

International Conference on Computational Science

Multi-Fidelity Design Optimization of Transonic Airfoils Using Shape-Preserving Response Prediction

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Abstract

A computationally efficient methodology for transonic airfoil design optimization is presented. Our approach exploits a corrected physics-based low-fidelity surrogate that replaces, in the optimization process, an accurate but computationally expensive high-fidelity airfoil model. Correction of the low-fidelity model is achieved by aligning its corresponding airfoil surface pressure distributions with that of the high-fidelity model using a shape-preserving response prediction technique. The presented method is applied to airfoil lift maximization in two-dimensional inviscid transonic flow, subject to constraints on shock induced pressure drag and airfoil cross-sectional area. More than a 90% reduction in high-fidelity function calls is achieved when compared to direct high-fidelity model optimization.

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1. Introduction

Aerodynamic and hydrodynamic design and optimization is of primary importance in several disciplines. In the design of turbines, such as gas, steam, or wind turbines, the blades are designed to maximize energy output for a given working fluid and operating conditions [1]. The shape of ship hull forms is optimized for minimum drag [2]. In aircraft design, for both conventional transport aircraft and unmanned air vehicles, the aerodynamic wing shape is designed to provide maximum efficiency under a variety of takeoff, cruise, maneuver, loiter, and landing conditions [3]. Constraints on aerodynamic noise are also becoming increasingly important [4]. The fundamental design problem, common to all these disciplines, is to design a wing shape that provides the desired lift for given operating conditions, while at the same time fulfilling the design constraints. However, design optimization normally requires a large number of analyses of objectives and constraints [5]. Therefore, a careful selection of computational methods, both for the fluid flow analysis and the optimization process, is essential for a fast and efficient design process.

Hicks et al. [6] began exploring the use of numerical optimization techniques for the design of aircraft components in the mid 1970s. These early studies focused primarily on airfoil and wing design at both subsonic and transonic conditions using low-fidelity flow analysis models and gradient-based optimization methods. Jameson [7] introduced control theory to optimal aerodynamic design. This method uses continuous adjoint methods to derive

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the gradient of a cost function with respect to the shape, and then approach the optimum using a gradient-based optimization method. Jameson and Reuther examined transonic airfoil design problems using both the full potential equation [8] and the compressible Euler equations [9]. Later, Jameson et al. extended the method to incorporate viscous effects using the Navier-Stokes equations, examining both two-dimensional high-lift airfoil design [10] and three-dimensional wing design [11]. Normally there is some uncertainty of the exact operating conditions, such as the Mach number and angle of attack, therefore, much research has been on optimizing aerodynamic shapes with respect to several operating conditions [12].

The aforementioned methods directly employ the computational code in the optimization loop. In the past decade or so, the drive had been towards including higher-fidelity analyses in the design process. As a result, design optimization, which requires large numbers of model evaluations, becomes prohibitively expensive. Surrogate-based optimization (SBO) methods use computationally cheap surrogate functions in lieu of the computationally more expensive high-fidelity models [13]. The overall objective of using SBO methods is to reduce the number of evaluations of the high-fidelity models, and thereby making the optimization process more efficient. The surrogates can be created by approximating the high-fidelity model data using, e.g., polynomial regression or kriging [13]. Another way of developing the surrogates is by using low-fidelity models, which are a less accurate but computationally cheap representations of the high-fidelity models [14] (multi- or variable-fidelity design).

Robinson et al. [15] presented a provably convergent trust-region model-management (TRMM) methodology for variable-parameterization design models. This is an SBO method which uses a lower-fidelity model as a surrogate and the low-fidelity design space has a lower dimension than the high-fidelity design space. The mathematical relationship between the design vectors is described by a mapping method, called space-mapping [16–19]. Since space mapping does not provide provable convergence within a TRMM framework, but any surrogate that is first-order accurate does, they correct the space-mapping to be at least first-order, called corrected space-mapping.

In summary, high-fidelity aerodynamic simulation is reliable but computationally far too expensive to be used in a direct, simulation-based design optimization, especially when using traditional, gradient-based techniques. There is a need to develop methodologies that would allow rapid design optimization with limited number of CPU-intensive objective function evaluations. SBO techniques currently used in aerospace engineering are either exploiting functional surrogate models (that require substantial computational effort to be set up), or adopt approaches such as classical space mapping (which does not guarantee sufficient alignment between the low- and high-fidelity models and requires enforced first-order consistency that required sensitivity data from the high-fidelity model). In either case, overall computational cost of the optimization process is still high. Development of a truly efficient SBO approach that would take full advantage of the low-fidelity model speed and high-fidelity model accuracy is still an open problem.

Here, a computationally efficient design methodology is introduced that exploits surrogates constructed using low-fidelity flow analysis models and shape-preserving response prediction technique [20]. We demonstrate that our approach allows a rapid design improvement of airfoils at a very low computational cost corresponding to a few evaluations of the high-fidelity model. Several examples of airfoil design at transonic flow conditions are provided.

2. Airfoil aerodynamics

A wing surface is defined by airfoil profiles located at several span stations, Figure 1(a). The airfoil profile is a streamlined surface of chord length c , as shown in Figure 1(b). The airfoil has a thickness t , which is a function of k (on the x -axis), and the ratio t/c refers to the maximum thickness of the airfoil divided by its chord. The curvature of the airfoil is called camber, and the mean camberline is the curve equidistant from the upper and lower surfaces. A few examples of airfoils parameterized according to so-called NACA convention [21] are shown in Figure 2.

