.067]^T$ mm ($|S_{11}| \leq -11$ dB for 3.1 GHz to 4.8 GHz, Fig. 6(b)) is obtained after four iterations of the SPRP-based optimization. The gain of the final design is shown in Fig. 6(c) which illustrates that the maximum of radiation points along the zero zenith angle closely over the bandwidth of interest. The total design cost corresponds to about 10 evaluations of the high-fidelity model (Table 2).

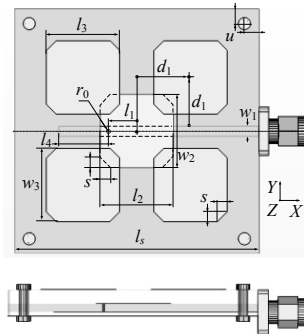


Fig. 5. Wideband microstrip antenna [24]: top and side views. The dash-dot line in the top view shows the magnetic symmetry wall (XOY).

Table 2. Wideband Microstrip Antenna: Optimization Cost

Algorithm Component	Number of Model Evaluations	Evaluation Time	
		Absolute [hours]	Relative to R_f
Evaluation of R_{cd}^*	$289 \times R_{cd}$	14.4	5.4
Evaluation of $R_f^{\#}$	$5 \times R_f$	13.3	5.0
Total optimization time	N/A	27.7	10.4

* Includes initial optimization of R_{cd} and optimization of SPRP surrogate.

Excludes evaluation of R_f at the initial design.

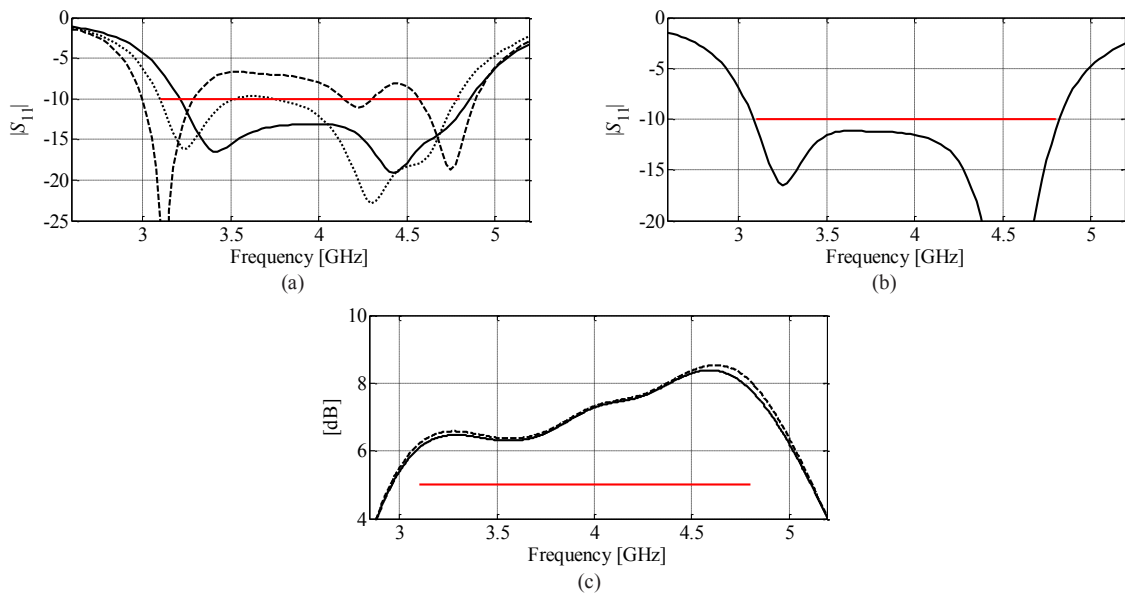


Fig. 6. Wideband microstrip antenna: (a) high-fidelity model response (dashed line) at the initial design $\mathbf{x}^{init}$, and high- (solid line) and low-fidelity (dotted line) model responses at the approximate low-fidelity model optimum $\mathbf{x}^{(0)}$; (b) high-fidelity model $|S_{11}|$ at the final design; (c) realized gain at the final design for the zero zenith angle (solid line, XOZ co-pol.) and realized peak gain (dash line). Design constrain is shown with the horizontal line at the 5 dB level.

