Silent video tasks – their definition, development, and implementation in upper secondary school mathematics classrooms

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SILENT VIDEO TASKS
THEIR DEFINITION, DEVELOPMENT, AND IMPLEMENTATION IN UPPER SECONDARY SCHOOL MATHEMATICS CLASSROOMS

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Thesis submitted in partial fulfilment of a
Philosophiae Doctor degree in Mathematics Education

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Abstract

This thesis introduces results from a design-based, task design research study in mathematics education, within which silent video tasks were defined, developed, and implemented in upper secondary school mathematics classrooms. It discusses a research problem concerning the identification of opportunities and challenges that arise from the use of silent video tasks. To tackle that problem, the researcher worked with seven teachers in six Icelandic upper secondary schools who implemented silent video tasks in their classrooms.

In short, silent video tasks involve the presentation of a short silent mathematics video clip that students are asked to discuss in pairs as they prepare and record their voice-over to the video. On the basis of students' recorded responses to the task, that are listened to by the whole group, the teacher leads a discussion with the aim to deepen and widen students' understanding of the mathematical topic presented in the video.

The idea of silent video tasks is grounded in social constructivist theories. It is considered important that interaction happens between teacher and learners and among learners themselves, who work together (support each other) toward richer understanding of mathematical content. The learner is seen as an active participant in the teaching and learning process and in the case of silent video tasks, learners get an opportunity to become aware of their own and their peers' current ways of describing or explaining mathematical phenomena.

Two implementation phases were conducted in 2017 and 2019, during which interview data on teachers' expectations and experiences of using silent video tasks was collected and analysed. In the first phase, four mathematics teachers in randomly selected upper secondary schools in Iceland assigned a silent video task to their 17-year-old students. Results from the first phase indicated that silent video tasks might be a helpful tool for formative assessment.

Thus, teachers in the second phase were purposefully selected to work at schools that aim for active use of formative assessment. One teacher assigned three silent video tasks to his 16-year-old students and two teachers assigned one silent video task to their 16-year-old students. Besides interview data, classroom observation protocols were collected during the second phase.

Influenced by theory and empirical results, the process of assigning a silent video task developed. To conclude the project, some characteristics that make a video suitable for use in silent video tasks were defined and the instructional sequence of silent video tasks was described. Together with the underlying theoretical and empirical arguments, they form design principles for silent video tasks.

Ágrip

Hljóðlaus myndbandsverkefni – skilgreining, þróun og beiting nýstárlegra verkefna í stærðfræðikennslu á framhaldsskólastigi

Í þessari ritgerð er greint frá niðurstöðum hönnunarmiðaðrar rannsóknar um skilgreiningu, þróun og notkun talsetningarverkefna í stærðfræðikennslu á framhaldsskólastigi. Rannsóknin var unnin í samstarfi við sjö stærðfræðikennara í sex íslenskum framhaldsskólum sem lögðu talsetningarverkefni fyrir nemendur sína. Augu rannsakanda beindust einkum að því að greina tækifæri og áskoranir sem felast í fyrirlögn verkefna af þessu tagi. Fræði og fagstarf voru fléttuð saman til að verkefnin gætu nýst kennara og nemendum sem best við námið.

Við fyrirlögn talsetningarverkefnis velur kennari stutt hljóðlaust myndband sem sýnir stærðfræði á kvikan hátt og kynnir það fyrir nemendum. Tvö og tvö saman undirbúa þau og taka upp talsetningu við myndbandið. Öll hlusta saman á afraksturinn, fjölbreyttar talsetningar, og ræða þær með það að leiðarljósi að nálgast sameiginlegan skilning á þeim stærðfræðilegu fyrirbærum sem fyrir koma í myndbandinu. Nákvæmni og ýmiss konar blæbrigði í orðalagi eru meðal þess sem getur orðið að umfjöllunarefni umræðnanna.

Í grunninn eru talsetningarverkefni sprottin úr félagslegri hugsmíðahyggju þar sem litið er á nemendur sem virka þátttakendur í lærdómsferlinu og smiði eigin skilnings. Talsetningarverkefnin gefa nemendum meðal annars tækifæri til að taka eftir og ígrunda eigin sýn og sýn félaga sinna á þeirri stærðfræði sem birtist í myndbandinu. Þeim er þannig ætlað að ýta undir samstarf og samtal jafnt milli kennara og nemenda sem og milli nemenda innbyrðis.

Í tveimur samstarfsköflum rannsóknarinnar haustin 2017 og 2019 var viðtalsgögnum safnað og þau greind með tilliti til væntinga og reynslu kennara af notkun talsetningarverkefnanna. Haustið 2017 lögðu fjórir slembivaldir stærðfræðikennarar, hver í sínum skóla, eitt og sama hljóðlausa myndbandið fyrir 17 ára nemendur sína. Niðurstöður bentu til þess að verkefni sem þessi gætu gagnast við leiðsagnarmat í stærðfræði.

Haustið 2019 var því leitað til framhaldsskóla sem leituðust við að nýta leiðsagnarmat. Einn kennari lagði þrjú ólík talsetningarverkefni fyrir 16 ára nemendur sína með nokkurra vikna millibili og tveir kennarar í öðrum skóla lögðu eitt slíkt verkefni fyrir 16 ára nemendur sína undir lok annarinnar. Auk viðtalsgagna voru skráðar áhorfsathuganir úr kennslustundum þessara þriggja kennara.

Á rannsóknartímabilinu þróaðist ferlið við fyrirlögn talsetningarverkefnanna og gætti þar áhrifa kenninga um kennslu og nám sem og niðurstaðna fyrri rannsókna. Verkefninu lauk með framsetningu hönnunarstaðals sem inniheldur lista yfir einkenni myndbanda sem henta við gerð talsetningarverkefna og lýsingu þess hvernig best þyki henta að leggja talsetningarverkefnin fyrir nemendahóp.

Preface

This thesis introduces silent video tasks as an innovative tool for teaching mathematics in secondary school. Still frames from an example of an animated video clip used in a silent video task are presented in figure 0.1. This particular example visualizes characteristics of the Cartesian coordinate system. It uses colors to highlight and distinguish between different parts of the coordinate system and draws the viewers' attention toward certain point coordinates in each of its four quartiles.

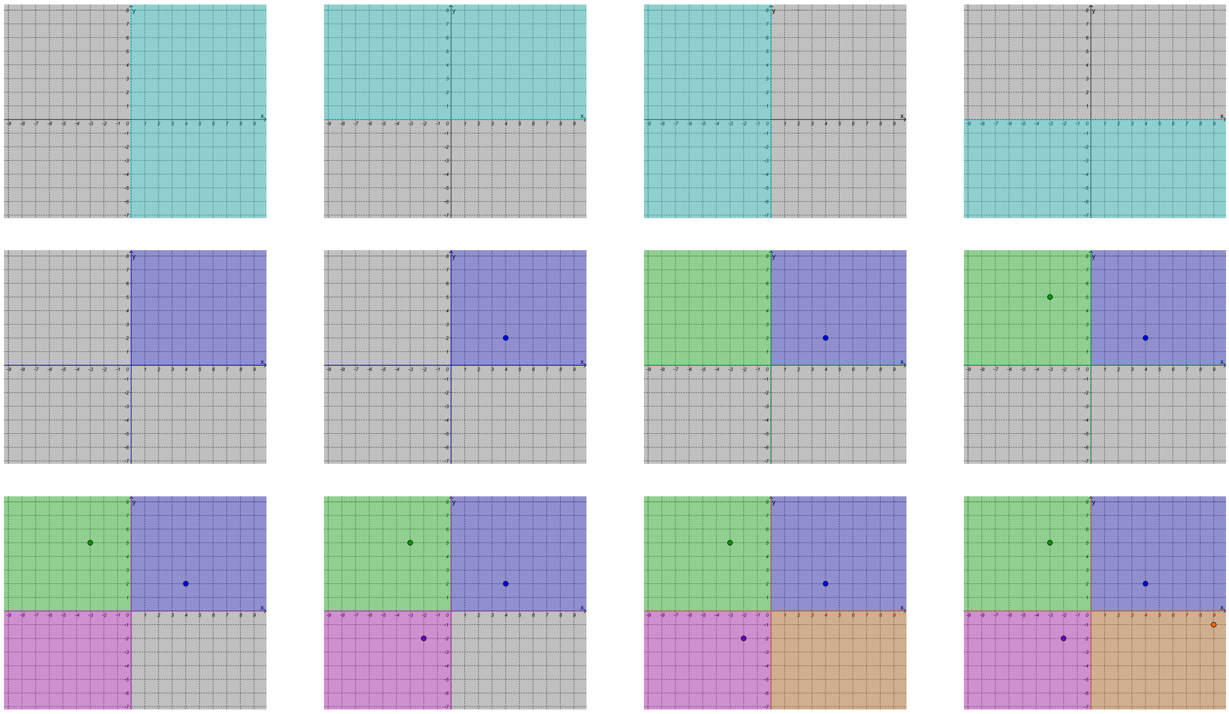

Figure 0.1. Stills from a 1:12 minute long silent video on the topic of the Cartesian coordinate system. Read from top-left to bottom-right the frames have the time stamps 0:06, 0:10:, 0:18, 0:24, 0:30, 0:38, 0:44, 0:48, 0:56, 1:00,1:06, and 1:10.

Like other video clips used in silent video tasks, the example shown in figure 0.1 (see https://youtu.be/8cLrbJM4F-I) includes no text or sound and is less than 2 minutes in length. Viewers are invited to describe, explain and narrate the video in their own words. By recording their voice-over and listening to other viewers' voice-overs, a group discussion can be orchestrated with the aim to deepen and widen students' understanding of the mathematics displayed in the video.

The reason why I embarked on the path toward defining and developing silent video tasks is closely connected to my own experiences with mathematics. Like most other students in Iceland, I experienced transmissive teaching in mathematics. Many other subjects offered constructivist learning environments but not mathematics. It was customary to listen to the teacher, take notes, and work individually on problems from the textbook. In 9th grade, my class–most of which had been classmates since first grade–was preparing for standardized exams but had a substitute teacher whom we for various reasons could not trust. This lead us to building a tree-structured system within the class: The root learnt from her parents, three stems learnt from her and explained to 2-3 classmates each, and these seven branches explained to two further classmates each, such that the whole class was covered.

After tenth grade our paths separated as we chose between several different upper secondary schools and mathematics classes went back to being individual work. I did not realize until much later how differently I must have learnt in tenth grade, being one of the three stems. We had transferred knowledge between each other and those of us who got to explain to others, surely benefited from that. Later, I realized and felt like I had been deprived of some opportunity of communicating in the other school years. I had always been a very conscientious student and the tight time schedule (especially in upper secondary school) got me running like a hamster in a wheel, focusing on technical calculation details rather than building understanding. Most of the problems we dealt with were drill and practice.

Facing challenging questions that required understanding during undergraduate studies in mathematics, I felt as if I would need to re-do the first year to build understanding. To get by without that repetition, I soon learnt the importance of collaboration and communication. The teaching was not much different, but in times of struggle I always had 4-5 fellow students to discuss problems and solution strategies with. Later, after completing a masters degree in applied mathematics and discovering that I liked teaching, I knew that I did not want to create the same learning-without-understanding hamster-wheel feeling in my students. I just did not know *how* to teach such that I could generate an environment that supported communication and building understanding.

At the same time, somehow, at the start of my teaching career, I felt like it was my job to simplify mathematics. Cover the curriculum by chopping it into consumable little bits that could easily be delivered and tested. Students happily accepted these bits and I did not realize that they could and would use them without too much thinking. Also, there was persistent lack of open problems and proper discussions. Even though I might discuss more than one way to a solution of an assigned problem, there was still a solution, that is, the problem was treated as closed, and students did not participate much in the discussion. In my classes, good questions were seldom being asked.

Several papers that I read in courses preparing for my teaching licence indicated that teaching in a constructive way would be the way to go, but there was frustratingly little material on what to do exactly. Our mentor emphasized how important it was to bridge between students pre-existing ideas and the often counter-intuitive reality in science, but I could only imagine pre-existing ideas being the biggest hurdle in a limited sector of mathematics. I wondered why mathematics education was not bundled with philosophy and foreign language education as teachers in those fields seemed to have more in common with the problems I was facing than the science teachers. To me, mathematics increasingly felt like teaching and learning a new language.

No matter how I searched, I could not find answers to my practical questions. Alongside the studies I was teaching full time at an upper secondary school and Bergþór Reynisson, one of my colleagues there, introduced me to John Mason's work. In his books, I found ideas for tasks that were meant to get students to think and connect, ideas that I connected with and I happily tried out a few of them. I also integrated the use of GeoGebra into my teaching, but mostly I was running to cope with a tight time schedule–being the teacher I did not want to be. After this first year of teaching and receiving my teaching licence, I went to Berlin for further studies at Humboldt University.

In Berlin, I continued my search for techniques or tools to address diversity in the mathematics classroom. How could I attend to different students' needs? Might the use of GeoGebra help in some way? During the year in Berlin I visited a course where Swiss teachers Urs Ruf and Gallin in Potsdam presented their method of supporting learning by dialogue (*dialogisches Lernen*). The way they treated mathematics as a language and gave students big-but-easy-to-understand (low floor, high ceiling) problems to explore fascinated me.

After returning back to Iceland, I focused on pacing (allowing students to think after prompts and questions), experimented with group work lessons, used GeoGebra for exploration, and started working with the Nordic-Baltic GeoGebra Network (NGGN)–a group of researchers and teachers interested in meaningful integration of technology in mathematics teaching. At the annual NGGN conferences I finally found some answers to my questions regarding how to attend to different students needs and what to search for in order to find interesting tasks or materials on the internet.

To name an example, I discovered the Mathematics Assessment Project and started both trying out and creating my own variations of tasks that Hugh Burkhardt and Malcolm Swan had developed with their MARS Shell Center team at the University of Nottingham. Freyja Hreinsdóttir and her Nordic-Baltic colleagues who planned the NGGN activities decided to start key topic groups where we would present and develop ideas of new ways of working with GeoGebra. At a brainstorming meeting, Freyja, Thomas, and Rokas[1] were discussing how screen recordings and GeoGebra

[1] Freyja Hreinsdóttir is my advisor at the University of Iceland, Thomas Lingefjärd at that time was an associate professor in the field of mathematics education at Gothenburg University in Sweden but now a politician in Gothenburg City Council, and Rokas Tamošiūnas at that time was working

might be used to support mathematics teaching in a multicultural classroom.

They came up with the idea of silent videos–an idea that we would all continue to discuss. Our Estonian, Swedish, and Lithuanian colleagues did the first teaching experiments. I was not quite convinced of the usefulness of asking students to add their voice-over to a silent video, but decided to give it a try. In the end we were around 21 teachers in Estonia, Iceland, Latvia, and Lithuania who tried out silent video tasks with our students. When I first used a silent video task in my own teaching in 11th grade, I was surprised by the variety of my students' responses to a video focusing on the area of a triangle.

A pair of students who normally did well on (procedural) exams seemed not to think much about or understand what was going on in the video. Another pair of students who were doing okay at the same exams created a comprehensive response. Aha! This information was helpful for both me and my students. Somehow, the silent video task informed me about what my students were thinking and made it clearer to me, what I needed to change. The question remained: Why did it work in that way? Would it work similarly for other teachers and their students? No matter how sceptical these teachers would be, would they also experience an aha! moment?

as a mathematics teacher in an upper secondary school but is now a senior data scientist and lecturer at the Mathematics and Informatics Faculty of Vilnius University in Lithuania.

Contents

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List of Publications

This thesis is based on the following publications, referred to by their Roman numerals:

I **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza, Z.(2018). Realizing students' ability to use technology with silent video tasks. In Weigand, H.G., Clark-Wilson, A., Donevska-Todorova, A., Faggiano, E., Grønbæk, N. & Trgalova, J. (Eds.), *Proceedings of the 5th ERME Topic Conference MEDA 2018* (pp. 163—170). University of Copenhagen.

II **Kristinsdóttir, B.**, Hreinsdóttir, F., Lavicza, Z., & Wolff, C.(2020). Teachers' noticing and interpretations of students' responses to silent video tasks. *Research in Mathematics Education, 22*(2), 135–153.

III **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza, Z.(2020). Using silent video tasks for formative assessment. In B. Barzel, R. Bebernik, L. Göbel, M. Pohl, H. Ruchniewicz, F. Schacht, & D. Thurm (Eds.), *Proceedings of the 14th International Conference on Technology in Mathematics Teaching* (pp. 189—196). Universiy of Duisburg-Essen. doi.org/10.17185/duepublico/70763

IV **Kristinsdóttir, B.**(under review). Opportunities and challenges that silent video tasks bring to the mathematical classroom. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The Matehmatics Teacher in the Digital Era (2nd ed.)*. Springer Nature.

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Co-authorship: The research project design, data collection, data analysis and paper writing in general was conducted by me, the doctoral candidate. Zsolt Lavicza supported my writing by providing a doctoral seminar for students working on design-based research in STEAM education. Within that doctoral seminar, I discussed my project design, data analysis, and writing with other doctoral students. Both Zsolt Lavicza and Freyja Hreinsdóttir supported my writing by discussing my findings with me and by giving feedback to final versions. Charlotte Wolff supported the writing of Paper II by discussing findings and pointing out to me a useful theoretical framework.