Transonic flow is characterized by regions of locally subsonic (Mach < 0.8) and supersonic (Mach > 1.2) flow that occurs over a body which is moving at Mach numbers near unity [22]. Assuming an inviscid, adiabatic flow with no body forces, the Euler equations are the most accurate description of the fluid flow. The Euler equations are a set of coupled, non-linear partial differential equations that represent the conservation of mass, momentum and energy, i.e.,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad \rho \frac{D\mathbf{V}}{Dt} = -\nabla p, \quad \rho \frac{Dh_0}{Dt} = \frac{\partial p}{\partial t} \quad (1)$$

where ρ is density, V is velocity vector, p is pressure, and h_0 is stagnation enthalpy. These equations hold, in the absence of separation and other strong viscous effects, for any shape of the body, thick or thin, and at any angle of attack.

Shock waves appear in transonic flow where the flow goes from being supersonic to subsonic. Across the shock there is almost a discontinuous increase in pressure, temperature, density, and entropy, but a decrease in Mach number (from supersonic to subsonic). The shock is termed *weak* if the change in pressure is small, and *strong* if the change in pressure is large. The entropy change is third order in terms of shock strength. If the shocks are weak, the entropy change across shocks is small, and the flow can be assumed to be isentropic. This in turn allows for the assumption of irrotational flow, i.e., $\nabla \times V = 0$. Then a velocity potential, $\Phi = \Phi(x, y, z)$, can be defined such that $V = \nabla \Phi$, and the governing Euler equations cascade to a single non-linear partial differential equation, called the full potential equation (FPE) [22],

$$\left(1 - \frac{\Phi_x^2}{a^2}\right)\Phi_{xx} + \left(1 - \frac{\Phi_y^2}{a^2}\right)\Phi_{yy} + \left(1 - \frac{\Phi_z^2}{a^2}\right)\Phi_{zz} - \frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy} - \frac{2\Phi_x\Phi_z}{a^2}\Phi_{xz} - \frac{2\Phi_y\Phi_z}{a^2}\Phi_{yz} = 0 \quad (2)$$

where a is the speed of sound. This equation is non-linear partial differential equation that holds for irrotational, isentropic flow past a body, thick or thin, and at any angle of attack. It is, however, an approximation to the Euler equations and does not hold when the shocks in the flow cannot be considered weak.

In the case of a slender body at small angle of attack, we can make the assumption of small perturbations and define the velocity potential to be $\Phi = V_\infty x + \phi$ where ϕ is the perturbation velocity potential and V_∞ is free-stream velocity. Then equation (2) becomes the transonic small-disturbance equation (TSDE) [22],

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = M_\infty^2 \left[(\gamma + 1) \frac{\phi_x}{V_\infty} \right] \phi_{xx}, \quad (3)$$

where $M_\infty = V_\infty / a_\infty$ is the free-stream Mach number and γ is the specific heat ratio.

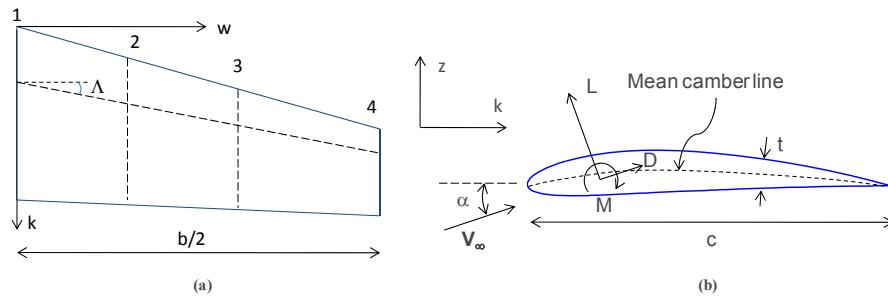


Fig. 1. (a) A schematic showing a wing planform of span b and quarter chord sweep angle A . Other parameters (not shown) of the wing are, e.g., taper ratio (ratio of tip chord to root chord) and twist distribution. At each spanstation (numbered 1 through 4) the wing is defined by an airfoil section with a straight line wrap between the stations. (b) Shown is a typical airfoil (NACA 2412) of chord length c , with thickness $t = t/c$, and a mean camberline, in a free-stream with speed V_∞ at an angle of attack α . The flow generates a lift force L , a drag force D , and pitching moment M acting at the center of pressure.

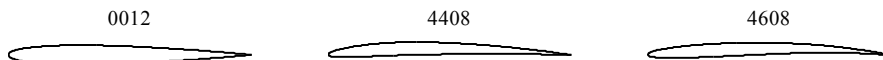


Fig. 2. Typical NACA four digit airfoils. A four-digit NACA airfoil is the simplest version and is defined by only three parameters m , p and t/c , where m is the maximum ordinate of the mean camber line as a fraction of chord, and p is the chordwise position of maximum ordinate. Airfoils are denoted by NACA $mpxx$, where xx is the thickness to chord ratio, t/c . The left most one is symmetrical with no camber ($m = 0$), the location is irrelevant ($p = 0$), and 12% thickness ($t/c = 0.12$). The middle one has 4% maximum camber ($m = 0.04$), located at 40% of the chord ($p = 0.4$), and 8% thickness ($t/c = 0.12$). The right most one has the same camber and thickness, but the maximum camber is located at 60% of the chord ($p = 0.6$).