5. SPRP for Aerodynamic Shape Optimization

The SPRP technique is illustrated here on the aerodynamic design optimization of airfoil sections at transonic flow conditions [17]. Consider the NACA four-digit airfoil with the shape defined by three parameters: m (the maximum ordinate of the mean camberline as a fraction of chord), p (the chordwise position of the maximum ordinate) and t/c (the thickness-to-chord ratio) [26]. The design variable vector is $\mathbf{x} = [m \ p \ t/c]^T$. Figure 7 shows examples of three NACA four-digit airfoil shapes.

The computational fluid dynamic (CFD) models are implemented using the ICEM CFD [27] grid generator and the FLUENT [28] flow solver. The high-fidelity model f is a two-dimensional steady-state Euler analysis with roughly 400,000 mesh cells and an overall simulation time around 67 minutes. The low-fidelity model c is the same as the high-fidelity one, but with a coarser mesh (roughly 30,000 cells) and relaxed convergence criteria (100 flow solver iterations). The ratio of simulation times of the high- and low-fidelity model is roughly 80.

In aerodynamic shape optimization, the SPRP technique is applied to the pressure distribution ($C_p(\mathbf{x})$) on the airfoil surface [17]. Figure 8 shows the pressure distributions of two different designs obtained by the low-fidelity model. Shown are the characteristic points (red circles) and the translation vectors (blue lines) at important areas of the distributions. The application of the translation vectors to the high-fidelity model distributions is shown in Fig. 9.

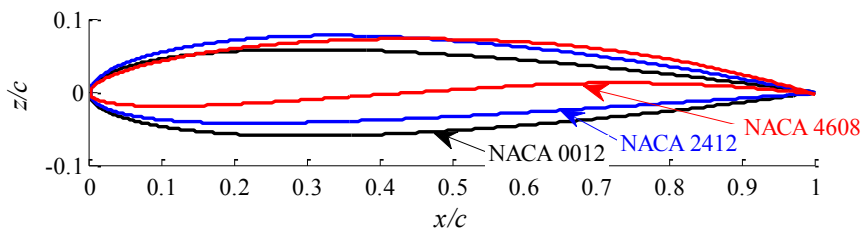


Fig. 7. Shown are three different NACA four-digit airfoil sections: NACA 0012 ($m = 0$, $p = 0$, $t/c = 0.12$), NACA 2412 ($m = 0.02$, $p = 0.4$, $t/c = 0.12$), and NACA 4608 ($m = 0.04$, $p = 0.6$, $t/c = 0.08$).

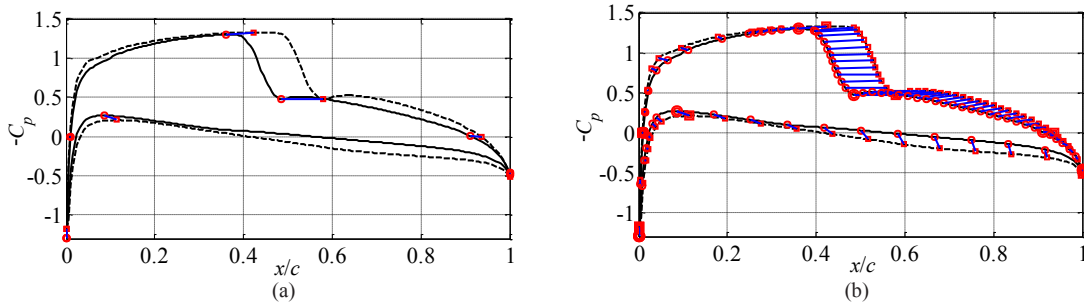


Fig. 8. An illustration of the SPRP technique applied to the pressure distributions obtained by the low-fidelity CFD models of two designs, (a) initial characteristic points and translation vectors, (b) additional points.