List of Related Publications

As a PhD student I have also authored or co-authored the following publications:

- Hreinsdóttir, F. & **Kristinsdóttir, B.** (2016) Using silent videos in the teaching of mathematics. In S. Ceretkova (Ed.), *Staircase to Even More Interesting Mathematics Teaching* (pp. 157–164). Constantine the Philosophers University Nitra.

- **Kristinsdóttir, B.** (2017). Workshop: Silent screencast videos and their use when teaching mathematics. In G. Kaiser (Ed.) *Proceedings of the 13th International Congress on Mathematical Education* (pp. 737–738). Springer.

- **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza (2018). Initiating student discourses with silent video tasks in mathematics classrooms. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.) *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education, Vol. 5* (p. 92).

- Gíslason, I., & **Kristinsdóttir, B.** (2018). Hvatt til hugsunar í stærðfræði. *Flatarmál, Rit Flatar Samtaka Stærðfræðikennara*, 25(1), 24–25.

- **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza (2019). Silent video tasks: Towards a definition. In M. van den Heuvel-Panhuizen, M. Veldhuis, & U.T. Jankvist (Eds.) *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 2709–2710). Freudenthal Institute & Utrecht University. https://hal.archives-ouvertes.fr/hal-02417067.

- Lieban, D., **Kristinsdóttir, B.**, & Lavicza, Z. (2019). Setting a Creative Math Task with SET 3D: Modeling Physical Pieces through Digital Resources. In S. Goldstine, D. McKenna, & K. Fenyvesi (Eds.) *Proceedings of Bridges 2019* (pp. 489–492). Tessellations Publishing.

- **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza (2020) Silent video tasks and the importance of teacher collaboration for task development. In A. Donevska-Todorova, E. Faggiano, J. Trgalova, Z. Lavicza, R. Weinhandl, A. Clark-Wilson, & H.-G. Weigand (Eds.), *Proceedings of the 10th ERME Topic Conference: Mathematics Education in the Digital Age (MEDA)* (pp. 415–416). Johannes Kepler University. https://hal.archives-ouvertes.fr/hal-02932218/.

- Fahlgren, M., Brunström, M., Dilling, F., **Kristinsdóttir, B.**, Pinkernell, G., & Weigand, H.G. (2021) Technology-rich assessment in mathematics. In A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, J. Trgalova, & H.G. Weigand (Eds.) *Mathematics Education in the Digital Age: Learning, Practice and Theory* (pp. 69–83). Routledge. http://dx.doi.org/10.4324/9781003137580-5.

- **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza (2021). Some lessons–regarding

inclusion and teacher change–learnt from developing silent video tasks. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.) *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education, Vol. 1* (p. 159).

- **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza (accepted for publication). Developing silent video tasks' instructional sequence. In *Proceedings of the 14th International Congress on Mathematical Education* (4 pages). World Scientific.

- **Kristinsdóttir, B.**, Hreinsdóttir, F., & Lavicza (accepted for publication). Developing silent video tasks' instructional sequence in collaboration with teachers. In *Proceedings of the 15th International Conference on Technology in Mathematics Teaching* (8 pages). Aarhus University.

Abbreviations

Clarification of certain terms and abbreviations in this thesis.

Term	Clarification
Pupils	learners (children) in compulsory school
Students	learners (youth) in upper secondary school
Learners	learners in general regardless of age or school stage
Teachers	teaching professionals in schools
Pre-service teachers	learners in teacher education programs at university level
In-service teachers	learners in professional development courses for teachers
Teacher educators	teachers in teacher education programs at university level

Abbreviation	Meaning
ATM	Association of Teachers of Mathematics
BTC	Building Thinking Classrooms
DGS	Dynamic Geometry Software
EQF	European Qualifications Framework
ICME	International Congress on Mathematical Education
ICMI	International Commission on Mathematical Instruction
ICT	Information and Communication Technologies
ICTMT	International Conference on Technology in Mathematics Teaching
IDPA	Icelandic Data Protection Authority
LMS	Learning Management System
NGGN	Nordic-Baltic GeoGebra Network
PME	Conference of the International Group for the Psychology of Mathematics Education
STEM	Science, Technology, Engineering, and Mathematics
TRU	Teaching for Robust Understanding
UNCRC	United Nations Convention on the Rights of Children

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Special thanks to all my climbing, hiking, and ski touring buddies that helped me stay sane! Also for your support when I was getting back on track after breaking my wrist.

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Elsku vinir mínir og fjölskylda, þið hafið ekki gefist upp á mér þrátt fyrir að hittingum hafi fækkað til muna meðan á þessari rússíbanareið stóð og fyrir það er ég afar þakklát. Takk fyrir skilninginn og stuðninginn, skilaboð og símtöl, og þessi fáu skipti sem við þó náðum að hittast!

Síðast en ekki síst, hjartansþakkir fyrir stuðninginn og hvatninguna, umhyggjuna, matinn og (mynd)símtölin frá mínum nánustu vinum og fjölskyldu sem fylgst hafa með öllum dölum og hæðum í ferlinu: Guðný, Bergur, Ketill, pabbi & mamma, Líney Halla & Sigurður Ægir, Baldur & Birte og amma Eyja. Ég segi kannski ekki að án ykkar hefði þetta ekki tekist því að eins og þið þekkið best þá hefði ég líklegast einhvern veginn bjargað mér en útkoman (bæði ritgerðin og mín eigin heilsa) væri alveg örugglega ekki jafngóð!

[2] I am lucky that this was possible. In Iceland, currently around 25% of doctoral research projects receive no funding and 23% receive only partial funding for 1-2 years. Some students who have a family to provide for cannot not start their studies even with funding provided, because it is so sparse. I do not mean to complain, my situation is good. In a rich country like Iceland, I just would wish for a government that would enable more students to pursue their research dreams.

1. Introduction

This chapter includes the background and aims of the task design study presented in this thesis. It also explains the research originality, its Icelandic context, and concludes with an outline of the thesis.

1.1. Background

This research project is a plant growing from a seed that started to sprout in a teacher-researcher collaboration project called the Nordic-Baltic GeoGebra Network (NGGN, see https://nordic.geogebra.no/). Within NGGN, teachers were encouraged to present and develop their own ideas regarding the use of technology in mathematics teaching. Rather than having researchers introduce ideas "from above", teachers were asked to share materials, exchange experiences and work in collaboration with teacher educators and researchers at annual conferences. At the third NGGN conference in Copenhagen in 2013, a key topic was presented with the initial question "How can we use screen recording technology with GeoGebra for mathematics teaching and learning?"–a question that a group of researchers and teachers set themselves to answer.

Four main ideas emerged, one of which involved the creation of short silent screen recordings made on the base of dynamical GeoGebra worksheets. Initially, teachers in each country were expected to add a voice-over to the video in their own language but soon, in the initial brainstorming stages, the idea evolved into students watching the video without sound. Questions regarding what students might learn and how their understanding might be brought to light pushed Freyja, Rokas, and Thomas (the initiators of the idea) toward suggesting that students would record a voice-over to the video. Thus, the idea developed into a tool that either could shed light on students' first ideas about a new mathematical topic (before or at the start of its introduction) or provide information on what students had to say about a mathematical topic that they had encountered or learnt about before. Together, the group developed some silent videos (see https://ggbm.at/JF8PqrNC) and selected three of them for teachers to try out with their students.

Mathematical understanding is not taken as something that can be assessed by a questionnaire. One can conclude about it, but not measure it directly. Indications of understanding are for example when students show that they are aware of relationships that exists between mathematical concepts or ideas. To guide teachers, Thomas suggested that we would introduce them to the structure of the observed learning outcome model, the SOLO-taxonomy model (J. Biggs & Tang, 2007; J. B. Biggs & Collis, 1982), which describes students' increased knowledge or deepened understanding in five levels of growing complexity from pre-structural over uni-structural, multi-structural, and relational, to extended abstract (see figure 3.4, p. 37).

For example, when working on a silent video about the area of a triangle (see https://youtu.be/qjzKQfhQ8NQ) at the pre-structural level learners might miss the point and instead talk about line slope. At the uni-structural level they might focus only on the changing area but never mention the dimensions of the triangle. At the multi-structural level learners might describe the changing area and dimensions of the triangle but make no connection between them, whereas at the relational level they would make a connection and explain how the triangle area depends on its height and base length. At the extended abstract level learners might generalize and explain why this connection applies to all triangles of whatever shape.

Together, the NGGN group agreed that assigning a silent video task would involve showing a silent video at the start of a lesson, dividing learners into groups of 2-4 that could watch the video as often as they liked whilst discussing what they saw in preparation to recording their commentary (voice-over) to the video they watched. This activity could be implemented either as an introduction to a new concept or as a debriefing session; the former intended for collecting preconceptions and the latter for assessing the learners' understanding of something previously studied in class.

The NGGN group decided to perform a teaching experiment in 2014. Participating teachers had three silent videos to choose from, one on each of the following topics: triangle area, line reflection, and point reflection. Around 450 learners from 5 countries in grade 5-13 participated with their teachers. Findings from the teaching experiment indicated that teachers might become better aware of their students' conceptual understanding by assigning a silent video task in their classrooms. Participating teachers experienced that the task stimulated learners' discussion about mathematics and that by reacting to their discussion, teachers were able to enhance their learners' motivation to think and learn (Hreinsdóttir & Kristinsdóttir, 2016).

1.2. Research aims

Based on my own experiences and what I heard from some of the other teachers in the 2014 teaching experiment, I had formed some hypotheses regarding the potentials of silent video tasks. During implementation of the silent video task, my students–who normally sat silently working on practice exercises from their textbook–effortlessly started talking about mathematics, I *heard* my students thinking and saw a potential to sequence some responses ranging from everyday language to formal mathematical language. Thus my hypotheses were that silent video tasks might

H1 Stimulate students' discussion about mathematics.

H2 Open teachers' ears to their students' thinking.

H3 Help students to bridge between everyday language used in informal settings and formal mathematical language used in school settings.

H4 Make students become aware of how being able to explain something to others implies their own understanding.

This sparked my interest in developing the idea of silent video tasks further. An opportunity to do so came in 2016 when I was offered to start the doctoral research project presented in this thesis. Although, in Malcolm Swan's words, "Ultimately, the goal is transformative; we seek to create new teaching and learning possibilities and study their impact on teachers, children, and other end users." (Swan, 2014), it seemed unlikely that a pre- and post-test would show differences in students' learning for such a short intervention as the presentation of a silent video task. I remembered that back in 2014, teachers seemed to find this idea useful, but information in regard to *why* they found it useful was limited–something I grew increasingly curious about. Did other teachers see the same or similar potentials as I did? Therefore, relatively soon in the research process, I set out to explore teachers' expectations and experiences of using silent video tasks.

In other words, I took the direction of focusing on teachers and the ways in which they implemented and made use of silent video tasks. Other aims regarding the definition of silent videos and the development of silent video tasks' instructional sequence were formed along the road, later in the research process. The following lists the aims of the research project:

A1 Gain insight into teachers' expectations and experiences of using silent video tasks in their mathematics classrooms.

A2 Define the characteristics of a silent video.

A3 Develop and describe the process of assigning a silent video task (silent video tasks' instructional sequence).

A4 Identify the opportunities and challenges that arise from the use of silent video tasks.

A1 was formed in 2016-2017 at the start of this research project whereas A2–A4 developed throughout the research project.

1.3. Research questions

This section states eight research questions that I set out to answer in this thesis. Even though they are presented here in the first chapter of the thesis, these research questions were neither ready nor set in stone before the research project started. On the contrary, they developed with time as I got familiar with the data and in discussion based on preliminary findings.

Early in the preparation process, while participating in a doctoral seminar, I decided to focus on teachers and their role in using silent video tasks. This emphasis is reflected in the following fundamental research questions:

Q1 What are participating teachers' expectations of using silent video tasks in their classroom?

Q2 What are participating teachers' experiences of using silent video tasks in their classroom?

These two questions, Q1 and Q2, shaped the pre-defined interview questions (see appendix B on p. 201) used in semi-structured interviews with teachers and accordingly influenced the collected data. They, therefore, made an impact on all the papers I, II, III, and IV. Questions Q3–Q8 took longer to develop as they emerged in the process of analysing, discussing and acting with data.

For clear communication with teachers and researchers, a definition of what silent videos are (and are not) was essential. Also, it was important to keep track of and be able to describe how the silent video tasks' instructional sequence developed. The following questions were a reaction thereto:

Q3 What are some characteristics of a video that can be used in a silent video task?

Q4 What are some factors that influence teachers' decisions regarding their implementation of a silent video task, how did these decisions impact the tasks' instructional sequence development, and why?

My first proper attempt at answering question Q3 was presented in a poster at CERME-11 in Utrecht in 2019 and the current answer to that question is given in chapter 5, listed from C1 to C9 on p. 71 as part of the silent video tasks' design principles. At the ICTMT-14 conference in Essen in 2019, I started discussing the instructional sequence with more focus on formative assessment (see paper III on p. 131) and my current answer to question Q4 is given in section 5.2.2 on p. 75. Also, the current suggestion of the task instructional sequence is listed from P1 to P16 on p. 72 as part of the design principles of silent video tasks.

At the PME 42 conference in Sweden in 2018, I presented my preliminary results from the first implementation phase of the research project. I focused on teachers' experiences of using the silent video task and their reactions to instances where the new situation caused them stress or anxiety – in other words, situations of teacher tension. At the conference, a group of researchers working on mathematics teachers' knowledge wanted to collect international perspectives on ways in which teachers' professional knowledge can be viewed via teachers' actions and reactions to certain classroom situations. They invited me to write a paper in a special issue of *Research in Mathematics Education* (see paper II on p. 109) and during the writing of that paper, the following questions were formed:

Q5 What do teachers notice when listening to student responses to the silent video task?

Q6 What do teachers notice about their own abilities to use the silent video task to support student understanding?

Both questions Q5 and Q6 are discussed in paper II. In retrospect, question Q6 could have been rephrased as "What do teachers notice about their own practices when they deal with the new situation created by the use of silent video tasks?", and that form of the question gave rise to the following final research questions, that I developed during the preparation and realization of the second implementation phase:

Q7 What are some opportunities that silent video tasks can bring to the mathematics classroom?

Q8 What are some challenges that silent video tasks bring to the mathematics classroom?

Questions Q7 and Q8 were addressed in paper IV on p. 141.

To answer the above listed questions, I ended up working with seven mathematics teachers in six upper secondary schools in Iceland who implemented silent video tasks with their students. Our collaboration was intended to keep my feet on the ground in terms of developing ideas that would be useful and could (practically) be used in the mathematics classroom.

1.4. Originality

Until recently, short educational videos about mathematics were mainly used in distance learning courses and by teachers using flipped classroom approaches. The COVID-19 pandemic pushed teachers worldwide to rapidly change from classroom teaching to remote teaching and suddenly everyone seemed to be creating their own educational videos or referring to existing open source videos. Silent videos, however, are different from most existing videos in the way that they do not include any text, music, narration, or recordings of classroom settings or human beings. They solely include dynamic representations of mathematical concepts.

The use of silent mathematics video clips in mathematics teaching is known since 1910 when the German mathematics teacher Ludwig Münch (1852–1922) produced and screened thirty short animated films about geometry and astronomy for his students. Among the topics was the Appolonius circle, i.e. a circle that touches three given circles in the plane. Out of Münch's thirty animated films only twenty are still known to exist in archives but they are not accessible to the general public (Kitz, 2013).

Despite them not being interactive, in a way, animated films were the predecessor of dynamic geometry software (DGS) that was introduced in the early 1980's. Similar to Ludwig Münch, Jean Louis Nicolet, a Swiss teacher, developed black-and-white animated mathematics films in the 1930's. His films were without sound or text and all had the title Animated Geometry (Tahta, 1981). Two of them are publicly accessible on the ATM YouTube channel (see https://youtu.be/gum9kvxR9K8 and https://youtu.be/4D3ttrC2Wdk). The teacher and mathematics educator Caleb Gattegno, a founding member of the Association of Teachers of Mathematics (ATM), was a pioneer in using Nicolet's films for teaching. He introduced these films to other teachers in the 1950's and demonstrated how they could be used to promote mathematics learning in the classroom (Tahta, 1981). His idea was to invite participants to mentally re-construct what they had seen in a short video and then collaborate to give a step by step description that was short and without judgement, explanation or speculation (Mason, 2017).

In 1979 Gattegno reconstructed seven of the Nicolet films, all of which focused on concepts related to the circle. The reconstructed Nicolet films differed from the original in that they were made in colour with computer animation and that Gattegno added frames to the films in such a way that repeated viewing was required to grasp all that was happening (Tahta, 1981). In addition to introducing the Nicolet films, Caleb Gattegno introduced films made by Trevor Fletcher. Fletcher was a pioneer in making mathematics films in the UK and in total made six films for university teaching in the years 1952-1979 (Tahta & Fletcher, 2004) some of which can be viewed on the ATM homepage. Gattegno underlined that the Nicolet films are not merely illustrations but tools for teaching and research that a teacher can use in many ways both in terms of explanations and follow-up work (Gattegno, 2007).

An example of teaching and research work done with the Nicolet films is the way in which Nathalie Sinclair, professor at Simon Fraser University in Vancouver, Canada, used the remade Nicolet film *Circles in the plane* to invoke gestures, teaching the mathematical concept of circle by imitation. Sinclair guided pre-service mathematics teachers through the process of watching the film a few times in a row, each time giving a new task to imitate what happened in the video: first by talking, then by moving their hands, and finally by drawing. She was inspired by methods developed by Caleb Gattegno and Dick Tahta in their work with Nicolet films; re-telling the stories of what happened in the film, but at the same time adding on to it the mobile, gestural activity (Sinclair, 2016). Her work might remind one of the eurythmic geometry movement exercises of Rudolf Steiner in Waldorf schools, drawing pictures in the mind and embodying them by dancing patterns or gestures, but the two differ in the way that each individual student worked on their own in Sinclair's approach whereas in eurythmy learners form patterns together (Ogletree, 1997).