These three different set of equations, i.e., the Euler equations, FPE, and TSDE, represent a hierarchy of models for the analysis of inviscid, transonic flow past airfoils. The Euler equations are exact, while FPE is an approximation (weak shocks) to those equations, and TSDE is a further approximation (thin airfoils at small angle of attack). The TSDE can be solved with a finite differencing scheme on a rectangular grid, without considering specifically the airfoil surface. Solving the FPE requires a more complicated grid that accounts for the body surface boundary condition. The Euler equations are even more complex since they require a time-marching solution method. The Euler equations can then be considered as a high-fidelity model (high accuracy and computationally heavy), and FPE and TSDE as low-fidelity models (reduced accuracy and computationally cheap).

The flow past NACA 0012 at $M_\infty = 0.75$ is analyzed in Figure 3. Pressure coefficient (C_p) and Mach number (M) contours at $\alpha = 2^\circ$, calculated using the Euler equations with FLUENT [23], are shown in Figures 3(a) and (b), respectively. The pressure coefficient is defined as

$$C_p = \frac{p - p_\infty}{1/2 \rho_\infty V_\infty^2} \quad (4)$$

where p is pressure in the flow-field and p_∞ is the free-stream pressure. The pressure coefficient on the surface of the airfoil as a function of chordwise position is shown in Figure 3(c) for the Euler equations with FLUENT (high-fidelity model) and the TSDE with TSFOIL [24] (low-fidelity model). The sharp drop in pressure around the mid chord is due to a shock on the upper surface, which is relatively strong in this case. Notice that TSDE predicts the shock to be stronger and located more aft. Nondimensional coefficients of lift (C_l) and wave drag (C_{dw}) are shown as a function of angle of attack in Figure 3(d). The section lift coefficient and wave drag coefficient are calculated from

$$C_l = -C_x \sin \alpha + C_z \cos \alpha \quad C_{dw} = C_x \cos \alpha + C_z \sin \alpha \quad (5)$$

Here, the force coefficients C_x and C_z are calculated by integrating the pressure counterclockwise around the surface of the airfoil as

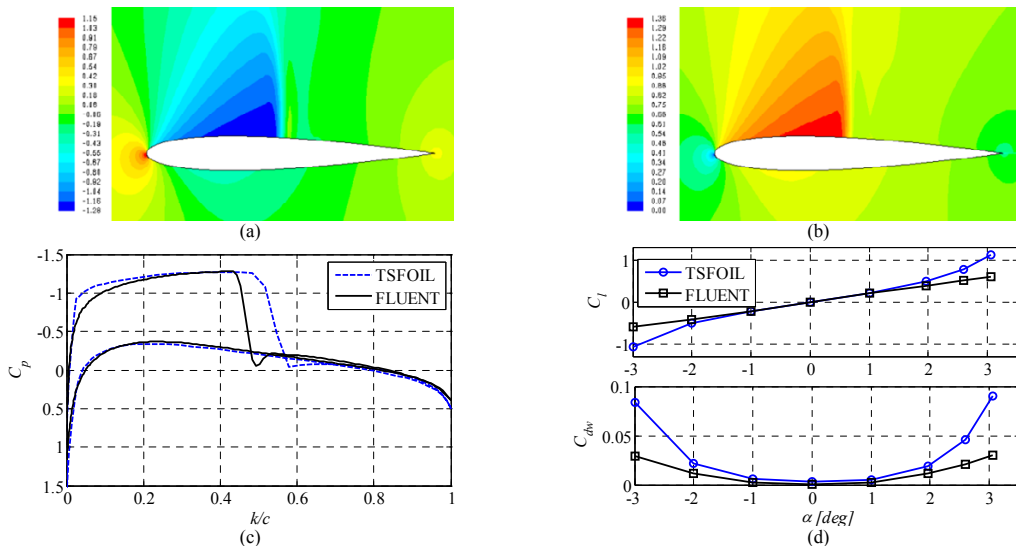


Fig. 3. (a) Pressure coefficient (C_p) contours of the flow past NACA 0012 at $M_\infty = 0.75$ and $\alpha = 2^\circ$ calculated with the Euler equations using FLUENT [23] (high-fidelity model). (b) Mach number (M) contours of the same airfoil, same conditions and same equations as in (a). (c) Pressure coefficient on the surface of the airfoil calculated with the Euler equations using FLUENT (high-fidelity model) and the transonic small-disturbance equation using TSFOIL [24] (low-fidelity model). The scale on the y-axis is reversed since the pressure coefficient is negative on the upper surface. (d) Nondimensional coefficients of lift (C_l) and wave drag (C_{dw}) as a function of angle of attack at $M_\infty = 0.75$.

$$C_x = \oint C_p \sin \theta \, ds \quad C_z = -\oint C_p \cos \theta \, ds \quad (6)$$

where ds is the panel length on the surface of the airfoil and θ is the angle the panel makes with the x-axis. We see that the lift changes linearly with angle of attack and the wave drag changes quadratically with angle of attack. Note that the low-fidelity model diverges from the high-fidelity model as the angle of attack increases or decreases.