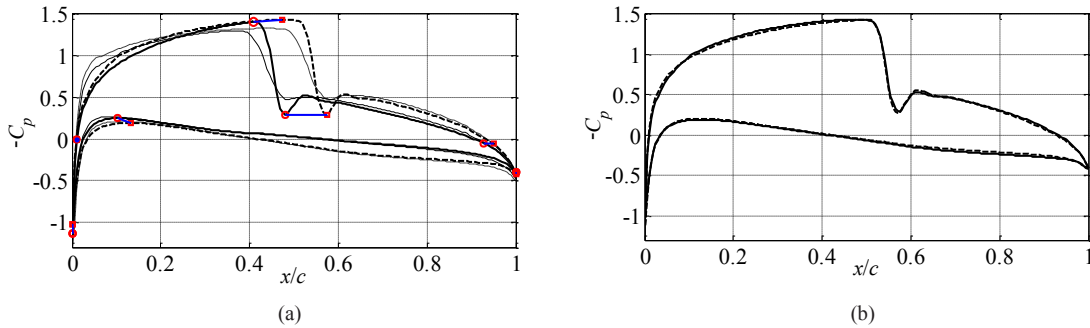


Fig. 9. Application of SPRP to the high-fidelity CFD model responses (thick lines) with (a) initial characteristic points and translation vectors (coarse model distributions are shown with thin lines), and (b) comparison of the actual and the predicted (dash) high-fidelity response.

The design objective is to maximize the section lift coefficient ($C_l(\mathbf{x})$) subject to constraints on the section drag coefficient ($C_{dw}(\mathbf{x})$) and the non-dimensional cross-sectional area ($A(\mathbf{x})$). The problem is formulated as minimization of the high-fidelity model $f(\mathbf{x}) = -C_l(\mathbf{x})$ subject to $g_1(\mathbf{x}) = C_{dw}(\mathbf{x}) - C_{dw,max} \leq 0$, and $g_2(\mathbf{x}) = A_{min} - A(\mathbf{x}) \leq 0$, where $C_{dw,max} = 0.0041$ is the maximum drag and $A_{min} = 0.065$ the minimum cross-section. The free-stream Mach number is set $M_\infty = 0.75$ and the angle of attack $\alpha = 1^\circ$. The design variable bounds are $0 \leq m \leq 0.1$, $0.2 \leq p \leq 0.8$, and $0.05 \leq t \leq 0.20$. The initial design is $\mathbf{x}^{init} = [0.03 \ 0.2 \ 0.1]^T$.

Due to unavoidable misalignment between the pressure distributions of the high-fidelity model and its SPRP surrogate, it is not convenient to handle the drag constraint directly, because the design that is feasible for the surrogate model, may not be feasible for the high-fidelity model. This problem is alleviated by implementing the drag constraint through a penalty function. More specifically, the objective function is defined as

$$H(C_p(\mathbf{x})) = -C_{l,s}(C_p(\mathbf{x})) + \beta [\Delta C_{dw,s}(C_p(\mathbf{x}))]^2, \quad (5)$$

where $\Delta C_{dw,s} = 0$ if $C_{dw,s} \leq C_{dw,s,max}$ and $\Delta C_{dw,s} = C_{dw,s} - C_{dw,s,max}$ otherwise. The cross-sectional area constraint is handled directly. We use $\beta = 1000$ in the numerical study. Here, the pressure distribution for the surrogate model is $C_p = C_{p,s}$, and for the high-fidelity model $C_p = C_{p,f}$. Also, $C_{l,s}$ and $C_{dw,s}$ denote the lift and drag coefficients for the surrogate.

The optimization problem is solved by the direct optimization of the high-fidelity model using the pattern-search algorithm, as well as by the SPRP algorithm. The results are presented in Table 3. It can be seen that both approaches are able to meet the design goals and produce similar optimized airfoil shapes. The direct approach requires 120 high-fidelity model evaluations (N_f). The SPRP algorithm requires 330 low-fidelity model evaluations (N_c) and 11 high-fidelity ones, yielding a total cost of less than 18 equivalent high-fidelity model evaluations.

Table 3. Numerical results for the airfoil design optimization.