Mathematical videos by Bruce and Katherine Cornwell are presented as a purely visual exploration of mathematical properties. However, their films include both sound (classical and jazz music) and text and thus differ from the films by Münch, Nicolet and Fletcher. Bruce and Katharine Cornwell started making animated educational films in 1956 because they wanted to do better than Disney's *Donald in Mathematic Land* (1959). They created films about numbers, sets and infinity in 1961-1962, the Mathematical Association of America (MAA) asked them to make a series of animated calculus films in 1965-1968, and they continued to work on such films until 1974 for the publisher Houghton Mifflin. The Cornwells are primarily known for their animated films on the subject of geometry that they made in 1976-1979 (Alexander, 2016). Many of the Cornwell films are accessible via a Vimeo channel belonging to Bruce and Katherine's son, Eric Cornwell (Cornwell, 2014).

Inspired by the question whether people imagine or see congruent things when they watch a specific mathematical idea or object, Helmut Linneweber-Lammerskitten, professor at PH Fachhochschule Nordwestschweiz in Switzerland has been collaborating with professors Marc Schäfer and Duncan Samson at Rhodes University in South-Africa on a project that they call VITALmaths. They made short silent stop-motion video clips with text in German and English to give students tasks to work on, i.e. instead of giving students a written problem, students watched a short film on the task. The project seeks to establish, disseminate and research the efficacy and use of short video clips designed specifically for the autonomous learning of mathematics (Linneweber-Lammerskitten et al., 2010). A follow-up project VITALmathsLIC aims at finding how the films call after, support and enhance the learning of mathematics in different settings and areas. In particular, the focus is on communication skills and how films and other teaching materials work together. The VITALmaths video clips along with student made video clips SANDBOXmaths and a few GeoGebra applets APPLETmaths can be viewed on the homepage VISUALmaths (http://visual-maths.com).

In none of the above mentioned projects, a connection to youth popular culture–such as the current TikTok, Instagram, and YouTube culture–is made as clear as when working with silent video tasks. Students' acquaintance with these cultures makes it so that it can seem straightforward for them to record a voice-over to a video even though it is in the context of the mathematics classroom.

1.5. Icelandic context

This section gives a short introduction to the structure of the Icelandic school system, to the relevant parts of the Icelandic national curriculum, and explains why the study is being conducted in Iceland.

1.5.1. The Icelandic school system

In 2019 out of around 364,000 inhabitants in Iceland, 86,182 were children and youth in schools: preschool/kindergarten (18,742), compulsory school (46,254) or upper secondary school (21,186) (Statistics Iceland, 2020). In Iceland, children spend ten years (6-16 years old) in compulsory school that is formally divided into grades 1-4 in primary school, grades 5-7 in middle school, and grades 8-10 in lower secondary school. In the lower grades most schools group children by age in mixed ability classes with one teacher for all core subjects, whereas in the upper grades many schools have a slightly modular scheduled system with teachers specialized in each subject. After compulsory school pupils have the right to go to upper secondary school to prepare for further studies and/or for vocational training. Upper secondary school used to be four years until a governmental decision in 2015-2016 shortened it to three years of normal studying time.

Out of 30 upper secondary schools that offer preparation for university studies, 13 are located in the capital area and 17 in rural areas, whereof 5 are in towns with more than 5000 inhabitants. Subject teachers are expected to have studied their subject (e.g. mathematics) and acquired a teaching licence. However, there is a shortage of formally qualified secondary school mathematics teachers and it is thus common that mathematics teachers in Iceland studied related fields such as engineering or science. Reports written 27 years apart in 1987 and 2014 for the Icelandic Ministry of Education, Science and Culture reach similar conclusions: based on their education, teachers at upper secondary school level lack mathematical content knowledge and pedagogical knowledge, and interview data from these governmental reports indicate that communication between teachers is sparse (Jóhannesson, 1987; Jónsdóttir et al., 2014).

In preparation for her upper secondary classroom research study on slow-pace courses for low-achieving students, Kristín Bjarnadóttir documented why upper secondary schools in Iceland got modularised in 1965-1985 and how that change influenced mathematics courses taught. Both economic and social reasons were given for the modularisation. For example, students should be flexible to move between schools and change their lines of study midway, and different lines of study should be equally respected and available under one roof. Courses in the credit-based modularised system tended to get more general (as opposed to focused and specialized) as a reaction to diversity when students with different future plans got mixed in one classroom (K. Bjarnadóttir, 2011, 2012). In 2017 (2019) there was a flexible credit-based modular system in 25 (26) of the 30 upper secondary schools and class-based year cohort system in five (four) schools.

In the credit-based modular system, students select and follow a certain line of studies that has some compulsory (core) courses, some compulsory-elective courses, and some elective courses. The school year (9 months) is split into two terms and for each term students need to select and sign up for courses according to a plan that they create freely within the frame set by their selected line of study. Thus, a student in a modular system will meet several different groups of students within each day–one group for each of the courses that they selected and signed up for. A student on a natural science line might, for instance, choose to set emphasis on chemistry and geology in their selection of compulsory-elective courses and take elective courses in drama, philosophy, programming, and psychology. If needed, they might decide to change their line of studies midway without this change causing much delay.

Students in a class-based year cohort system select a certain line of studies and one out of three languages (German, French, or Spanish) as their third language. Within their line of studies, they follow a fixed schedule with pre-defined courses, many of which are specialized for their line of study. Therefore, it can prove difficult for students in the class-based cohort system to change between lines of study midway. Apart from the third-language courses, students will normally attend all classes with the same group of students (their class) according to the pre-defined plan. Sometimes, students are allowed to select one or two courses from a list of elective courses as a part of their third-year schedule, but not all class-based schools offer such (limited) flexibility.

Each course is measured in upper secondary school credits (hereafter referred to as credits), with each such credit corresponding to 18-24 hours of work for an ordinary student and a selected line of study usually being composed of 200-205 credits. The mathematics courses usually are ranked 5 credits each and requirements range from 5-10 credits (one or two courses) for students on non-mathematical-oriented lines and up to 25-35 credits (five to seven courses) on mathematically oriented lines of study. Students who aim for further studies in mathematics, engineering, or science are usually advised to take 5-15 additional credits of elective mathematics courses. A study by Valgerður S. Bjarnadóttir indicates that teachers have low expectations of students enrolled in non-mathematical oriented lines regarding students' ability to succeed in mathematics. Conceptual demands were lowered in courses tailored for theses students (V. S. Bjarnadóttir, 2018). Even though Valgerður's study only included four mathematics teachers, its results might be characteristic. At least, the aforementioned research by Kristín Bjarnadóttir indicated that course contents for low-achieving students in upper secondary schools was not in accordance with the National Curriculum, built on too much repetition and in general not useful for students (K. Bjarnadóttir, 2011).

1.5.2. The Icelandic National Curriculum for upper secondary schools

Icelandic upper secondary schools have freedom in selecting content and teaching methods for their mathematics courses. All schools have liberty to decide their own school curricula within the frame of the National Curriculum Guide if they report the number of credits and content in each course for approval by the Ministry of Education, Science and Culture ("Lög um framhaldsskóla nr. 92/2008 [Law on upper secondary schools number 92 of 2008]", 2008). In the years 1999-2008 an attempt was made to standardise university preparatory education in Iceland to a greater extent than before, but a study by Atli Harðarson (2010) indicates that despite detailed lists of mathematical content to be covered in each course, it did not get standardised to the extent that was aimed for (Harðarson, 2010). In Iceland, there are no standardised exams at the end of upper secondary school. On the contrary, each teacher formulates their own assessment–be it formative or summative assessment–based on goals made by the general curriculum and school curriculum ("Lög um framhaldsskóla nr. 92/2008 [Law on upper secondary schools number 92 of 2008]", 2008).

Upper secondary school mathematics teachers that Atli Harðarson interviewed in 2009 were generally quite unhappy with the changes from 1999, which decreased the number of compulsory credits from 35-45 to 25 credits for any mathematical or science-oriented line of studies and from 20-25 credits to 10 credits for any non-mathematical-oriented line. According to teachers, the long lists of things to cover in each standardised course made it impossible to put emphasis on mathematical proofs because of time constraints. Three of the teachers tried to make up for the "running down lists of things to cover" by digging deeper with their students in elective courses. Teachers made clear that if they could change the curriculum, they would increase the number of compulsory courses again, slow down the pace, and shorten the list of things to cover in each course (Harðarson, 2010).

The current general mathematics curriculum for upper secondary schools in Iceland describes goals regarding the knowledge, skills, and competences in mathematics that students are expected to reach partly or fully at each of four competence levels. Knowledge, skills and competences refer to terms by the European Qualifications Framework (EQF) where knowledge is described as a body of facts, principles, theories and practices, skills as the cognitive and practical ability to apply knowledge and use know-how to complete tasks and solve problems, and competences in terms of responsibility and autonomy mean the proven ability to use knowledge, skills and personal, social and/or methodological abilities ("ESCO - ESCOpedia - European Commission", n.d.). The four levels in the Icelandic general curriculum range from general education at level 1, which is shared with the highest lower-secondary school level, to specialization at level 4 that is shared with the lowest university level. For example, a course on algebra and linear functions would be at level 2 and a course on differentiation would be on level 3.

At all levels, schools are expected to provide students with a foundation for understanding in mathematics, for instance, the aim is not only to become proficient in calculation methods (procedural fluency), but also to attain conceptual understanding, i.e. the specific comprehension of mathematical concepts, operations, and relations. The competence goals also include some adaptive reasoning as students are expected to understand and interpret the explanations and arguments of others without prejudice, showing respect and tolerance (Ministry of Education, Science and Culture, 2011).

In spite of these curricular objectives, interviews with teachers and observations in classrooms in nine Icelandic upper secondary schools revealed that mathematics teaching practice was mostly limited to expository methods and recitation, drill, and practice (Jónsdóttir et al., 2014). This finding was confirmed in a more recent study by Sigurgeirsson et al. (2018) where mathematics lessons stood out among other subjects taught in upper secondary schools for its lack of diversity in the teaching methods used. Notably, discussions were never practised in mathematics classes visited during the time of study (Sigurgeirsson et al., 2018). Also, according to a study that explored upper secondary school teachers' pedagogic practice in terms of student autonomy, mathematics teachers were the only subject teachers to report on fixed pedagogic practices where students had little or no possibility to influence their learning and they were also the only teachers who did not participate in interdisciplinary collaboration (V. S. Bjarnadóttir, 2018).

Given these conditions, junior teachers might like to implement changes in the teaching practice. In Iceland, however, a study by Eiríksdóttir and Jóhannesson (2016) revealed that if school policy did not support changes in the teaching practice, senior faculty in the group of mathematics teachers determined whether changes would be implemented or not. In other words, senior faculty sometimes stood in the way, limiting possibilities for change and causing frustration for junior teachers. This aligns with findings from other international studies (Thurlings et al., 2015).

1.5.3. Why in Iceland?

Silent video tasks were originally designed to be used across cultures and languages as section 1.1 on p. 1 describes. For this research project, I decided to work with teachers in Iceland. It is interesting that in a school system where no compulsory standardised examination is implemented at the end of upper secondary school and where individual upper secondary schools have the possibility to be independent in designing their school curriculum in mathematics, teaching practice still seems to be mostly limited to expository methods and drill exercises. By introducing a silent video task in mathematics classrooms with rather teacher-oriented instruction and where discussion seldom or never finds place, I was curious to see if something similar to my teaching experiment experience from 2014 would happen. Also, the following facts played a role in my decision: i) I finance my doctoral studies mostly by a grant that was made available as a reaction to the evaluation report of mathematics teaching in Icelandic upper secondary schools by Jónsdóttir et al. (2014), ii) I have better access to teachers in Iceland than elsewhere, and iii) I know the Icelandic school system from my own experience as a learner and as a teacher.

One of the results from the international comparative study PISA 2015 with 65 participating countries was that inquiry-based rather than teacher-directed instruction practices supported a successful school learning environment (OECD, 2016). When 12 European countries were compared in the PRIMAS project (http://primas-project.eu) regarding the use of inquiry-based teaching methods in mathematics classrooms; Germany, the Netherlands and UK had the highest proportion of teachers using teacher-oriented teaching methods whereas Eastern European countries like Hungary, Romania and Slovakia rather used student-oriented teaching methods (Engeln et al., 2013). Teaching methods at upper secondary school level in Iceland are rather teacher-directed like in Germany, UK, and the Netherlands. In Icelandic compulsory schools (primary and lower secondary level), the teaching methods have partly developed towards inquiry-based methods. This happened with influence from researcher-teacher collaborations where teacher educators support mathematics teachers dealing with growing diversity in mainstream schools or use lesson study to create communities of practice (Guðjónsdóttir et al., 2010; J. V. Kristinsdóttir, 2016; Pálsdóttir & Gunnarsdóttir, 2012). It remains to be shown whether such researcher-teacher collaborations will be formed at upper secondary level also.

1.6. Outline of the thesis

This thesis is organized into six chapters, starting with an introduction to the background, aims, originality and context of the research presented in this thesis. Chapter 2 begins by acknowledging that a comprehensive review on task design research in mathematics education had recently been published at the start of my doctoral studies. It continues to give a short account of theories that informed the design and instructional sequence of silent video tasks, influenced the research design, and were used for data analysis. Chapter 3 explains why a design-based approach was chosen for this research project, describes the research design, how participants were selected, what data was collected and why, and addresses ethical aspects of the research project. Chapter 4 summarizes what each of the four reviewed papers I–IV, that this thesis is based on, was about and what their findings were. Chapter 5 offers a discussion of the research results in light of the eight research questions Q1–Q8. Chapter 6 concludes the thesis by discussing how this research has contributed to the bigger aims of the thesis and provides a reflection on what has changed for me as a researcher in the process of leading this research project.

2. Conceptual and Theoretical Framework

On the research journey which the project presented in this thesis has taken me, I certainly have felt like a handywoman or *bricoleuse* (e.g. Gravemeijer, 1994), inventing pragmatic solutions in practical situations, trying out different tools and research frameworks, many of whom *worked* but maybe did not give me the information that I was seeking after, so I continued the search. I do not necessarily view it in such a way that I have found *the* frameworks, but rather that I found *some* frameworks that helped in the tinkering, modifying the work until it got as it is presented in this thesis.

As Cobb et al. (2003) pointed out, theories play a central role to inform prospective design. In this chapter I will introduce the underlying theories and existing empirical results that influenced the design of silent video tasks, the development of their instructional sequence and the data analysis. Furthermore, this chapter defines what I mean by a task and an instructional sequence.

The chapter starts with some underlying constructivist theories and a short introduction to *task design*. It continues with the *Building Thinking Classrooms* framework, which informed the task design, and the introduction of the concept of *classroom norms*, which was helpful when analysing data. After a long search of frameworks for data analysis, I decided to analyse the first bulk of interview data using *teacher noticing* frameworks. Also, the definition of *formative assessment* informed data analysis and empirical results regarding strategies used for formative assessment practices influenced the instructional sequence development. Finally, the *Teaching for Robust Understanding* framework was used as a lens in the process of analysing the second bulk of interview data.

2.1. Constructivism and encouraging mathematical discourse

To create a constructivist environment in their classrooms, teachers are required to reach out to listen to, acknowledge and value students' perspectives. By asking questions such as "Explain your thinking, please?" and stimulating discussion, teachers create opportunities to listen to students and to make them heard in the classroom. In such an environment, teachers care about the next steps after students answer more than whether their answer was correct or not. This is not easy for teachers because they are kept on their toes, eliciting student feedback and altering instruction as needed, but it can certainly lead to more interesting lessons than presenting mathematics as a set of procedures to be learnt in order to pass a test (Battista, 1999; Schunk, 2011).

Opportunities for learning arise when the unpacking of mathematics becomes a shared responsibility via discussion and any cases of disturbances (e.g. running into some obstacles) are a shared experience that might trigger development (Mason, 2002). For example, it might involve *conceptual reorganisation*, a negotiation process between students and teachers including 1) an exposing event where students explore how they use conceptions, 2) a discrepant event which serves to create a cognitive conflict, and 3) a resolution where students make a conceptual shift (Romberg, 1993).

Leung and Bolite-Frant (2015) expressed a need for the design of tasks that encourage discourses for mathematical knowledge mediated by tools in the classroom. Any discourse is placed within a social, historic, and cultural context. When explaining or describing to others what they have seen, students tend to point to objects they want to refer to in speech, avoiding or ignoring the mathematical concepts and using general everyday language (Pimm, 1989). Without the need to express ourselves precisely, e.g. to make sure that others understand what we mean, we do not need to rethink the way in which we say something.

A dualistic view of the everyday world and the mathematical world is not uncommon. Normally, a child can grow up unaware of the mathematics at play around it and do everything it needs to do without using mathematics. Why should it "mathematise", as Freudenthal called it (Freudenthal, 1973, p. 49), and move on toward mathematical ideas? At the *13th International Congress on Mathematics Education* in Hamburg in a plenary for *ICME-13* Early Career Researcher Day, Anna Sfard conceptualized mathematical objects as discursive objects. She described how the idea of *objectification* might help us to understand why so many people do not understand mathematics:

With teachers juggling mathematical objects, students see them throwing things around, but they do not grasp what they are throwing about. It is a constructivist idea to make the students be the jugglers and the teachers be supporters of students' juggling. Because in mathematics our students need support to learn how to objectify. Without objectification they will stop at a certain point and cannot go any further. To assist learners on this path could thus be seen as a fundamental task for teachers (Sfard, 2016).