In general, the objective of aerodynamic shape optimization is to, for a given operating condition, design an airfoil shape that maximizes lift (or minimizes drag, or maximizes lift-to-drag ratio), subject to several constraints, such as a limit on maximum drag (or a limit on minimum lift), maximum pitching moment, minimum pressure, and minimum cross-sectional area [5].

3. Optimization methodology

The main focus of this work is to perform aerodynamic shape optimization in a computationally efficient way. The high-fidelity model f is computationally expensive, so it is desirable to reduce the number of evaluations of f as much as possible. The method presented here follows the general principles of surrogate-based optimization (SBO) [13], where the optimization burden is shifted to the low-cost surrogate model s , whereas the high-fidelity model is referenced occasionally for verification purposes and to obtain data necessary to update the surrogate. From now on x will denote a vector of design variables (parameterization of the airfoil shape).

3.1. General Considerations

A surrogate model can be built using approximation techniques such as low-order polynomials, radial-basis functions or kriging [25]. These popular approaches require, however, considerable amount of training data to build a good surrogate model. Here, we build a surrogate model using a physics-based low-fidelity model c .

The low-fidelity model has to be corrected in order to become a reliable representation of the high-fidelity one. The problem is that the figures of interest in an aerodynamic analysis (lift, drag, pitching-moment, cross-sectional area, etc.) are scalars, which results in non-uniqueness of any alignment procedure that could be applied in order to match the low-fidelity model with the high-fidelity one at any given design. Here, the model alignment is performed using intermediate simulation results, more specifically, the pressure distribution, whose dimensionality can be made as large as necessary by selecting sufficient number of control points along the airfoil chord. As the objectives and constraints are uniquely determined by the pressure distribution (for an inviscid flow analysis), alignment of corresponding distributions for the low- and high-fidelity models will result in (unique) alignment of the figures of interest.

One of the popular SBO methods exploiting physics-based surrogates is space mapping (SM) [16], [18]. SM surrogate is a composition of the low-fidelity model and simple, usually linear transformations that re-shape the model domain (input-like SM [16]), correct the model response (output-like SM [18]) or change the overall model properties (implicit-like SM [18]). SM is able to yield a satisfactory design after a few evaluations of the high-fidelity model but its performance depends on the proper selection of the SM transformations and their parameters. Also, simple SM transformations are not able to ensure good alignment of the high- and low-fidelity models in our case.

Another generic SBO approach is approximation and model management (AMMO) [14]. AMMO is based on ensuring zero- and first-order consistency conditions between the high-fidelity model and the surrogate (i.e., agreement between the function values and first-order derivatives at the current iteration point) by using a suitable correction term [14]. Unfortunately, AMMO requires high-fidelity model sensitivity. Therefore, it can be efficient if the derivatives can be computed cheaply, e.g., using adjoint methods.

3.2. Surrogate Model Based on Shape-Preserving Response Prediction

Here, we adopt a shape-preserving response correction (SPRP) methodology introduced in [20] in the context of microwave engineering. SPRP is easy to implement, unlike space mapping it does not need any auxiliary transformations or extractable parameters [20]. Also, it does not require high-fidelity model derivative information.

As mentioned before, the surrogate model is constructed based on the pressure distribution C_p rather than directly on the figures of interest such as lift (C_l) or wave drag (C_{dw}). The reason is that these figures are not uniquely determined by the design variable vector x . The pressure distribution, on the other hand, is uniquely determined by

x , and uniquely determines lift and drag at the same time. The pressure distribution for the high- and low-fidelity models will be denoted as $C_{p,f}$ and $C_{p,c}$, respectively.

The surrogate model is constructed assuming that the change of $C_{p,f}$ due to the adjustment of the design variables x can be predicted using the actual changes of $C_{p,c}$. The change of $C_{p,c}$ is described by the translation vectors corresponding to certain (finite) number of its characteristic points. These translation vectors are subsequently used to predict the change of $C_{p,f}$, whereas the actual $C_{p,f}$ at the current design, $C_{p,f}(x^{(i)})$, is treated as a reference.

Figure 4(a) shows the pressure distribution $C_{p,c}$ of the low-fidelity model at $x^{(i)} = [0.02 \ 0.4 \ 0.12]^T$ (NACA 2412 airfoil) for $M_\infty = 0.7$ and $\alpha = 1$ deg, as well as $C_{p,c}$ at $x = [0.025 \ 0.56 \ 0.122]^T$; $x^{(i)}$ will denote a current design (at the i th iteration of the optimization algorithm; the initial design will be denoted as $x^{(0)}$ accordingly). Circles denote characteristic points of $C_{p,c}(x^{(i)})$, here, representing, among others, k/c equal to 0 and 1 (leading and trailing airfoil edges, respectively), the maxima of $C_{p,c}$ for the lower and upper airfoil surfaces, as well as the local minimum of $C_{p,c}$ for the upper surface. The last two points are useful to locate the pressure shock. Squares denote corresponding characteristic points for $C_{p,c}(x)$, while small line segments represent the translation vectors that determine the “shift” of the characteristic points of $C_{p,c}$ when changing the design variables from $x^{(i)}$ to x .

In order to obtain a reliable prediction, the number of characteristic points has to be larger than illustrated in Figure 4(a). Additional points are inserted in between initial points either uniformly with respect to k/c (for those parts of the pressure distribution that are almost flat) or based on the relative pressure value with respect to corresponding initial points (for those parts of the pressure distribution that are “steep”). Figure 4(b) shows the full set of characteristic points (initial points are distinguished using larger markers).