Variable	Initial	Direct	SPRP
m	0.0300	0.0080	0.0090
p	0.2000	0.6859	0.6732
t/c	0.1000	0.1044	0.1010
C_l	0.8035	0.4641	0.4872
C_{dw}	0.0410	0.0041	0.0040
A	0.0675	0.0703	0.0680
N_c	N/A	0	330
N_f	N/A	120	11
Total cost	N/A	120	< 18

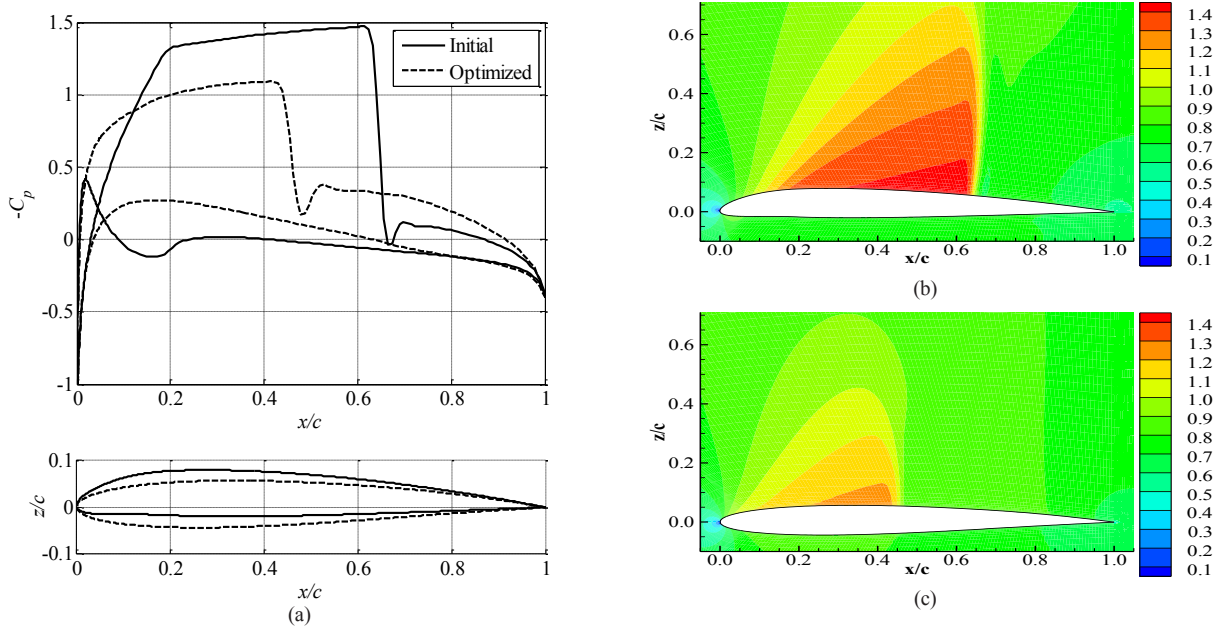


Fig. 10. Results of the optimization showing both the initial and optimized airfoils (a) pressure distributions and shapes, and Mach contours of the (b) initial, and (c) optimized airfoils.

To meet the design goals, the optimizer does three fundamental shape changes: (1) the maximum ordinate of the mean camber line (m) is reduced, (2) the location of the maximum ordinate of the mean camber line (p) is moved aft, thus increasing the trailing-edge camber, and (3) the thickness-to-chord ratio (t/c) is reduced. Shape changes (1) and (3) reduce the shock strength and, thus, reduce the drag coefficient. The associated change in the pressure distribution reduces the lift coefficient. However, shape change (2) improves (or recovers a part of) the lift by opening up the pressure distribution behind the shock. These effects can be seen in the pressure distribution plot in Fig. 10(a), and the Mach contour plots in Figs. 10(b) and (c).

6. Conclusion

A review of shape-preserving response prediction (SPRP) and its applications to solving simulation-driven design problems in various engineering disciplines has been presented. SPRP exploits the knowledge embedded in the low-fidelity model of the structure under consideration in order to predict the response of the expensive high-fidelity model. As a result, SPRP is capable of yielding a satisfactory design at a low computational cost as demonstrated using several examples involving design problems in electrical and mechanical engineering.

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