This story describes a shift from *acquisitionist* to *participationist* perspectives, moving from learning as the act of transferring knowledge (private possession) to learning as being part of a team, participating in a dialogue that takes place in context (Sfard, 1998). Sfard developed a theory on *commognition*, which suggests that we can think of *cognition as a process of communication* (Sfard, 2008). It can either be communication within the individual (intercommunication) or between individuals (social communication) although both are shaped by the culture and society where they take place.

For example, the task to create a narrative from a visual mediator like a video might first spark an inner dialogue that later develops into discussion with others. The narrative might reveal how students interpret relations between objects and processes with objects that are shown dynamically in the video. Thus it might give teachers and students insights into students' conceptual understanding and make it possible to refer to and refine the vocabulary used. Not with an end goal in mind where everyone ends up with the same concept definition, but with a constant flow of taking part in mind where learners become members of the classroom community. Sfard (2008) also describes a *commognitive conflict*, which is not the same as the above mentioned (see the second paragraph of this section) cognitive conflict. To illustrate the difference, she describes how the commognitive conflict acts between two conflicting discourses whereas the cognitive conflict happens between the speaker and the world. Further, the role of a commognitive conflict in learning is that it is crucial for learning at the level of discourse about discourse (Sfard calls this *metalevel* learning) whereas a conceptual conflict creates an opportunity for resolving a situation involving misconceptions (by possibly removing the misconception). The commognitive conflict is solved by students' acceptance and by rationalizing the discursive ways of an expert speaker whereas the cognitive conflict is resolved by students' rational effort (Sfard, 2008, pp. 254-258).

Essentially, when an approach to teaching and learning involves tasks that encourage learners to take on an active role in their learning and participate in the classroom discourse, it is a constructivist approach. Knowledge is not passively received by learners when they are asked to interpret what they see in a mathematical visualisation. On the contrary, learners' explanations, descriptions or narratives can be viewed as a result of their mental images.

By sharing their thoughts with a group of students, students add various meanings to the visualisation. Via discussion, the teacher might support learners to build shared understandings and to get a feeling of belonging. In this way learners construct meaning on the basis of their own experience (von Glasersfeld, 1995) but at the same time learners come to know through engaging in a practical, object-oriented activity and in social interaction with others (Wertsch, 1981).

According to von Glasersfeld (1992, p. 443),

> it is one of the primary duties of the teacher to create an atmosphere in the classroom that not only allows but is also conducive to conversation, both between student and teacher and among students.

As Confrey (1993, p. 306) points out, teachers cannot approach this duty by simply encouraging more talk or asking more questions. Teachers who previously put no emphasis on student discussion or listening to students' mathematical ideas can get quite surprised when they open their ears to what students say. They will need practice and support to recognize and make effective use of students' ideas in their classroom.

The prompts to facilitate discussion should be of the *think and wonder* form, which can be an additional challenge for a teacher who is transitioning from factual questions and is maybe afraid of students' expressing "wrong ideas". It is, however, important to challenge teachers to prevent the kind of silencing that Confrey (1993) describes as "situations in which teachers have overlooked opportunities to explore rich student ideas" or where teachers "unintentionally distort students' statements to fit with their own understanding of the content" (Confrey, 1993, p. 306–307). After all, we want participants in the discussion to feel that they are heard, that their ideas are valued, and that it is worthwhile to engage in the discussion–we want them to feel that they belong.

Confrey (1993) notes that the teachers' view of knowledge can critically influence the course of events. Even though teachers are aware of the value of their students' input, and mean to do well, they still might silence students, because they are not prepared to cultivate opportunities that come up when students offer insights into their own thinking. By offering learners opportunities to discuss their mathematical ideas, teachers acknowledge the learners' perspective and thus might be able to understand issues that learners run into when dealing with certain mathematical topics. Also, when mathematics becomes an issue of communication and interpretation, then learners' own work and ideas get a more important role (receive status); the students' voice can be heard (Confrey, 1993).

Underlying here is the requirement that teachers pay attention to and understand their students' mathematical thinking in order to orchestrate a mathematical discussion. Pirie and Schwarzenberger (1988) define mathematical discussion to be purposeful talk on a mathematical subject in which there are genuine pupil contributions and interaction. This might sound simple, but supporting learners in making sense of and respecting each other's explanation, description or narration is a complicated task. This is made clear in a review by Walshaw and Anthony (2008) on research in mathematics classrooms dealing with the teachers' role in classroom discourse.

Fortunately, researchers were not all discouraged by results regarding the complexity of orchestrating productive mathematics discussions. Stein et al. (2008) conducted empirical research aiming to find ways that might support teachers' facilitation of mathematical discussions based on students work on cognitively demanding tasks. Their findings are summarized in the following five instructional practices:

1. anticipating likely student responses to cognitively demanding mathematical tasks,

2. monitoring students' responses to the tasks during the explore phase,

3. selecting particular students to present their mathematical responses during the discuss-and-summarize phase,

4. purposefully sequencing the student responses that will be displayed, and

5. helping the class make mathematical connections between different students' responses and between students' responses and the key mathematical ideas (Stein et al., 2008, p. 321).

In other words, teachers should not only try out solving the problems themselves before assigning them to students, but also try to visualize different possibilities of how to respond to the task, and in order to make the step of selecting and sequencing student responses for discussion easier, they should walk around the classroom and listen to their students preparing their task responses.

These principles have been introduced in Iceland in various professional development courses for pre-service and in-service teachers on the lower secondary level. In recent research, Kooloos et al. (2020) built a community of practice (Wenger, 1998) among Dutch teachers who were inexperienced in leading group discussions and they developed their practices into effectively using the five instructional practices by Stein et al. (2008) within the course of only one year.

It should not be belittled how demanding it is for teachers to work in this way and lead group discussions in a fruitful direction based on their knowledge of students' thinking about the mathematical content. According to Kooloos (personal communication, December 2020) what remains is to find ways in which such changes in teacher practice can be made longer lasting and draw in more teachers to make the impact sustainable.

2.2. Task design

In an introduction to the proceedings of ICMI study group 22, Watson et al. (2013, p. 12) define a *task* or a *task sequence* to be "anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something . . . also anything that students decide to do for themselves in a particular situation". This definition is based on discussions with 80 international scholars who gathered to share experiences and by comparing and summarizing these, they hoped to create a baseline and a springboard for future research and practice.

To be more precise, Watson and Mason (2007) describe that a *task* in the full sense

> includes the activity which results from learners embarking on a task, including how they alter the task in order to make sense of it, the ways in which the teacher directs and redirects learner attention to aspects arising, and how learners are encouraged to reflect or otherwise learn from the experience of engaging in the activity initiated by the task. Reflection, discussion, individual and group work, time to ponder and the use of resources such as ICT [information and communication technologies] or other apparatus are integral to the ways of working on tasks (Watson & Mason, 2007).

That is also the way in which a task is viewed in this research project. Furthermore, in this thesis the term *instructional sequence* is used to describe the process by which a teacher can implement a silent video task. It involves the task assignment, organization of students' work on the task and a whole group discussion. The term *learning sequence* is used to describe every act of teaching and learning that happens within the time frame of focus on a certain mathematical topic, e.g. the intended focus on the coordinate system and linear functions stretching over some weeks within a course schedule.

Paying attention to teachers' and students' work on tasks can inform us about many aspects of teaching and learning. That is also the reason why Sierpinska (2004) considers the design and empirical research on mathematical tasks as one of the most important responsibilities of mathematics education. Some tasks are designed for teaching and other for research. Either way, tasks can be used as research tools, e.g. when they are used to consolidate knowledge or evaluate students' understanding. Indirectly, assessment tasks probe the effectiveness of the teaching approaches used (Sierpinska, 2004). Tasks can also be designed to point students' attention in certain directions. As Mason (2002) notes, *what* learners attend to and *how* they attend to it defines what learners are able to make sense of and in the end what they internalize.

From their review on task design research that involves teachers as partners, Jones and Pepin (2016) conclude that i) task design is complex and many variables must be defined (what is designed, which tools are necessary, under what conditions), ii) digital task resources offer particular affordances and constraints, and iii) teachers have an important role to play (Jones & Pepin, 2016, p. 115). Teachers tend to adapt tasks to their needs and preferred ways of working. Without their input in the task design process, such important aspects of practicality are more likely to be neglected. Jones and Pepin (2016) also point out that teachers often benefit from being partners in task design in terms of their professional learning.

2.3. Building Thinking Classrooms

Learners do not necessarily come to the classroom with the same intention as teachers expect of them. For example giving an effort just above the minimum requirements to pass the course is not likely to be a teachers' goal for her learners. John Holt describes how in traditional school settings that "run on right answers", the so called *producers* get encouraged in the school system whilst *thinkers* get depressed (Holt, 1982).

It is important to avoid dialogues degenerating into a guessing game of the kind "What does the teacher want me to say?" as often happens in classrooms where the teacher is viewed as the holder and evaluator of knowledge. On the contrary, everyone's voice is important and this can be emphasized by setting focus on the way in which each learner interprets and responds to a task.

Furthermore, by engaging students in discussion based on their responses it becomes visible that it is expected of learners to think for themselves and in collaboration with peers. Working in such a way in the mathematics classroom requires a certain classroom culture that Peter Liljedahl calls a *thinking classroom*: a classroom where students are expected to think and given opportunities to think, think individually or collectively, constructing knowledge and learning to understand together via activities and discussion (Liljedahl, 2016).

For over a decade, via empirical research in mathematics classrooms, Liljedahl along with his collaborating teachers has tried out a spectrum of different teaching practices in hundreds of classrooms. He collected the practices which, based on results from his research, would build and support a *thinking classroom*. Altogether, teachers are invited to implement 14 practices that are organized in four toolboxes–toolbox by toolbox–in their own classrooms. It depends on each teachers' will and ability (in terms of outer conditions) how many of the practices will be implemented, but their implementation follows a certain time schedule as can be seen in figure 2.1.

Liljedahl's work arises from his collaboration with a teacher who asked for support when starting problem-solving practices in her teaching. Despite best intentions, it remained hard for her to implement problem-solving tasks in her classroom and she eventually gave up. Based on observations and reflections, Liljedahl noticed that seemingly students in the classroom were not expected to think and he set out to do empirical research with many teachers, aiming to define what supported a thinking classroom (Liljedahl, 2020).

Although silent video tasks are different from problem solving tasks, I could relate to the teachers' struggles and intentions as I read a chapter (see Liljedahl, 2016) on Building Thinking Classrooms. I had an aha! moment realizing that what I experienced when my students worked on the silent video task was a thinking classroom.

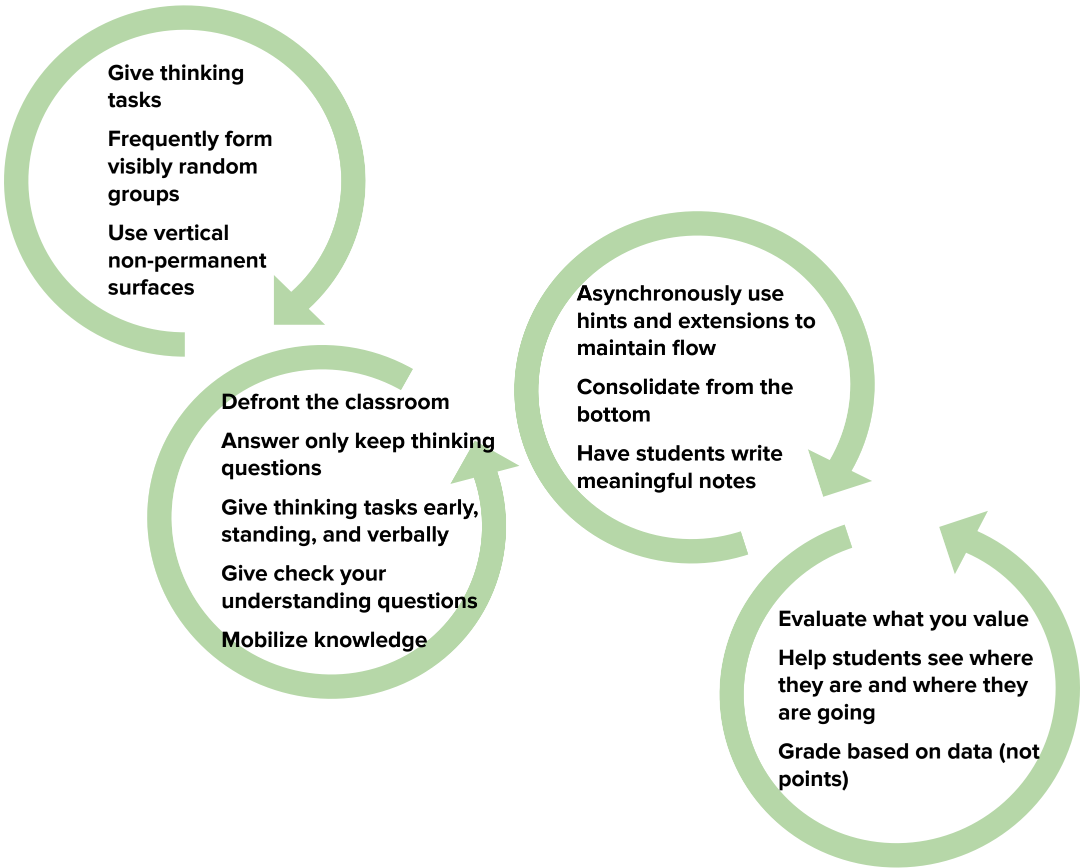

Figure 2.1. The fourteen Building Thinking Classrooms teaching practices, organized into four toolboxes. In the first box (top left), all three practices must be implemented simultaneously. The practices' order of appearance within the second toolbox (second left) is irrelevant, they can be implemented one at a time or concurrently. In the third toolbox (second right), each of the practices is best implemented one at a time in the order of appearance, establishing each one before starting the next. In the fourth toolbox (bottom right), the last suggested practice on grading based on data should not be implemented until the formative assessment practices have been established. (Liljedahl, 2020, pp. 282–287).

From the 14 practices collected by Liljedahl I will refer mainly to the following:

- *Assign students visibly randomly into groups*. By assigning students into groups such that they clearly see that it is done in a random way, results indicate that:
  - students rather agree to work in any group they get assigned,
  - social barriers within the classroom get eliminated,
  - the mobility of knowledge between students increases,
  - students rely less on the teacher for answers,
  - students rather rely on answers constructed in collaboration with peers in their group or other groups, and
  - students' enthusiasm and engagement in mathematical tasks increases (Liljedahl, 2014).
- *Give short oral instructions*. This is a preferred way in which tasks are given to students due to results indicating that
  - rather than decoding instructions from paper,
  - students will discuss their interpretation of the task among themselves, within their groups (Liljedahl, 2016).
- *Answer only keep-thinking questions*. All questions must be acknowledged. Students asked three types of questions, only the last type of which should be answered:
  - proximity questions, asked by students only because the teacher is close-by, often for the sake of acting like they are working,
  - stop-thinking questions, asked by students only because they find it hard to think for themselves (e.g. if something "is right"), and
  - keep-thinking questions, asked for clarification or to get approval of extensions that students would like to try out (Liljedahl, 2020).
- *Build student autonomy*. Student autonomy is the ability to take charge of one's own learning. For the teacher it relates to the delicate task of *keeping students in flow*, challenging them such that they do not get bored and yet not so much that they give up (Csikszentmihalyi, 2014). Instead of students being dependent upon the teacher, e.g. for getting hints and extensions for tasks that they are working on, teachers need to

  - show students that autonomy exists, by pointing students toward other groups when they are stuck or need extensions, and
  - make students feel the value of autonomy, e.g. by feeling how rewarding it is to have figured things out without any help from the teacher (Liljedahl, 2018)
- *Notes to my future forgetful self*. Such notes are more meaningful than mindless copy-from-teacher notes. Students will not write notes on
  - something that they do not find important or interesting,
  - something that they know that they can find elsewhere (e.g. in the textbook),
  - things that they think they will remember.
- *Formative assessment*. Rather than focusing on products, assessment in a thinking classroom needs to communicate to students where they are and where they are going in their learning. This can be done by
  - gathering information for the purposes of informing teaching and learning,
  - using navigation instruments (rubrics) where students self-evaluate their performance on a quiz, review test or set of check-your-understanding questions based on data–which is different from self-evaluating based on opinions (Liljedahl, 2020).

Several of these changes in teacher practice involve a change in classroom social norms (Yackel & Cobb, 1996). For example, if a teacher falls into the trap of doing always more for students, it is more likely that students will expect their teacher to do things for them (Mason, 2002). When students expect that something will take great effort and time and foresee themselves doing it rather slowly and inefficiently then it is of course much more convenient to get someone else to think and do the task, but in such an environment student autonomy is not nurtured. Changes in practices suggested in the list above are intended to support mathematics classroom atmosphere that welcomes and expects thinking to take place–to create a new norm. The next section introduces the concept of classroom norms briefly.