The pressure distribution of the high-fidelity model at the given design, here, x , can be predicted using the translation vectors applied to the corresponding characteristic points of the pressure distribution of the high-fidelity model at $x^{(i)}$, $C_{p,f}(x^{(i)})$. This is illustrated in Figure 5(a) where only initial characteristic points and translation vectors are shown for clarity. Figure 5(b) shows the predicted pressure distribution of the high-fidelity model at x as well as the actual $C_{p,f}(x)$. The agreement between both curves is very good.

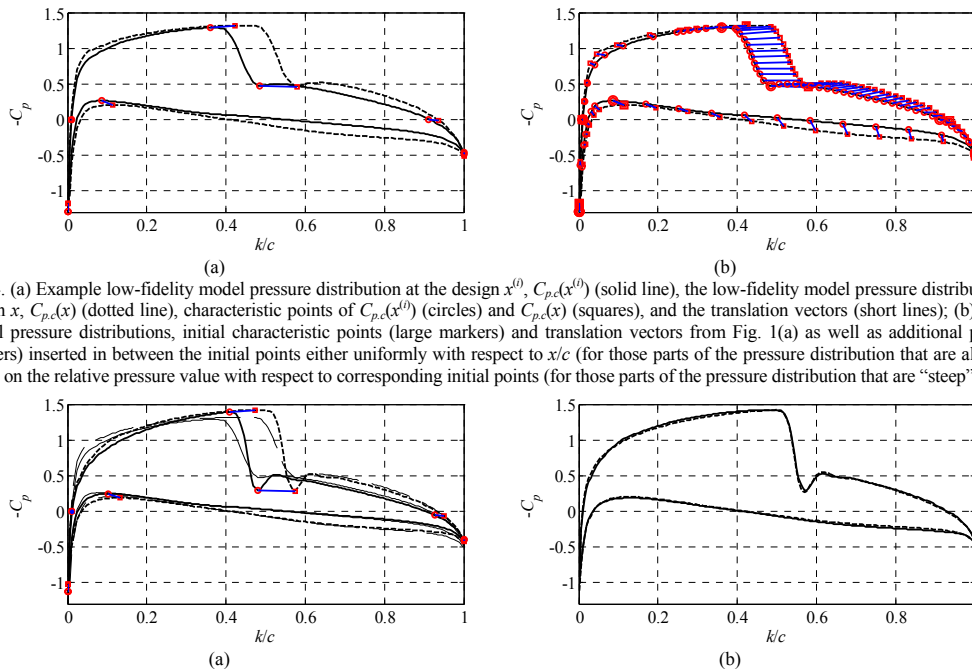


Fig. 4. (a) Example low-fidelity model pressure distribution at the design $x^{(i)}$, $C_{p,c}(x^{(i)})$ (solid line), the low-fidelity model pressure distribution at other design x , $C_{p,c}(x)$ (dotted line), characteristic points of $C_{p,c}(x^{(i)})$ (circles) and $C_{p,c}(x)$ (squares), and the translation vectors (short lines); (b) low-fidelity model pressure distributions, initial characteristic points (large markers) and translation vectors from Fig. 1(a) as well as additional points (small markers) inserted in between the initial points either uniformly with respect to x/c (for those parts of the pressure distribution that are almost flat) or based on the relative pressure value with respect to corresponding initial points (for those parts of the pressure distribution that are “steep”).

Fig. 5. (a) High-fidelity model pressure distribution at $x^{(i)}$, $C_{p,f}(x^{(i)})$ (solid line) and the predicted high-fidelity model C_p at x (dotted line) obtained using SPRP based on characteristic points of Fig. 4(b); characteristic points of $C_{p,f}(x^{(i)})$ (circles) and the translation vectors (short lines) were used to find the characteristic points (squares) of the predicted high-fidelity model pressure distribution (only initial points are shown for clarity); low-fidelity model distributions $C_{p,c}(x^{(i)})$ and $C_{p,c}(x)$ are plotted using thin solid and dotted line, respectively; (b) high-fidelity model pressure distribution at x , $C_{p,f}(x)$ (solid line), and the predicted high-fidelity model pressure distribution at x obtained using SPRP (dotted line).

SPRP can be rigorously formulated as follows. Let $C_{p,f}(x) = [c_{p,f}(x, y_1) \dots c_{p,f}(x, y_m)]^T$ and $C_{p,c}(x) = [c_{p,c}(x, y_1) \dots c_{p,c}(x, y_m)]^T$, where $y_j, j = 1, \dots, m$, are control points on the k/c axis (we assume that $y_{j+1} > y_j$ and $0 \leq y_j \leq 1$ for all j). To simplify the notation we assume that $C_{p,f}$ ($C_{p,c}$) is the pressure distribution for the upper surface only. Formulation for the lower surface is identical. Let $p_j^f = [y_j^f \ r_j^f]^T$, $p_j^{c0} = [y_j^{c0} \ r_j^{c0}]^T$, and $p_j^c = [y_j^c \ r_j^c]^T$, $j = 1, \dots, K$, denote the sets of characteristic points of $C_{p,f}(x^{(i)})$, $C_{p,c}(x^{(i)})$ and $C_{p,f}(x)$, respectively. Here, y and r denote the k/c and magnitude components of the respective point. The translation vectors of the low-fidelity model pressure distribution are defined as $T_j = [y_j^f \ r_j^f]^T, j = 1, \dots, K$, where $y_j^f = y_j^c - y_j^{c0}$ and $r_j^f = r_j^c - r_j^{c0}$.