2.4. Classroom social norms and socio-mathematical norms

In general, classroom social norms describe what teachers and students expect from each other in terms of participation in the classroom (Yackel & Cobb, 1996). They define what counts as acceptable behaviour and are influenced not only by teacher and students but also by culture and what has been experienced earlier (constructed by the community). For example, students who have come to understand that their teacher only values "correct" answers, will be more reluctant to answer teachers' questions than students who have experienced that their answers do not get evaluated but rather are taken as an important input into the classroom discussion. Like Sfard (2000) points out, classroom social norms can affect students' participation in the classroom discourse. The norms can be modified, usually when students start to work with a new teacher or at the start of a semester (Sfard, 2000).

Yackel and Cobb (1996) extended their work on classroom social norms by defining *socio-mathematical norms*, which are more specific to mathematics as a subject and highly influence students' learning of mathematics and what they think of as being mathematical. For example, what counts as *enough* when answering a mathematical question (e.g. explaining ones solution and ones ways of thinking) is a social norm, but what is understood to count as acceptable mathematical explanation is a socio-mathematical norm. Changes in the socio-mathematical norms can create new learning opportunities, especially if these are in the direction of supporting student autonomy (encouraging and facilitating student participation and contribution), and supporting students in making sense of and respecting each other's responses to a task (comparing, finding similarities) (Yackel & Cobb, 1996). For example, Sfard et al. (1998) found that the practice of supporting students in explaining their task interpretations affected the socio-mathematical norm of what counts as acceptable such that within a few weeks most students gave conceptual explanations when needed and asked others clarifying questions that concerned their underlying task interpretations (Sfard et al., 1998).

2.5. Teacher noticing

Like mentioned at the start of this chapter, the search for frameworks to analyse data was not without its challenges. By looking at what the data was telling me, with the words of John Mason in mind, that "What is noticed, marked, or recorded is necessarily being attended to" (Mason, 2011), it seemed important and of interest to attend to in what ways teachers reacted to students' responses to the silent video task. Teachers of course might notice more things than they verbally express. Still, a limited view based on interview data might provide some glimpse into what teachers noticed.

Researchers have pointed their attention to teacher noticing with the aim to understand how mathematics teachers make sense of the complex setting of their classrooms. This is based on the presumption that since teachers cannot pay attention to everything, what they seem to point their attention to must tell something about their practices. This is extensively discussed by Mason (2002) with focus on *intentional* noticing, i.e. noticing which is specific to the profession of teaching and involves both what is noticed and a *reflection* of what was noticed, usually with the aim to improve practice (Mason, 2002). It can be described as a continuous process of professional development and might even happen outside the classroom when reading a book, similarly to what happened when I first read about Building Thinking Classrooms.

Mason (2002) distinguishes between ordinary noticing (perceiving), marking and recording. Ordinary noticing is easily lost from memory later on unless one is reminded by a similar situation, whereas when marking something, we assign importance to what we noticed, such that it can be reconstructed in our mind as we reflect on it and therefore more likely to be accessible to us later on. Recording takes yet more motivation and energy since it not only includes mental note taking but literally requires one to write down and thus record what was noticed (Mason, 2002).

By interviewing teachers, incidentally, I had asked teachers to reflect on–at least a part of–what they had marked. It was not a measurement of success, because noticing offers no criteria to measure whether some action in the classroom has had a certain effect or not (Mason, 2002). It does, however, inform us about decisions that teachers make–decisions based on their knowledge and resources, goals and orientations (Schoenfeld, 2011).

In the turbulent time of searching for frameworks to use for the analysis of data, I accidentally ran into Charlotte Wolff, a new colleague of mine at the the University of Iceland School of Education. After a short exchange in the library hall introducing ourselves and what we were working on, Charlotte realized that our research areas partly overlapped. We stayed in touch and when working on the first version of paper II, Charlotte introduced me to a teacher noticing framework by van Es and Sherin (2008) on teachers learning to notice–a framework that seemed well-suited for analysing my interview data.

This teacher noticing framework that Charlotte introduced me to, is made up of three parts where a teacher

1. identifies a significant event (some noteworthy aspect) in a teaching situation,
2. uses knowledge about the context to reason about the situation, and
3. makes connections between these specific events and broader principles of teaching and learning (van Es & Sherin, 2008).

It was intended for use in instructional or professional development settings where teachers in a video club watched and discussed video clips from their own classrooms (van Es & Sherin, 2008). Based on frameworks by Mason (2002), van Es and Sherin (2008) and others, Jacobs et al. (2010) developed a framework on professional noticing of children's mathematical thinking where teacher noticing is applied to settings where teachers observe and interpret students' responses to mathematical tasks. Teachers interpreting students' responses has also been studied by Morgan and Watson (2002) in terms of equity and Crespo (2000) focusing on paying attention to students' thinking.

I considered using the framework by Morgan and Watson (2002) because it addresses some factors that might influence teachers assessment practices. These factors are:

1. Teachers' professional knowledge of mathematics and the curriculum, including affective aspects of their personal mathematics history,
2. Teachers' beliefs about the nature of mathematics, and how these relate to assessment,
3. Teachers' expectations about how mathematical knowledge can be communicated,
4. Teachers' experience and expectations of students and classrooms in general,
5. Teachers' experience, impressions, and expectations of individual students, and
6. Teachers' linguistic skills and cultural background (Morgan & Watson, 2002).

Although I noticed some of these factors at play in my data (see section 5.1.2), I was not sure if they would assist in answering my research questions. In the first version of paper II, I used the framework by van Es and Sherin (2008), but I was not explicit enough about why and how I was using it. In the extensive review process which followed, I read further literature on teacher noticing. In this process I decided to refer briefly to the notion by Scheiner et al. (2019) on a teacher moving toward making the unpacking of mathematics become a shared responsibility (Scheiner et al., 2019). Also, I found out that based on their previous work, van Es (2011) had continued to develop another framework with focus on teachers learning to notice students' mathematical thinking. This framework describes four levels of what and how teachers notice:

1. *baseline* where teachers attend to whole class environment, behaviour and learning, and to teacher pedagogy, form general impressions of what occurred, and provide descriptive and evaluative comments, usually without evidence to support analysis,
2. *mixed* where teachers primarily attend to teacher pedagogy but do begin to attend to particular students' mathematical thinking, form primarily evaluative

comments but include some interpretive comments referring to specific events and interactions as evidence,

3. *focused* where teachers attend to particular students' mathematical thinking, highlight noteworthy events and provide interpretative comments referring to specific events and interactions, elaborating on these events and interactions

4. *extended* where teachers relate particular students' mathematical thinking to teaching strategies, highlight noteworthy events, provide interpretative comments, not only referring to specific events and interactions as evidence but also making connections between events and principles of teaching and learning (van Es, 2011).

Again, in the second round of review process of paper II, I was asked to clarify why and in what ways I used teacher noticing. Based on my further discussion with Charlotte, I decided to focus on the frameworks of teacher noticing by Mason (2002) and van Es (2011) and van Es and Sherin (2008). They aligned well with the design-based research methods that I was using and helped to investigate the influence of teachers' knowledge when using silent video practices (innovative practices) in their teaching. We rephrased the research questions and I partly re-analysed the data, refined other parts of the data analysis, and restructured the paper. In this process, I defined a framework variation for the purpose of analysing data from interviews where teachers reflected on their experience of using a task that has the potential to reveal some of students' mathematical thinking, listened to students' responses to the task and commented on what they noticed. This variation consists of three levels where:

1. teachers provide descriptive and evaluative comments,

2. teachers provide some interpretative comments,

3. teachers connect students mathematical thinking to principles of teaching and learning.

Paper II on page 109 presents analysis based on this framework.

2.6. Formative assessment

In section 2.3, formative assessment appeared in the fourth toolbox of the Building Thinking Classrooms framework. It also comprises the fifth dimension of the Teaching for Robust Understanding framework that will be introduced in the next section 2.7. According to Suurtamm et al. (2016), we should view it as the purpose of an assessment to improve student learning of mathematics. I agree with that, and it also fits well with Wiliam's definition of formative assessment:

> An assessment functions formatively to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have made in the absence of that evidence.

(Wiliam, 2011, p. 43).

In other words, assessment is formative when *what students do* influences *what happens next*, e.g. the teacher might modify her/his teaching to meet learners' needs. The feedback gives students information regarding *where they are at, where they should go next, and what is needed to get there*. This is quite different from summative assessment, which is meant to certify or confirm certain achievements. In a summative assessment final examination situation, students might even receive a grade (or pass/fail) without being able to view or learn from their exam solutions.

It can be hard to use formative assessment practices. Mason (2002) notes that teachers' knowledge can get in the way, especially if the teacher is used to sharing his knowledge and to receiving positive feedback thereto. The trouble lies in how hard it is not to give direct step-by-step instructions on how students could proceed from *where they are* to *where they are heading*, i.e. how hard it is to refrain from giving hints that decrease challenge (Mason, 2002, p. 206).

Wiliam and Thompson (2008) list the following five key strategies for formative assessment:

1. Clarifying and sharing learning intentions and criteria for success,

2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding,

3. Providing feedback that moves learners forward,

4. Activating students as instructional resources for one another, and

5. Activating students as the owners of their own learning (Wiliam & Thompson, 2008, p. 67).

Tasks that give students an opportunity to explain to others and/or to receive explanations from their classmates make direct use of strategies 4 and 5 in the list above. Based on students' input to the discussion, teachers could make decisions about the next steps in instruction, provide feedback, and students might reflect on their own understanding. Such actions can support students' learning of mathematics.

Building on the base of Wiliam and Thompson (2008)'s key strategies for formative assessment, Wright et al. (2018) list six potentials that the use of technology in formative assessment has for supporting learning:

1. Provide immediate feedback,
2. Encourage discussion
3. Provide a meaningful way to represent problems and misunderstandings,
4. Give opportunities to use preferred strategies in new ways,
5. Help raising issues that were previously not transparent for teachers, and
6. Provide different outcomes feedback (Wright et al., 2018, p. 219).

These six potentials were used in the analysis of data from the first implementation phase of the research project presented in this thesis. It also influenced the second implementation phase (see paper III on p. 131).

2.7. Teaching for Robust Understanding in Mathematics

At the previously mentioned ICME-13 Early Career Researcher Day, Alan Schoenfeld (2019) briefly introduced the *Teaching for Robust Understanding* of Mathematics framework (TRU framework) in a plenary lecture[1]. The TRU framework is built on empirical evidence of meaningful learning occurring in interaction with others. Instead of sharing best practices, the framework acknowledges that several different ways of working can be used to make the classroom supportive of students' mathematical learning (Baldinger et al., 2018).

Thus, the TRU framework is not a toolbox. It is arranged into five partly overlapping dimensions that describe classroom characteristics identified by research as critical when it comes to fostering students' understanding when teaching mathematics. These five dimensions are:

i. *Mathematics*, the richness of the mathematical content,

[1] Although I was not completely aware of it back then, it is hard to stretch how influential ICME-13 was. Luckily for me, it was the first time that an Early Career Researcher Day was organized at an ICME conference. Initiated by Gabriele Kaiser and Norma Presmeg, and organized by Armin Jentsch and Thorsten Scheiner with workshops and plenary talks similarly scheduled as a summer school, it gave an excellent start for a "theory-clueless" doctoral student like me.

ii. *Cognitive demand*, an opportunity for students to engage in productive struggle,

iii. *Equitable access to content*, all students participate and are involved in meaningful ways,

iv. *Agency, ownership, and identity*, opportunities for students to develop a sense of agency, i.e. to make mathematics their own, and to develop productive mathematical identities as thinkers and learners, and

v. *Formative assessment*, the degree to which students' ideas and interpretations are made public and responded to in productive ways (Schoenfeld, 2018).

Each dimension is meant to highlight different aspects of the same practices and experiences made in the classroom. To clarify further what is meant by each dimension, questions that can be seen in figure 2.2 are provided (Baldinger et al., 2018; Schoenfeld, 2018).

The Five Dimensions of Mathematically Powerful Classrooms	
The Mathematics	How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections?
Cognitive Demand	What opportunities do students have to make their own sense of mathematical ideas? To work through authentic challenges? How can we create more opportunities?
Equitable Access to Content	Who does and does not participate in the mathematical work of the class, and how? How can we create more opportunities for each student to participate meaningfully?
Agency, Ownership, and Identity	What opportunities do students have to see themselves and each other as powerful mathematical thinkers? How can we create more of these opportunities?
Formative Assessment	What do we know about each student's current mathematical thinking? How can we build on it?

Figure 2.2. A summary in the form of questions asked within each of five dimensions of mathematically powerful classrooms according to the Teaching for Robust Understanding (TRU) framework (Baldinger et al., 2018, p. 2).

Used mostly for professional development, it was not obvious that the TRU framework might be useful for task design. The reason why it proved to be useful when developing silent video tasks, was that it puts emphasis on conversations between teachers and students and on an ongoing individual reflection thereof, which eventually can foster awareness of and learning from experience. The intentions of silent video tasks seemed to align well with the five dimensions of the TRU framework

By viewing data based on teachers' experiences of silent video tasks and researcher observations in classrooms using the tasks, I hoped that using the TRU framework as a lens might help to identify opportunities and challenges that silent video tasks bring to the mathematics classroom. As such, it offered criteria that proved to be helpful when developing a task that was meant for supporting students' understanding of mathematics. Special attention was directed toward the fifth dimension on formative assessment, connecting with key strategies that were introduced in section 2.6.

2.8. Summary

This chapter gave an account of what is meant by a task, task design, instructional sequence, and learning sequence in this thesis. It provided a brief definition for concepts that will be referred to and an overview of the mathematics education theories that informed task design and/or informed data analysis. To analyse data, a local framework for situations where teachers reflect on students' task responses that give insight into students' mathematical thinking was introduced in section 2.5. The way in which the TRU framework, which was introduced in section 2.7, was used as a lens to analyse data will be discussed further in the next chapter. The next chapter starts by describing the design-based research approach that was used in this research project.

3. Methods and Methodology

This chapter explains why a design-based approach was chosen for this research project, how participants were selected, what data was collected and why, and addresses the project's ethical aspects. The silent video tasks used in the study are also described in this chapter.

3.1. Design-based research

To describe silent video tasks' instructional sequence more generally and to develop it in connection with existing theories in mathematics education, it seemed straightforward and of interest to stay close to practice and collect data in real classrooms, collaborating with other teachers with various beliefs and views on mathematics teaching and learning. It was therefore in accordance with the research aims to use a qualitative approach and to do design-based research. In other words, it was a pragmatic decision: since what I was interested in seeing happening in the classroom was and is not already happening there, an intervention was needed to make that change possible.

Design-based research is "a formative approach to research, in which a product or process (or 'tool') is envisaged, designed, developed, and refined through cycles of enactment, observation, analysis, and redesign, with systematic feedback from end users" (Swan, 2014) and the main goal for a design experiment is to improve the initial design (Cobb et al., 2003). In 2014, the idea of silent video tasks had been draft designed and handed out to teachers to "see what happens". However, design-based research must be driven by theory (Swan, 2014) and the theory not only guides practice, but is also seen to emerge from practice, i.e. the two exist in a reflexive relationship (Cobb, 2000).

This means that design-based research is intended not only to design and test interventions, but also to contribute to our understanding of the relationship between theories, designed artefacts and practices, and even contribute to new theories that can impact learning and teaching (Bakker, 2018; Barab & Squire, 2004). Therefore, it was a logical next step to strengthen the link to theory, to describe and explain - in light of theories and practice - how silent video tasks can be used for teaching and learning in mathematics classrooms at upper secondary school level.

3.2. Participants

This section describes how participants for the first and second implementation phase of the research project were selected randomly and purposefully, respectively.

For the first phase–out of the thirty upper secondary schools in Iceland that prepare their students for university studies–I randomly selected ten to contact for participation in the first implementation phase. The selection included schools of different sizes, types, and locations. Five schools agreed to participate, four in the capital area and one in the countryside. Interestingly, three of the five schools used a class based system, but the majority of upper secondary schools in Iceland uses a modular system. I therefore decided to randomly select more schools of the modular type until I found one that was ready to participate. When the school year started, one of the schools withdrew their participation because not enough students had signed up for the planned course about trigonometric functions. A teacher in another school quit participation after the first interview due to time constraints. In the end four teachers in four schools participated in the first implementation phase.

Based on findings from the first phase, participants were purposefully selected in the second phase to have experience with the use of formative assessment (see p. 26), the use of DGS in their mathematics classroom, and some use of group discussion. Therefore, I contacted schools that are known for using formative assessment and DGS. Three teachers in two schools accepted participation. The four (first phase) and three (second phase) participating teachers are listed in table 3.1 on p. 32 with pseudonyms used for both teachers and schools. Information about teaching experience was rounded to 5 years when possible (I did not round 2 years experience down to zero).

Table 3.1. Four teachers participated in the first implementation phase (listed above the line) and three teachers participated in the second implementation phase (listed below the line) of the research project. Both teachers and schools were given pseudonyms. Teaching experience was rounded to 5 years unless it would have rounded to zero. The school size refers to small (S) with less than 500 students, medium (M) with up to 1000 students, and large (L) with more than 1000 students.