The SPRP surrogate model is defined as follows

$$C_{p,s}^{(i)}(x) = [c_{p,s}^{(i)}(x, y_1) \dots c_{p,s}^{(i)}(x, y_m)]^T \quad (7)$$

where

$$c_{p,s}^{(i)}(x, y_j) = \bar{c}_{p,f}(x^{(i)}, F(y_j, \{-y_k^f\}_{k=1}^K)) + r(y_j, \{r_k^f\}_{k=1}^K) \quad (8)$$

for $j = 1, \dots, m$. $\bar{c}_{p,f}(x, y)$ is an interpolation of $\{c_{p,f}(x, y_1), \dots, c_{p,f}(x, y_m)\}$ onto the interval $[0, 1]$. The scaling function F interpolates the data pairs $\{y_1, y_1^f\}, \{y_1^f, y_1^f - y_1^f\}, \dots, \{y_K^f, y_K^f - y_K^f\}, \{y_m, y_m\}$, onto the interval $[0, 1]$. The function r does a similar interpolation for data pairs $\{y_1, r_1\}, \{y_1^f, r_1^f - r_1^f\}, \dots, \{y_K^f, r_K^f - r_K^f\}, \{y_m, r_m\}$; here $r_1 = c_{p,c}(x, y_1) - c_{p,c}(x^f, y_1)$ and $r_m = c_{p,c}(x, y_m) - c_{p,c}(x^f, y_m)$. Note that $C_{p,s}^{(i)}(x^{(i)}) = C_{p,f}^{(i)}(x^{(i)})$ as all translation vectors are zero at $x = x^{(i)}$.

Our prediction method assumes that the high- and low-fidelity model pressure distributions have corresponding sets of characteristic points. This is usually the case for the practical ranges of design variables because the overall shape of the distributions is similar for both models. In case of lack of correspondence, original definitions of characteristic points are replaced by their closest counterparts. The typical example would be non-existence of the local minimum of the pressure distribution for the upper surface for the high- and/or low-fidelity model at certain designs. In this case, the original point (local minimum) is replaced by the points characterized by the largest curvature.

3.3. Objective Function

Due to unavoidable misalignment between the pressure distributions of the high-fidelity model and its SPRP surrogate, it is not convenient to use a drag constraint directly, because the design that is feasible for the surrogate model, may not be feasible for the high-fidelity model. In particular, the design $x^{(i+1)}$ obtained as a result of optimizing the surrogate model $C_s^{(i)}$ will be feasible for $C_s^{(i)}$. However, if $x^{(i+1)}$ is not feasible for the high-fidelity model, it will not be feasible for $C_s^{(i+1)}$ because we have $C_{p,s}^{(i+1)}(x^{(i+1)}) = C_{p,f}(x^{(i+1)})$ by the definition of the surrogate model. In order to alleviate this problem, we shall use the penalty function approach to handle the drag constraint.

More specifically, the objective function is defined as

$$H(C_p(x)) = -C_{l,s}(C_p(x)) + \beta[\Delta C_{dw,s}(C_p(x))]^2 \quad (9)$$

where $\Delta C_{dw,s} = 0$ if $C_{dw,s} \leq C_{dw,s,max}$ and $\Delta C_{dw,s} = C_{dw,s} - C_{dw,s,max}$ otherwise. In our numerical experiments we use $\beta = 1000$. C_p is a pressure distribution ($C_p = C_{p,s}$ for the surrogate model and $C_p = C_{p,f}$ for the high-fidelity model); $C_{l,s}$ and $C_{dw,s}$ denote the lift and wave drag coefficients (both being functions of the pressure distribution).

3.4. Optimization Algorithm

Our optimization algorithm exploiting the SPRP-based surrogate model can be summarized as follows:

1. Set $i = 0$;
2. Evaluate $C_{p,f}(x^{(i)})$;
3. Obtain $x^{(i+1)} = \text{argmin} \{l \leq x \leq u, A(x) \geq A_{\min} : H(C_{p,s}^{(i)}(x))\}$;
4. Set $i = i + 1$;
5. If termination condition is not satisfied, go to 2.

In this paper, the algorithm is terminated if the current iteration does not bring further improvement of the high-fidelity model objective function or $\|x^{(i+1)} - x^{(i)}\| < 0.001$. It should be noted that optimization of the surrogate model is normally performed under relaxed tolerance requirements: finding the accurate optimum of the surrogate is not necessary due to its limited accuracy. In practice, step 3 of the above procedure requires about 30 evaluations of the surrogate model.

4. Verification examples

Three test cases are presented for design algorithm. The design formulation is the same for all cases, except that the operational parameters, initial design, and constraint values are different. The objective is to maximize the lift coefficient C_l , subject to a constraints on wave drag ($C_{dw} \leq C_{dw,limit}$) and airfoil cross-sectional area nondimensionalized with the chord squared ($A \geq 0.075$). Details of the formulation are given in Section 3.3. The airfoil shape is parameterized using a NACA four digit airfoil with m , p , and t/c as the design variables. The side constraints on the design variables are $0 \leq m \leq 0.03$, $0.3 \leq p \leq 0.6$, and $0.09 \leq t/c \leq 0.13$. Details of the test cases and optimization results are given in Table 1. Because the low-fidelity model (TSFOIL) evaluates quite fast (about 1 to 3 seconds depending on the design) and the high-fidelity model (FLUENT) evaluation takes a few minutes, the total cost of evaluating the low-fidelity model in the whole optimization run corresponds to roughly 1-2 evaluations of the high-fidelity model.