Teacher	Gender	Teaching experience	School	School size
Gauti	m	10	Common Raven	M
Lilja	f	15	Whooper Swan	L
Magni	m	5	Arctic Tern	L
Snorri	m	40	Rock Ptarmigan	M
Andri	m	10	Mallard	L
Edda	f	20	Mallard	L
Orri	m	2	Blackbird	S

In the first implementation phase, Gauti, Lilja, Magni, and Snorri were asked to use a silent video task in their classrooms, to report on their experiences, and to make suggestions for changes. In the second implementation phase, Andri, Edda, and Orri were asked for video topic ideas, to implement up to three silent video tasks in their classrooms, to reflect on their experiences, and to make suggestions for changes in the instructional sequence of the tasks. I created custom made silent videos based on teachers' topic suggestions. At the start, teachers were mainly viewed as practitioners implementing the task with their students. They, however, also had some influence on the task design itself. Especially in the second implementation phase where teachers made suggestions for fundamental change in the instructional sequence and thus took on the roles of co-designers.

3.3. Research design

In this section I will describe how I worked with teachers to learn about their experiences of using silent video tasks and to further develop the silent video tasks' instructional sequence.

As I was interested in teachers' expectations and experiences of using a silent video task in their classrooms, I conducted teacher interviews. In the preparation, realization and post-processing of the first phase, I followed the seven stages of an interview inquiry by Brinkmann and Kvale (2009), that is

i) formulated the purpose and theme of the study,

ii) designed the study taking aims and ethical considerations into account,

iii) conducted the interviews based on an interview guide for semi-structured interviews (Brinkmann & Kvale, 2009; Drever, 1995) taking into account the interpersonal relation of the interview situation,

iv) prepared the interview recordings for analysis by transcribing them verbatim,

v) decided in what way to analyse the data based on the nature of the interview material,

vi) discussed the analysis of data with my supervisors and a group of PhD students at the Johannes Kepler University in Linz, Austria checking for validity and reliability, and

vii) reported on preliminary findings at conferences, making various attempts to connect findings to theory.

The interview guides can be found in appendix B on p. 201.

3.3.1. First implementation phase

In preparation for the first phase, I made a flow-chart showing the process of assigning a silent video task (see figure 3.1 on p. 34). Later, I saw that creating the flow-charts assisted me in what Ruthven calls *keeping track of the phasing of the task activity* (Ruthven, 2015). I continued updating the flow-chart throughout the research project. All flow-charts can be found in appendix D on p. 213.

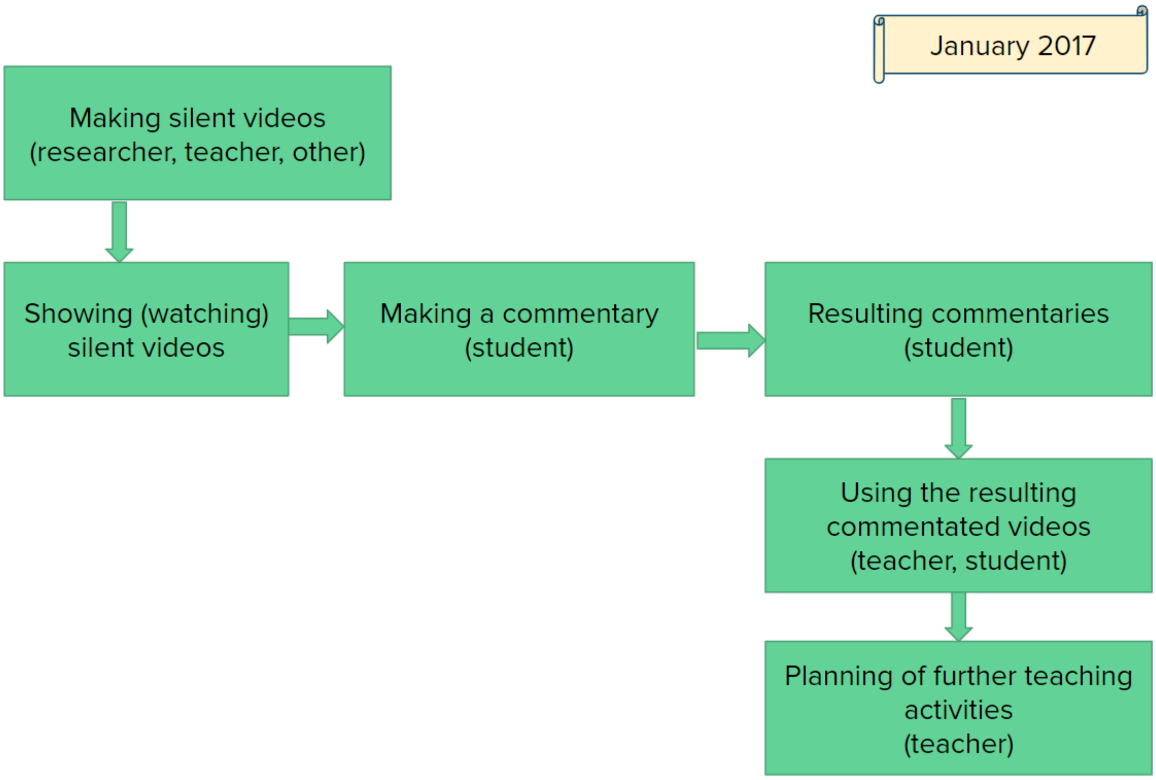

Figure 3.1. A flow-chart showing the process of assigning a silent video task as it was visualized in January 2017 during preparation for the first phase of the design-research project.

When I first contacted teachers via phone, I informed them about what their participation would entail and what silent video tasks are. After the phone call, I sent them a link to a website that contained instructions for teachers, consent letters, and information for teachers, parents, guardians, and school principals. A translation of all these documents can be found in appendix A on p. 181.

During the first phase of the research project, three semi-structured teacher interviews were conducted with each of the four participating teachers. Figure 3.2 on p. 35 visualizes the order in which the interviews took place and what the main themes of each interview consisted of. The intervention took 25-40 minutes of class time for students working on their voice-over and 10-15 minutes of class time for group discussion in the follow-up lesson. No further silent video task interventions were implemented in the four participating classrooms. As can be seen in figure 3.2 on

p. 35, students (N = 74) answered two short anonymous student questionnaires that teachers sent to them electronically; first on their experiences after they recorded their responses to the silent video task and second after their experiences of the group discussion in the follow-up lesson. Students were not obliged to answer any of the questionnaire questions.

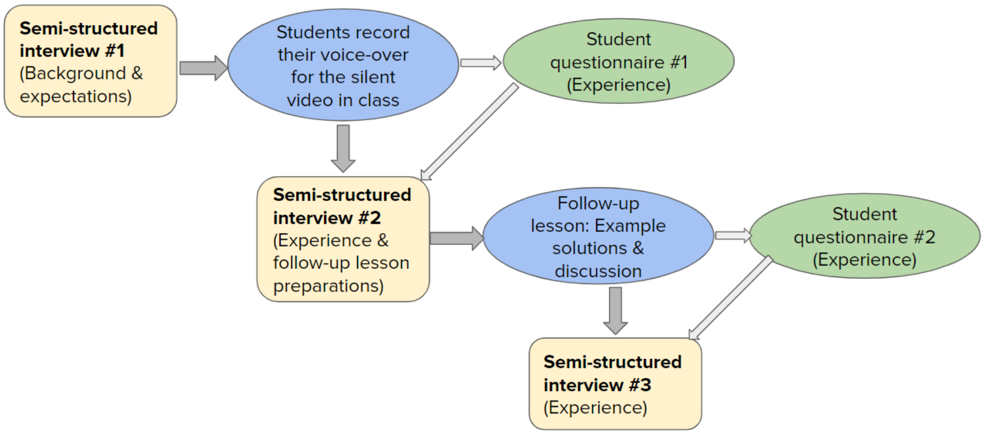

Figure 3.2. This figure demonstrates the respective sequence of interviews for each participating teacher in the first main phase of the research project. Teachers were interviewed before the intervention, and before and after the follow-up lesson.

The reason for including the student questionnaires was firstly that they provided an efficient way to give students information about my research project. Secondly, that they provided students with an opportunity to reflect on their experiences and be honest in their answers (due to the anonymity). Thirdly, in my own experience (e.g. from back in 2014 when I implemented a silent video task in my own classrooms), students' anonymous reflection on their experiences sometimes could provide useful information for the teacher. Thus, I prepared the questionnaires (see appendix C on p. 209) and teachers forwarded the link to their students via email or their Learning Management Systems (LMS). Based on students' responses to the questionnaire, it was possible to refer to students' voices in the second and third teacher interview.

In design-based research it is in the interest of all participants to be informed of the latest feedback about possibilities for ways of working with the tool currently being developed and re-designed. It was therefore a convenient coincidence that due to different course syllabi and time schedules, teachers did not all assign the silent video task at the same time. Throughout the fall of 2017 I was able to communicate experiences from one teacher to the next when needed. Figure 3.3 on p. 36 shows the sequence of interviews with each teacher and how teachers could influence each other.

All four teachers taught a trigonometry course that introduced the definition of the unit circle and a general definition of the trigonometric functions. In this course, they were asked to implement one silent video task after a learning sequence on the topic of the unit circle. Instructions sent out to teachers on how to implement the silent video task can be found in appendix A on p. 181. Teachers were introduced

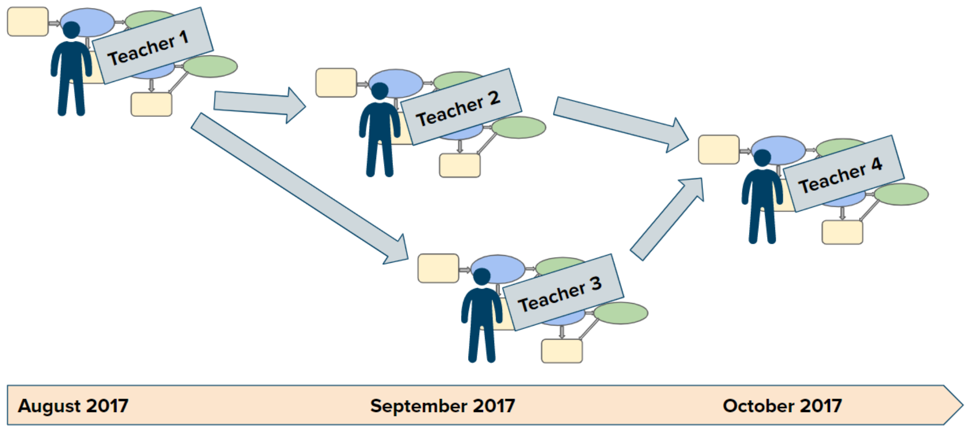

Figure 3.3. The figure shows approximately at which point in time the researcher worked with each of the four teachers who participated in the first implementation phase. Participating teachers implemented the task at different points in time. Experiences were communicated from one teacher to the others by the researcher when needed.

to the structure of observed learning outcomes model (SOLO-taxonomy model) by J. Biggs and Tang (2007) and J. B. Biggs and Collis (1982) as an example tool to evaluate students' responses to the task in terms of five stages of conceptual understanding. The five levels are shown in figure 3.4 on p. 37 and range from pre-structural to extended abstract. The SOLO-taxonomy model had been used in the teaching experiment in 2014 and it was interesting to see if it was helpful for teachers when preparing the next steps of teaching.

Teachers adjusted the idea presented to them to their local contexts, which helped in seeing if it was robust enough to be generalizable. After implementation, teachers met with me to share their experiences of implementing the silent video task and to listen to students' responses in order to prepare a follow-up lesson. Teachers were asked to select and sequence some example student responses to the task if possible. In an interview after the follow-up lesson, teachers were asked about their experiences and if they had ideas regarding enhancement, change or re-design of the silent video task and its instructional sequence.

3.3.2. Second implementation phase

Using silent video tasks is not straightforward for teachers. They are using new technology (video and recording software) and a type of tasks (open tasks) that is seldom used and rarely included in the course material. For learning to happen in such circumstances some effective pedagogy is needed (Bell & Bull, 2010; Klavir & Hershkovitz, 2008) but such a pedagogy was not ready at the start of the research project. Results from the first phase indicated that the use of new technology had not troubled students (see chapter 4 on p. 55 and paper I on p. 99) but to support teachers using open tasks in the second phase of the research project I decided

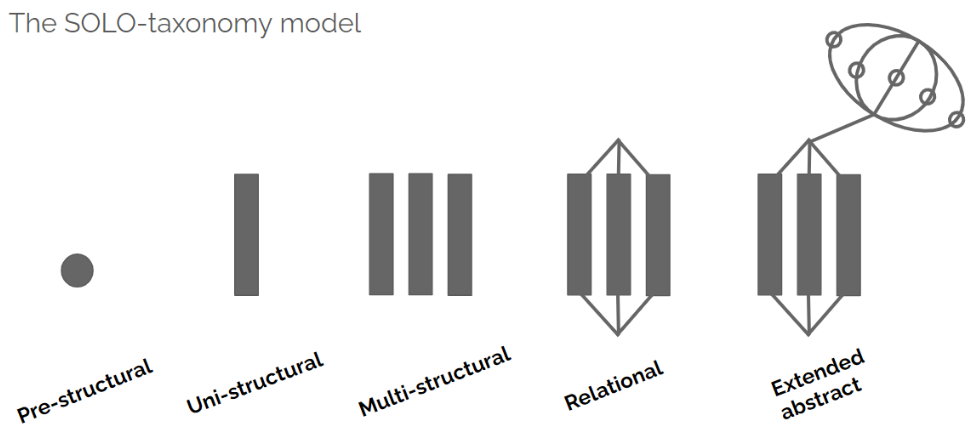

Figure 3.4. The SOLO-taxonomy model describes students' understanding in five levels of growing complexity from pre-structural to extended abstract: learners miss the point or do not understand the task properly (pre-structural), learners focus only on one relevant aspect of the task (uni-structural), learners focus on several relevant aspects but do not connect them (multi-structural), learners connect the several relevant aspects into a coherent whole (relational), and learners conceptualize to a higher level of abstraction and generalize ideas to new areas or topics (extended abstract). One could say that at the multi-structural level, learners see the trees but not the wood. The relational level marks adequate understanding of a topic, and on the extended abstract level, students go beyond what was given (J. Biggs & Tang, 2007; J. B. Biggs & Collis, 1982).

to use suggestions from Peter Liljedahl's research regarding the organisation of the classroom environment (Liljedahl, 2016) to build a thinking classroom. His suggestions along with results from the first phase were used to refine and improve teacher instructions and the process of assigning silent video tasks.

It was foreseen to depend on each teacher how many of the conditions feasible for building a thinking classroom would be used, but suggestions were made for them to give short oral instructions, assign students into visually random pairs, offer (when needed) students to use vertical non-permanent surfaces for drafting of their ideas, use a flexible room organisation, and to answer keep-thinking questions only (Liljedahl, 2016, 2018). At first, I thought to include also the possibility of student created notes at the end of the group discussion, but since we already were trying out many new things, I decided to wait and see if that component would be suggested or not.

It might influence the motivation of teachers and students if they carry out a silent video task more than once. Therefore, I planned to work with two or more teachers who would assign a silent video task at least twice during one semester. Instead of searching for teachers to use silent videos on certain pre-defined topics, I decided to create silent videos based on the content in the courses that participating teachers taught during fall 2019. This was important to not again run into problems with teachers quitting participation due to certain courses not being taught.

In the second phase, in addition to conducting semi-structured interviews with teachers and giving students a possibility to answer short questionnaires, I planned to visit teachers' mathematics classrooms, make some video recordings and write observation protocols; focusing the protocol writing on observations of teachers, their work with student groups, and how they approached group discussions. Student questionnaires were not a priority so I did not include them in figure 3.5 on p. 38, where the intended work flow is shown with relative timings of planned interviews.

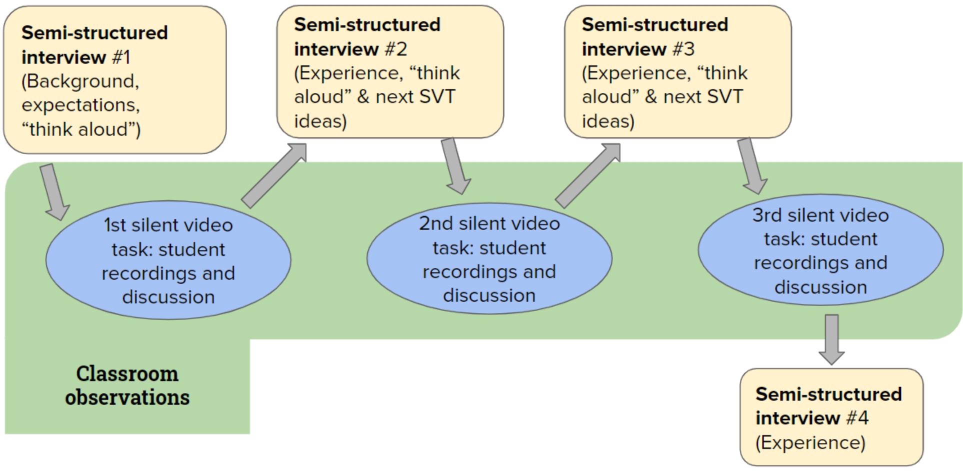

Figure 3.5. This figure demonstrates the planned respective sequence of interviews for each participating teacher in the second main phase of the research project. An interview was planned before and after each intervention. After an initial meeting with Andri and Edda at Mallard High School, a decision was made to include the group discussion directly after collecting students' responses to the silent video task, which is the reason why no follow-up lessons are shown in this diagram.

For the second implementation phase, teachers were purposefully selected. Results from the first implementation phase suggested that silent video tasks might be a good tool for formative assessment (see paper III on p. 131). To make the transition to using silent video tasks smoother, I thus wanted teachers participating in the second phase to be familiar with using formative assessment. It was also considered helpful if they had previous experiences with using DGS and group discussions in their classrooms.