The initial design of the first test case is NACA 0012 with both constraints not active. The optimum design obtained by direct optimization has a 62.5% higher C_l than the initial design. The wave drag constraint is active, but the cross-sectional area constraint is not. The number of high-fidelity model evaluations is 102. The optimum design obtained by the algorithm proposed in this work has a 70% higher C_l than the initial design, but requires only 5 high-fidelity model evaluations and 126 low-fidelity model evaluations, with a total equivalent optimization cost being less than 7 high-fidelity model evaluations. The design variables of the optimum designs are very similar. The maximum camber has increased from zero to approximately 0.5% and location of maximum camber hits the upper bound of 60% of the chord. As can be seen from Figure 6 the shock on the upper surface has become stronger, hence the wave drag constraint becomes active, and it has moved rearward. The pressure distribution has opened up, yielding the increase in lift.

The second test case starts with a NACA 2412 airfoil and both constraints being not active. Using direct optimization the lift is improved by 40.8%, whereas the proposed algorithm improves it by 36.7%. However, direct optimization requires 110 high-fidelity model evaluations, and the proposed algorithm less than 7 equivalent high-fidelity model evaluations. In this case, as for the first case, the maximum camber is increased, from 2% to approximately 2.5%, and the location of maximum camber is moved rearward, from 40% to 60% in the direct optimization and 56.5% for the proposed algorithm. Hence, the aft camber of the airfoil is increased and the pressure distribution opens up in the rear, yielding more lift, as can be seen in Figure 7. The shock moves rearward and becomes stronger and, therefore, the wave drag constraint becomes active.

The third case also starts with NACA 2412, but at a higher Mach number and lower angle of attack. The limit on the wave drag is lower, and so the wave drag constraint is violated at the initial design. Both optimization methods reduce the maximum camber and move the location of it rearward. By reducing the camber the flow velocity decreases on the upper surface and the shock strength is reduced, as can be seen in Figure 8. By moving the maximum camber rearward, the aft camber increases and the pressure distribution opens up the in the rear, behind the shock, and increases lift.

Table 1. Numerical results for three test cases. Shown are results for the initial design, direct optimization and optimization using SPRP. All the numerical values are from the high-fidelity model (FLUENT). N_e is number of low-fidelity model (TSFOIL) evaluations and N_f is the number high-fidelity model (FLUENT) evaluations.

Variable	Case 1 $M_\infty = 0.75$, $\alpha = 1^\circ$, $C_{dw,limit} = 0.0032$			Case 2 $M_\infty = 0.70$, $\alpha = 1^\circ$, $C_{dw,limit} = 0.0058$			Case 3 $M_\infty = 0.75$, $\alpha = 0^\circ$, $C_{dw,limit} = 0.0040$		
	Initial	Direct [#]	This work [§]	Initial	Direct [#]	This work [§]	Initial	Direct [#]	This work [§]
m	0.0	0.00435	0.00489	0.02	0.0251	0.0253	0.02	0.0160	0.0173
p	0.0	0.6	0.6	0.4	0.5993	0.5653	0.4	0.5999	0.5930
t/c	0.12	0.1199	0.1180	0.12	0.12	0.1197	0.12	0.1199	0.1163
C_l	0.2072	0.3367	0.3521	0.6006	0.8454	0.8210	0.4732	0.4770	0.5085
C_{dw}	0.00097	0.0032	0.0032	0.0035	0.0058	0.0057	0.0100	0.0040	0.0041
A	0.0807	0.0807	0.0794	0.0808	0.0808	0.0806	0.0808	0.0808	0.0783
N_e	N/A	0	126	N/A	0	135	N/A	0	161
N_f	N/A	102	5	N/A	110	5	N/A	130	5
Total cost [*]	N/A	102	< 7	N/A	110	< 7	N/A	130	< 7

[#] Design obtained through direct optimization of the high-fidelity FLUENT model using the grid-search algorithm.

[§] Design obtained using the algorithm proposed in Section 3; surrogate model optimization performed using the grid-search algorithm.

^{*} The total optimization cost is expressed in the equivalent number of high-fidelity model evaluations.

Direct optimization improves the lift 0.8%, but the proposed algorithm improves it by 7.5%. With increased lift there should be an increase in wave drag. However, the optimum design obtained by proposed algorithm reduces the thickness from 12% to 11.6%, and therefore, is able to reduce the wave drag (since thinner airfoils have, in general, lower wave drag). The proposed algorithm requires less than 7 equivalent high-fidelity model evaluations, whereas direct optimization requires 130 high-fidelity model evaluations.