Three teachers in two schools that put emphasis on the use of formative assessment were willing to participate in the project. It so happened, that all three teachers had slow paced courses for low-achieving students on their teaching schedule. Students in slow paced courses are usually quite diverse and different from the rather homogeneous (in terms of grades and socio-economic background) groups of students that had participated in the first phase. Teachers had more flexible course schedules in these courses and one of them managed to try out three different silent video tasks within one semester, as can be seen in figure 3.6 on p. 39.

To enrich the discussion, especially when students' responses were similar and less suitable for sequencing, I had received a suggestion regarding the use of pseudo-

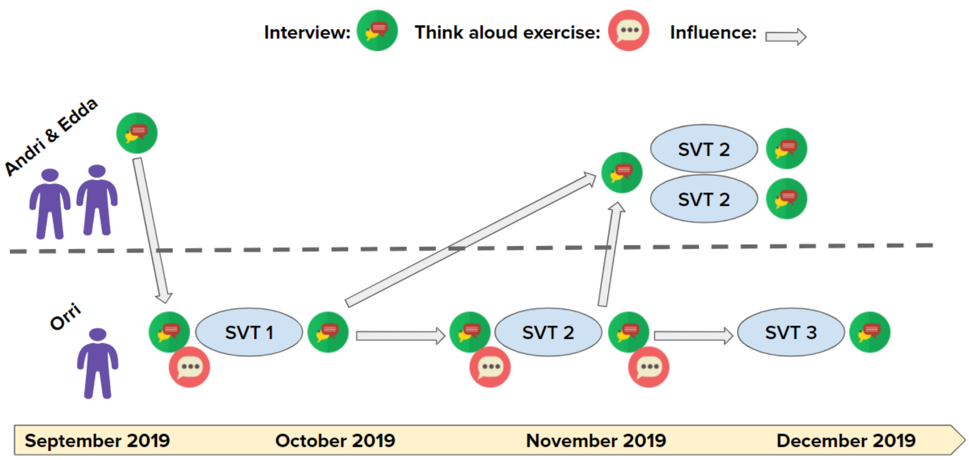

Figure 3.6. The figure shows approximately at which point in time the researcher worked with Andri and Edda at Mallard High School and with Orri at Blackbird High School. Silent video task (SVT) implementation is indicated by a grey parabola. Experiences were communicated between teachers (indicated by grey arrows that cross the dotted line school border) by the researcher when possible. A green icon with yellow and red talk bulbs indicates that an interview took place. A red icon with grey talk bulb indicates that a "think aloud exercise" took place.

responses for teachers to use in a mixture with their students' responses. This idea of "the use of pre-designed student responses to unstructured mathematics problems as a possible resource for teachers to develop their capacity of acting contingently in the mathematics classroom in a productive way, whilst teaching" (Evans & Ayalon, 2016) also seemed to me to be a possible workaround in case of tensions coming up regarding the use of student task responses as a basis for discussion–a tension that three teachers had experienced in the first implementation phase. Being willing to create some pseudo-responses for teachers was therefore one of the ideas that I introduced to teachers in the first interview.

3.4. The silent videos used in the study

This section gives account of the technical side of how the silent videos used in this research project were made. It describes the contents of the silent videos that were used in the first and second implementation phases of the research project.

3.4.1. Tools used to create a silent video task

To create a dynamic representation of mathematical concepts, I used the dynamic geometry software GeoGebra. During brainstorming, ideas for the silent video tasks were drafted on paper as frames, similar to the process of preparing longer animated films. GeoGebra was used to realize the static paper written frames. Some of the videos used scripts or background code written in GeoGebra. In other cases, direct manipulation of variables was used. By direct manipulation I mean that sliders created in GeoGebra's Algebra view were moved to using the computer arrows (up and down) to create motion in the GeoGebra Geometry view.

Attention was paid to pacing. The design aimed to provide enough time for students to describe, explain or narrate what they saw in the video within the limit of 1-2 minutes. This means that pauses in movement were included to make space for the foreseen narration. In some cases, labels were used to make it easier for viewers to distinguish between two objects of the same type (e.g. two points, labelled A and B) when referring to them in their voice-over.

Videos were recorded using the following screen recording software: Screencast-o-matic, Screencastify, and Xbox Game Bar for Windows 10. VLC media player and Handbrake were used to crop and edit the videos. All of the videos used in this study were created by me, the researcher, but teachers had influence on the video topics and content in the second phase.

3.4.2. First phase video: The unit circle and trigonometric functions

In the first phase, the video topic (the unit circle) was decided and the video created before contacting teachers for participation. What follows is a short description of the video contents but a detailed description of the silent video is given on p. 6–7 in paper II on p. 109 and the video can be viewed at https://ggbm.at/BfRqGSKq.

The video first shows a circle and a point moving on the circle. Line segments that cut the circle in quarters appear one after the other. The line segments transform into axes of the coordinate system, marked such that the circle has its center at $(0, 0)$ and radius 1. The point on the circle continues moving throughout the video. One after the other, two line segments showing the projection of the moving point onto the axes of the coordinate system appear in distinct colors.

3.4.3. Second phase videos: The coordinate system and linear functions

In the the second phase, the topics for three videos were decided based on discussion and brainstorming with teachers. Teachers' feedback on draft versions of the videos was considered when final versions were prepared.

- **SVT1** The first silent video focuses on properties of the coordinate system. It highlights different zones (e.g. $x < 0$) one after the other. It then continues to highlight the quartiles and one point appears in each of the quartiles (see https://youtu.be/8cLrbJM4F-I).

- **SVT2** The second silent video focuses on line slope. It shows the graph of a line, defined by two points. One of the points rotates around the other point, pausing when the line slope is $\frac{1}{2}, 1, 2$, undefined, $-2, -1, -\frac{1}{2}, 0$ (see https://youtu.be/-snC4JLe63g).

- **SVT3** The third silent video focuses on the graph of a line as a function of x. It shows the graphs of two lines, $y = x$ and $y = 2x + 4$, in two different representations: a discrete representation where points with integer coordinates belonging to the lines appear one after the other, and a continuous graph representation. Then, points belonging to the discrete representation of each of the lines "drop down" to the x-axis and "jump up" to their linear graph positions (see https://youtu.be/aBtlIVTcs8M).

A detailed description of the three silent videos used in the second implementation phase is given in paper IV on p. 141.

3.5. Data collection

Data collection was prepared in summer 2017 and conducted in fall 2017 for the first implementation phase. In winter 2018-2019 a second implementation phase was prepared and conducted in fall 2019. This section describes the preparation and data collection for each of the implementation phases.

3.5.1. First implementation phase

In preparation for the first data collection, I created a website with information about the research project for principals, teachers, students, and their guardians. This website also included links to questionnaires and to the silent video on the unit circle that I made with GeoGebra. To see if anything was unclear, I asked three students in grade 10, 12, and 14 to watch the silent video, give feedback and to add their voice-over. One of them created a voice-over and no changes were made to the silent video.

After the drafting of prepared questions for the semi-structured teacher interviews, I invited an experienced mathematics teacher (a former colleague and a friend of mine) to take part in mock-versions of all three interviews in one day. By mock-version I mean that I guided the teacher through a story of assigning a silent video task giving an experience-based fictive description of how the class reacted and what problems were encountered. During the interviews, I made some notes regarding questions to add or refinements to make, part of which were suggestions and comments from the teacher. These pilot interviews were recorded and transcribed as an exercise for the interviews to come.

As mentioned previously, I worked with four teachers (Gauti, Lilja, Magni, and Snorri) who implemented a silent video task in their mathematics classrooms. Data collection included teacher interviews, student questionnaires and students' responses to the task:

1. I conducted three semi-structured interviews with Gauti, Magni, and Snorri, and two with Lilja. Lilja skipped the interview before the follow-up lesson as she thought that only two interviews were planned. Teachers were informed that the information sought after in the interviews mainly focused on their expectations and experiences of the implementation of silent video tasks. The interviews' pre-defined questions are presented in appendix B on p. 201.

2. Students' responses (voice-over recordings) to the silent video task. These audio and screen recordings were listened and reflected to during the second teacher interview.

3. Students' anonymous answers to two short questionnaires (see appendix C on p. 209). The reason for including students' questionnaires was that at that time it seemed important to me to be able to refer to students' voices in the second and third teacher interviews.

3.5.2. Second implementation phase

After analysing the interview data from the first phase, presenting its initial results at conferences, and discussing them further in doctoral seminars in Linz and Montpellier (YERME Summer School), I updated the instructional sequence of silent video tasks accordingly. Then, with focus on the use of open tasks and formative assessment, I prepared a second implementation phase for fall 2019. As described in section 3.2 on p. 32, teachers that already had experience with using formative assessment were selected purposefully to participate in the second implementation phase. At least three new silent videos were planned for the second implementation phase and I decided not to prepare the videos beforehand. This way, I could adjust the video topics to participating teachers' course schedules.

At the point of data collection the General Data Protection Regulation (GDPR) had recently been introduced and school leaders in Iceland were increasingly aware of complexities regarding data collection in their teachers' classrooms. They preferred that I take field notes rather than record video data in teachers' classrooms. What goes on in classrooms involves speech, gestures, and mimes, many of which happen simultaneously in different corners of the classroom or school building. Even though these actions might seem obvious to the participants involved, they can be less obvious and require more careful examination for visitors. Therefore, to grasp what goes on in classrooms in practice, researchers normally aim to collect video recordings rather than only classroom observation notes. To build trust and positive correspondence needed for research that is done in collaboration with teachers, I decided to follow school leaders' advice and take field notes.

The interview question schemes (see section B.2.2 in appendix B on p. 205) were updated and adjusted for the next round of implementations. Two questions were inspired by Schoenfeld (2017, p. 6), one of which addressed whether the task sufficed in making students' voice be heard and another one whether the task sufficed in providing teachers with information regarding common misunderstandings.

In the second phase, I worked with three teachers: Andri and Edda who implemented one silent video task each in their mathematics classrooms and Orri who implemented three silent video tasks over the course of one semester in his mathematics classroom. Data collection included teacher interviews, students' self and/or peer evaluation and reflection responses, students' responses to the task, classroom observation notes, email/chat communication, and a research journal:

1. I conducted two preparation interviews with Andri and Edda, and one interview with each of them after they tried out a silent video task in their classrooms. With Orri, I conducted five interviews; before and after using the first two silent video tasks, and after using the third silent video task. Teachers were informed that the information sought after in the interviews mainly focused on their experiences of the implementation of silent video tasks. The interviews were semi-structured and their pre-defined questions are presented in appendix B on p. 201.

2. Students' responses (voice-over recordings) to the silent video task. These audio and screen recordings, were listened and reacted to by teachers in the silent video task lesson and again during an interview after the lesson.

3. After Orri's first task implementation, his students answered an anonymous questionnaire and a self/peer assessment form, to which I do not have access other than that some of its answers were read out loud by Orri during an interview. Andri and Edda's students answered a reflection sheet that was prepared in collaboration with them prior to task implementation. Both the questionnaire and the reflection sheet can be viewed in section C.1.2 in appendix C on p. 210.

4. During the implementation of silent video tasks I visited Andri, Edda, and Orri in their classrooms and wrote observation notes.

5. Email and WhatsApp communication with Orri regarding new ideas that we discussed in between interview/meeting sessions.

6. My research journal included reflection notes written after school visits and phone calls regarding the research project.

As compensation for their participation, I offered participating teachers in the second phase some support meetings in case they were working on any changes in their practice. Orri accepted the offer and we met six times to discuss ways to build a thinking classroom (e.g. Liljedahl, 2018) at meetings that were recorded but not transcribed or analysed.

3.5.3. Think aloud exercises

According to Gravemeijer (1994), "the elaboration of an educational design is, in practice, constituted via a thought experiment". In addition to setting a mathematical learning goal before starting developmental research, the thought experiment is intended for developers to envision how the teaching-learning process might proceed in the classroom. I assume that I first read of thought experiments in texts by Freudenthal (1973) during studies in Germany in 2010, then forgot about them but naturally started using them on my own in the first implementation phase and then with Orri, during the second implementation phase. With Orri, I called these "think aloud exercises" but they were indeed the same as the thought experiments Freudenthal (1973) had described and I agree with him that these exercises or experiments lay the ground work for instructional design.

In the cases where I asked Orri to do a thought experiment, I asked him to imagine how he would implement a silent video task and "think aloud" what he would do and why. It was a free-flow and in-the-moment exercise, meaning that he could change his mind "on the go", wonder about and discuss different variations as needed. I thus normally did not interrupt unless something needed immediate clarification. In the first interview, I wanted to hear his initial ideas and then in later interviews the exercise was a means to reflect on his experiences and keep a record of his ideas for the next implementation round. It requires training to remember and reconstruct our own interpretations–what were we thinking in the moment? Were we making sense of something? Or rather thinking about something? Despite no training, Orri reflected on what he thought about in-the-moment and related it to planned actions for the next implementation. In my mind and during data analysis, I related all of this to my own experiences as a secondary teacher and a PhD student.

3.6. Researcher and designer roles

The mathematics teacher community in Iceland is small so participating teachers knew that I am "one of them" (an insider) simultaneous to being a designer and a researcher (an outsider). Within the research project I mainly held the roles of a designer and a researcher, although my actions were surely influenced, informed, and affected by my previous experiences as a pupil, student, and teacher. My teaching experience, especially that of implementing a silent video task in my classrooms, made me not a purely objective observer or listener when it came to interviewing and visiting teachers in their classrooms.

Certainly, I was aware of the fact that I was influenced by my past experiences. Without these previous experiences I claim that teachers would have had a harder time trusting me and I would have had a harder time understanding their reactions, reflections, situations and considerations. For example, it varied how strictly teachers followed my suggestions–I only influenced what they did up to a certain degree. They made their own decisions based on the suggestions and their own experiences and expectations.

An example of the challenge of being an insider is that during interviews, I often had to stop myself from jumping from the role of the listener to the role of the story-teller. I strived to set myself aside and focus on interviewees' stories. Another challenge that I was aware of was that in some cases interviewees might tend to give socially desirable responses. To partly address this, teachers were informed that their ideas and suggestions for change were welcome. When they freely described challenges that they had faced when implementing the silent video task, I considered that to be an act of trust. To sum up, it is in this case rather an advantage than a disadvantage to be an insider.

Teachers also understood my situation. I had been struggling with time pressure–a feeling they all were familiar with–and it was not until I had an opportunity to take a break from teaching and focus on this research project that I finally had proper time to reflect on and develop my ideas about silent video tasks. I am very grateful for their trust and willingness to take some time from their tight course schedules to assist me in exploring possibilities of using silent video tasks in the upper secondary school classroom. In all cases, I experienced surprises and learnt something new about the teachers and culture within each school.

3.7. Ethical considerations

Before data collection started, I contacted Persónuvernd, the Icelandic Data Protection Authority (IDPA) to inform them about the research project and consult with them to ensure that my plans would comply with "Lög um persónuvernd og meðferð persónuupplýsinga nr. 77/2000 [Law on the protection of privacy as regards the processing of personal data number 77 of 2000]" (2000). Both teachers and school principals received an information sheet explicitly stating and/or explaining the research aims, the intended data collection, treatment of data and in what ways results would be communicated. I also informed teachers about the research project via telephone. Teachers and principals signed an informed consent stating that they were willing to take part in the research project and that they were aware of that they could quit participation any time.

In the first implementation phase students were informed about the research project orally by their teachers and in a written preface to the anonymous questionnaires. In the second implementation phase, students were informed about the research project orally by their teachers and an email with information was sent out both to students and their guardians. In both phases, students were informed that they could deny participation in the research project at any point in time, meaning that only their teacher would listen to their voice-over recording and not the researcher. None of the students decided to deny participation.

No personal or personal identifiable information was collected from students. Student questionnaires used in the first implementation phase were anonymous and free of any identifiable information. If teachers mentioned students by their names during interviews, these were changed to numbers in the transcript and I never had access to students' full names. Regarding students' voice-over recordings, firstly the United Nations Convention on the Rights of Children (UNCRC) ensures youth the right to express themselves, and secondly expression on the topic of mathematical concepts such as circles, lines and points in a coordinate system is not considered to be personal. Strictly speaking, the recordings could have been considered personal identifiable because teachers can possibly tell their students apart, but since the topic was not personal, the IDPA lawyer confirmed that signed informed consent forms for students would not be necessary.

Teacher interviews were recorded and transcribed verbatim in Icelandic using a pseudonym for each participating teacher and school, and a number for each student. There can be a thin line between professional and personal when it comes to teachers sharing their expectations and experiences of their work in classrooms. Thus, when personal identifiable information was shared in the interviews, it was deleted or made unidentifiable in the transcript. In publications such as conference presentations, written papers or on posters I refer to participants and schools using the pseudonyms. Teacher pseudonyms were chosen such that they were short, Icelandic, and involved only letters from the English alphabet. School pseudonyms were chosen from a list of names of common bird species in Iceland.

3.8. Mistakes

At the time when I started the research project presented in this thesis, great effort had been put into gathering all the known information about task design by the mathematics education research community (Margolinas, 2013; Watson & Ohtani, 2015). Looking back, re-reading some of the papers that I struggled with back then, I realize that I in many cases could not make proper use of the information gathered. Just like a student who makes mistakes in mathematics class and learns from them, I made mistakes in my research that could have been avoided if I had understood at the start what information was relevant for my project and what people I should contact or courses I should visit.