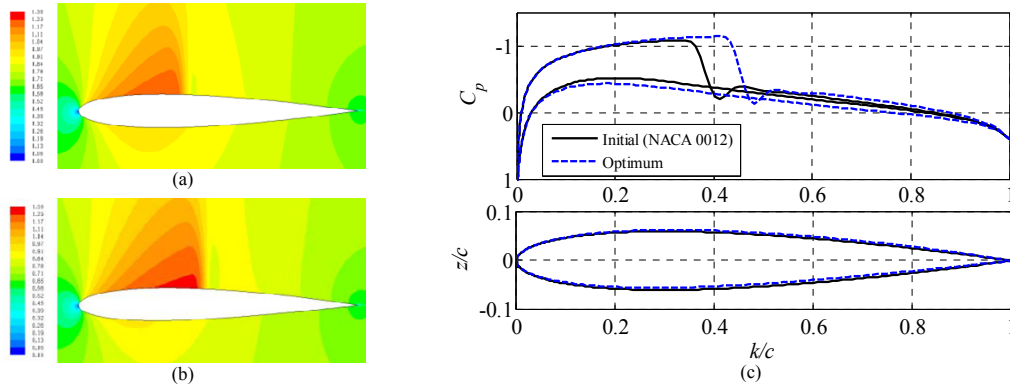


Fig. 6. (a) Mach number contours for the initial design (NACA 0012) of Case 1 ($M_\infty = 0.75$, $\alpha = 1^\circ$). (b) Mach number contours of the optimum design obtained using the proposed algorithm. (c) Pressure coefficient on the surface of the initial and optimum designs, and the airfoil shapes.

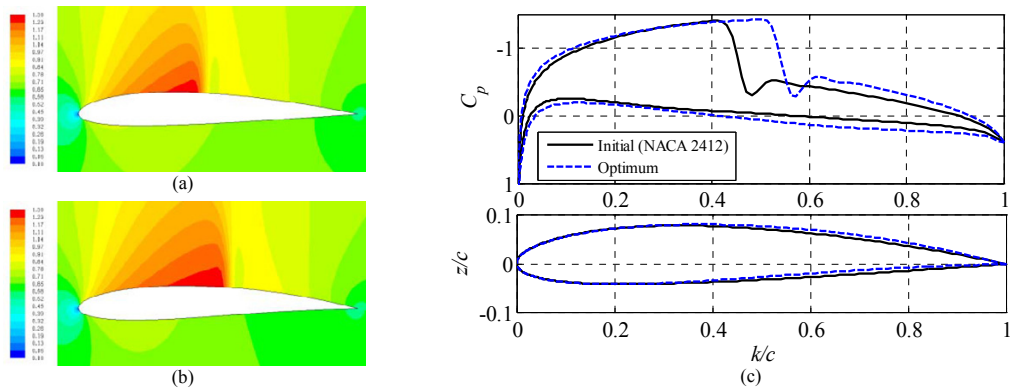


Fig. 7. (a) Mach number contours for the initial design (NACA 2412) of Case 2 ($M_\infty = 0.70$, $\alpha = 1^\circ$). (b) Mach number contours of the optimum design obtained using the proposed algorithm. (c) Pressure coefficient on the surface of the initial and optimum designs, and the airfoil shapes.

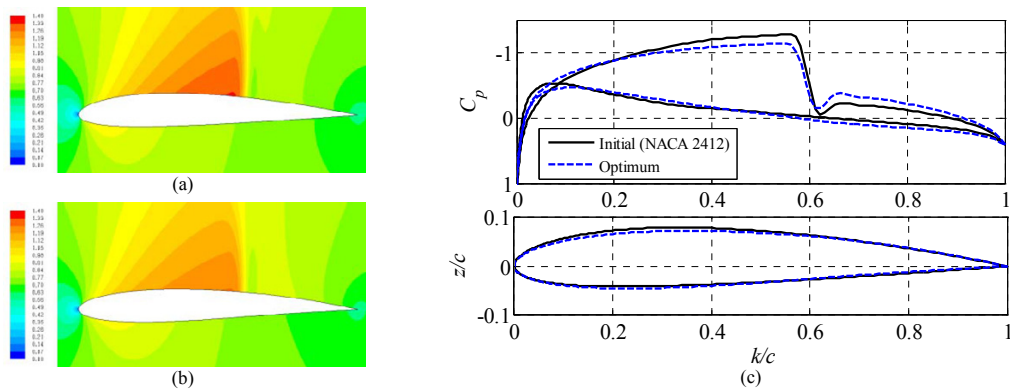


Fig. 8. (a) Mach number contours for the initial design (NACA 2412) of Case 3 ($M_\infty = 0.75$, $\alpha = 0^\circ$). (b) Mach number contours of the optimum design obtained using the proposed algorithm. (c) Pressure coefficient on the surface of the initial and optimum designs, and the airfoil shapes.

5. Conclusions

We have presented a design optimization methodology for transonic airfoils that uses a computationally cheap, physics-based low-fidelity model to construct a surrogate of an accurate but CPU-intensive high-fidelity model. The low-fidelity model is corrected by aligning its airfoil surface pressure distribution with the corresponding distribution of the high-fidelity model. The alignment is carried out using a shape-preserving response prediction methodology and ensures good generalization capability of the surrogate model with respect to both objectives and constraints (lift and wave drag). Several optimization case studies demonstrated computational efficiency of the proposed method. More specifically, our approach brings over 90% reduction of the number of high-fidelity model evaluations when compared to the direct optimization using grid-search.

Further verification studies of the proposed method in two-dimensions are required before extending it to three-dimensions. Cases with shock on both the upper and the lower surface should be examined. Also, better control of the airfoil shape can be realized by using other parameterization method.

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