My focus at the start was set on whether anyone had done similar things before–if this research was original enough to be accepted as a research project in mathematics education. However, my vision was so narrow and close to what I was aiming at working on that I probably hardly would have recognized similar or comparable projects. In retrospect, the process of hitting walls and walking into dead ends seems to be a part of a necessary process. Not only as students but also as teachers we make mistakes. It is rather the question whether we continue without thinking about our experiences or if we notice them, think deeply about them and try to learn from them. Fortunately, I soon discovered books by Malcolm Swan and John Mason that helped along that road (Mason, 2002; Swan, 2006).

To name an example, one of my mistakes was that I did not integrate Building Thinking Classroom strategies explicitly into the instructions for teachers in the first implementation phase even though I had started to realize that such practices might make implementation of silent video tasks easier. Instead, I wondered if such practices might "appear". In the end the teacher who integrated six of these strategies in her class, was a teacher who had visited a professional development course where I had given a short introduction into Peter Liljedahl's work on Building Thinking Classrooms. It is not very surprising in retrospect.

Another naïve idea of mine was that other teachers would or might experience a similar aha! moment as I did myself when I first used a silent video task in my teaching. Probably these ideas could have been blocked or corrected from the start by my supervisors or the group of doctoral students. However, I am grateful that they did not stop me, because the mistakes along the way taught me a lot. I was also fortunate enough to continue my training in listening to other teachers' experiences.

3.9. Data analysis

This section describes what part of the data was analysed and which analysis processes or procedures were used.

3.9.1. Data storage and transcripts

Data was stored on a Dropbox drive with read-only backup files stored on the secured drive of the central servers of The Educational Research Institute at the University of Iceland School of Education. Teacher interviews were recorded and transcribed verbatim in Icelandic using pseudonyms for teachers and schools, and numbers for students. Words that were emphasized in speech were underlined in the transcript. For transcription, I used the freely available oTranscribe (https://otranscribe.com. Personal identifiable information was deleted or altered in the transcript. In total around seven hours of interviews from the first phase and around nine hours of interviews from the second phase were transcribed.

3.9.2. First phase: Thematic analysis

During data collection and after finishing transcribing the data in January 2018, I decided to use an inductive approach and open coding of the data in the same manner as I had practised in a workshop on thematic analysis (Braun & Clarke, 2006). This seemed to be a good choice because I was not sure which theoretical frameworks to use for the data analysis (I needed a theoretically flexible approach) and because my underlying research questions about teachers' experiences of and views on using an innovative teaching material suited thematic analysis.

To guide me in the coding process, I used a book by Braun and Clarke (2013). First, I read across all the data, taking note of every part or text that seemed to me to hold interesting information of any kind. Alongside, I tried to make sense of some broader meaning of the data. Despite my intention to use an inductive approach, in the second iteration of coding, I in particular paid attention to anything that seemed to me to be connected to the study aims and kept in mind my underlying research questions regarding expectations and experiences of silent video tasks (with indirect focus on teacher change). In other words, I partly coded as directed by existing ideas from my research questions. Thematic analysis can be either inductive or deductive but I was not completely aware of (at this point) that I was heading in a deductive direction with my analysis. During this next round of coding, I, however, also made some improvements as I revised my initial coding. For example, I threw away "Student reaction" because it was too general and used instead codes such as "Students ask for being spoon-fed" and "Students react as if they never had much autonomy before". Another example is that I threw away "Explains that students are confused by the subject matter and gives examples of reasons" because it included too many ideas. Instead, I used more focused codes such as "Students find the subject hard to understand" and "Students learning mathematics by heart causes frustration for the teacher".

After the third iteration of coding, I started choosing what codes to collect in a theme and color-coded developing initial themes. In retrospect, this process was connected to what Virginia Braun called "throwing in different bins" and is not the preferred way of practising thematic analysis. My bins, so to say, were marked:

i Thinking classroom related

ii Collaboration between teachers

iii Expectations of the silent video task

iv Experiences connected to student reactions

v Experiences connected to the teacher role

vi Teacher actions in the instructional sequence

vii Teacher beliefs

viii Criticism of the silent video task

For example, theme i "Thinking classroom related", in a way, is deductive (looking for existing ideas in the data), even though I was aiming at using an inductive approach. Even though the process of realisation started earlier in the process it was at this point, that I (in hindsight: non-surprisingly) started accepting that the data was telling me a lot more than I had intended or expected. After discussing the "bins" against "what the data is telling me" in a group of PhD students within the frame of a research methods course at the Johannes Kepler University in Linz, Austria, I revised my work toward an inductive approach, and constructed the following new themes that better captured the story that I noticed – the story of teacher tension in regard to assigning the silent video tasks:

I Teachers fear that technology will fail.

II Teachers worry that students are sensitive to peers listening to their responses to the task.

III Teachers are stressed about leading a group discussion.

IV Teachers using summative assessment consider it a prerequisite for using innovative teaching material that it prepares students for final exams.

V Teachers have a tendency towards restricting an intentionally-designed-to-be-open task.

A thematic map connected to the above themes I–V is included in figure E.1 on p. 223 and some results of this early work regarding the use of technology in the mathematics classroom and on initiating student discourses were discussed in paper I on p. 99 and a PME short communications paper (B. Kristinsdóttir et al., 2018a). After presenting my initial findings at PME in Umeå in July 2018, a group of researchers focusing on teacher knowledge asked me to take a look at my findings from a different angle and form new research questions regarding the influence of teacher knowledge on how teachers used the silent video task in their classrooms.

This proved to be quite a challenge for me and I ended up partly re-analysing the data. Again, I participated in a workshop on thematic analysis and after reading the new article on reflexive thematic analysis (Braun & Clarke, 2019) I was left wondering whether I had managed to construct themes underpinned by central organizing concepts. In the re-analysis process, I drew upon theories about teacher noticing and teacher knowledge (Mason, 2002; Scheiner et al., 2019; van Es, 2011). Some themes were kept and reformulated, and new themes built:

1. Teachers expect technology to fail.
2. Teachers notice what is missing.
3. Teachers notice what is unclear.
4. Teachers have the best intentions for their students.
5. Discussing student responses (solutions) with the whole group is a delicate matter.

In some cases, themes 2 and 3 were overlapping so in the end they were brought together into "Teachers notice what is missing or unclear". A thematic map to illustrate these themes can be found in figure E.2 on p. 224. Examples for semantic (close to interviewee's language) and latent (informed by underlying concepts) codes are given in paper II on p. 109. Results from the first phase influenced the design, work flow and process of assigning a silent video task. Work flow charts showing both concrete ways in which teachers implemented silent video tasks and some ideas of ways in which the tasks might be implemented as suggested to teachers at different stages can be found in appendix D on p. 213.

It is worth noting that in spring 2019, after a discussion with John Mason and Anne Watson, I stopped using the word *solution* to describe learners' response to the silent video task. Instead, *response* or *voice-over* seemed to be more fitting and less likely to cause misunderstanding when communicating results. From the very start of the design process, I used drawings on a whiteboard to express my intentions and vision. For CERME in Utrecht in February 2019, I also started drawing comics to display the instructional sequence in an accessible way to teachers. These comics also proved to be helpful when communicating my work to colleagues at conferences. A part of these comics can be seen in section A.2.2 of appendix A on p. 187.

3.9.3. Second phase: Hermeneutic phenomenological approach

By asking teachers to implement silent video tasks in their classrooms and reflect on their expectations and experiences, I took a hermeneutic (interpretive) phenomenological stance (Van Manen, 2016) towards answering the question of how and why silent video tasks could be used for teaching and learning in the mathematics classroom. Especially in the case of Orri, I studied teachers' actions–here the think aloud exercises described earlier in this chapter played a key role–and reasons given for their actions with the aim to make the instructional sequence generalizable. In other words, to describe the silent video tasks' instructional sequence in such a way that it could be helpful for teachers aiming to use such tasks in their own teaching.

It was important that this happened in the busy setting of participating teachers' own classrooms where I could observe their work and make notes preparing for the next interview. In the interviews I referred to their actions and when possible, teachers would give their reasons for their actions. Moreover, they gave their personal insight as to whether and how they could use this tool for the teaching and learning of mathematics. Directly after our meetings, I reflected on teachers' insights in writing and then repeated that process when I analysed the transcripts from our interviews. Thus, I used iterative cycles of writing notes and reflections, considering how excerpts from the data contributed to evolving understanding of the way silent video tasks could be used in the mathematics classroom.

All interviews were transcribed verbatim in Icelandic. When possible, I transcribed directly after the interview took place and thus was able to add some extra notes in parentheses. Analysis started immediately after the first interview and in that first familiarization phase, I focused on the instructional sequence design, making sure that any important idea would be transferred between the participating teachers and paying attention to how these ideas developed over the semester. Also, I regularly updated the previously mentioned flow charts and comics that described ways in which the silent video tasks were implemented. After transcribing the last interview, I underwent a second familiarization phase of the data not only focusing on aspects related to instructional sequence design but using a bottom-up approach of open coding in Icelandic on anything that I found interesting in the data.

In the third iteration, I read through the transcripts again writing detailed notes in English where I summarized and deepened my thoughts. On the basis of these detailed notes, I created a condensed overview from the five interviews with Orri on a large sheet of paper (630×891 mm) studying how Orri's ideas, experiences, and expectations developed in time. Then, in a fourth iteration, I used a deductive approach where I once again read through my detailed notes with questions (see figure 2.2 on p. 29) from the TRU framework (Baldinger et al., 2018; Schoenfeld, 2018) in mind. Similar to what was done in the first phase, initial findings from the second phase were discussed in a doctoral seminar with the PhD student group in Linz, Austria.

Some opportunities identified at this stage in the analysis process are listed here:

- v1 Learners put their own ideas into words that get heard and discussed by the whole group.
- v2 Learners get an opportunity to make sense of mathematical ideas and to get aware of their own understanding (meta-cognition).
- v3 Learners and teachers become aware of the variety of descriptions, explanations, and/or narrations that can be made.
- v4 Learners might create meaningful connections between mathematical ideas.
- v5 Some conceptual obstacles or misunderstandings–previously unnoticed by teachers and students–related to the video clip's mathematical topic might be uncovered in students' responses to the task.
- v6 It becomes more tangible to learners why precision in word use is important in mathematical discussion.
- v7 Previously unnoticed imprecision in teachers' own ways of wording might get noticed by teachers.
- v8 Some common understanding of the video clip's mathematical topic might be reached.
- v9 It demonstrates that every member should get an opportunity to participate meaningfully in the mathematical communication of the class and thus might create a sense of belonging.
- v10 Learners might be supported in moving from detailed descriptions toward generalizing about patterns.
- v11 An opportunity for teachers to gain insight into previously unnoticed things regarding themselves and their own teaching.

Some challenges identified at this stage in the analysis process are listed here:

- x1 The shift toward assigning open tasks, utilizing whole-class discussions, and making use of formative assessment all imply changes regarding what is valued as important in mathematics classrooms. It changes classroom norms, and thus creates tension.
- x2 It is hard for teachers to refrain from their previous role of being evaluators of the correctness of students' work.

x3 Facilitating a meaningful group discussion built on students' responses is very demanding and it requires both training and support.

x4 Teachers need support to develop their awareness of what students are saying and to be better prepared to reflect in the moment.

x5 Existent ideas about what mathematical practice entails and established classroom norms can get in the way.

Due to the Covid-19 pandemic, my plans to discuss these findings with colleagues at ICME-14 in Shanghai, China and PME-44 in Khon Kaen, Thailand were postponed until July 2021 and planned to take place in an online setting. Helpful comments from reviewers of paper IV made me take a new look at the data analysis in March 2021. I had initially used the questions from the TRU framework directly to determine the opportunities and challenges that silent video tasks could bring to the mathematics classroom. In this process I had confusingly mixed intentions and observed practices.

In accordance with reviewers' suggestions, I took a fresh look at what I had written, making a clearer distinction between what silent video tasks were *intended for* and what opportunities and challenges were *encountered in practice*, i.e. appeared explicitly or implicitly in the data. I kept the TRU framework questions as a lens to view the data through, but now organized the findings by themes that described opportunities and challenges encountered. Then, coming back to the TRU framework questions, I discussed the findings along the dimensions of the TRU framework. Based on data, I identified two opportunities, four challenges, and one phenomenon which I considered to give rise to both an opportunity and a challenge. After the review, editors suggested further minor changes to the manuscript of paper IV in July 2021. These suggestions were very helpful and served toward more clarity. The findings from paper IV will be listed in the next chapter and described in chapter 5.

3.10. Summary

This chapter explained why a design-based approach was chosen for this research project and described how participants were randomly chosen in the first phase and purposefully selected in the second phase. An overview of the research design, data collection, data analysis, and ethical aspects of the study were given. This included a description of the reflexive thematic analysis which was useful when making sense of data from the first phase, because it a) gave me a feeling of freedom as I could view the data from different angles, testing out and drawing on different theories, and b) it allowed me to dig into and engage actively with the data. Finally, it explained another useful procedure of data analysis used in the second phase where repeated note-taking, extracting information from interview and classroom observation data step-by-step helped to i) gain overview of the development of participating teachers ideas regarding task implementation–a process in which think aloud exercises played a key role and flow charts and comics were continuously used to document changes in the instructional sequence, and ii) identify opportunities and challenges that arise from the use of silent video tasks–a process in which the TRU framework supported the data analysis.

4. Short Summary of Papers

4.1. Paper I

This paper introduces silent video tasks and the story behind them. It is a peer-reviewed conference proceedings paper written for the 5th ERME topic conference Mathematics Education in the Digital Age (MEDA), which allowed me to discuss three preliminary findings from initial stages of analysing data from the first implementation phase. For data analysis, I was using open coding and identifying themes and even though I at the time of writing the paper intended to use a grounded theory approach in my data analysis, I later (for papers II and III), re-analysed the data using thematic analysis. The main points from the project's initial findings are the following:

- Teachers who seldom use technology fear that technology will fail. This can cause anxiety and tension.
- Teachers might not be aware of their students' technological reality and popular culture (e.g. YouTube, Instagram, SnapChat), at least
  - they were surprised when the task of adding a voice-over to a silent video was observed to be easily understood and tackled by students,
  - they were hesitant toward showing examples of students' task responses to the whole group, and
  - in the only case where technology created problems, there was discrepancy between how the researcher and how the teacher viewed the best way of assigning the task. This teacher decided to create detailed (more than one A4 page) written instructions for students.
- A teacher who was inclined toward being a facilitator of learning found it easier than transmission-oriented teachers to move away from traditional (transmission-oriented) teaching methods toward creating a constructivist learning environment, as manifested by
  - her giving students more autonomy (in this case to transfer the responsibility of tackling potential technological issues to students) and

– her steadfast assumption that by trusting students to get past possible obstacles brought by the silent video task, she was giving her students an opportunity to experience success.

These initial findings counted toward answering research questions Q1 and Q2, by providing insights into tensions created by the use of technology in silent video task practices (see section 5.1 on p. 65.

4.2. Paper II

Compared to the first paper, this second paper describes in more detail what some characteristics of silent video tasks are. The paper is a journal paper, which underwent two demanding review cycles and one easy review cycle. During the demanding review cycles, I re-analysed data from the first implementation phase using teacher noticing frameworks by Mason (2002) and van Es (2011) and van Es and Sherin (2008) and reflexive thematic analysis (Braun & Clarke, 2019).

In the review process, I also reformulated the research questions addressed in the paper until they took the form of research questions Q5 and Q6, and (eventually) based on discussion with one of my co-authors, Charlotte, I defined a framework for the purpose of analysing data from interviews where teachers reflect on their experiences of using a task that has the potential to reveal some of students' mathematical thinking, listen to students' responses to the task, and comment on what they notice. This framework is a variation based on the teacher noticing frameworks by van Es (2011) and van Es and Sherin (2008) and it consists of the following three levels where:

1. teachers provide descriptive and evaluative comments,

2. teachers provide some interpretative comments,

3. teachers connect students mathematical thinking to principles of teaching and learning.

The RME journal special issue where paper II was published, aimed to collect international perspectives on ways in which teachers' professional knowledge might become visible in teachers' actions and reactions to certain classroom situations–which in this case was a situation created by teachers' implementation of a silent video task. This new type of task forced teachers to transit from a transmission-oriented to a socio-constructive teaching approach.

The paper describes characteristics of silent video tasks in detail and gives account of what influenced teachers' decisions regarding the implementation of silent video tasks, thus partly informing the answer of research questions Q3 and Q4. More precisely, it gives insights into turning points when teachers who are trying out new practices suddenly, consciously or subconsciously, turn back to previous practices. Therefore, demonstrating how challenging it can be for teachers to change their practices and the classroom norms.

Its findings also provide answers to research questions Q5 and Q6 by identifying the following themes:

- When listening to student responses to silent video tasks,
  - *teachers notice what is missing or unclear*. In other words, teachers pointed their attention mainly in the direction that they were used to (evaluate the correctness of students' responses).
  - *teachers have the best intentions for their students*. This theme captured when teachers expected students to "know better" than their responses to the silent video task showed, often drawing upon previous evaluation of students' work on other (procedural) tasks in class.
- The biggest challenge that teachers encountered when it came to implement silent video tasks in their classroom was that
  - *discussing student responses with the whole group is a delicate matter*. Teachers admitted stress and/or anxiety towards the possibility of losing control of the learning process. Their experiences were shaped by the classroom culture because normally discussions were seldom or never practised in their classrooms.

The above findings also provided a foundation for the later answering of research questions Q7 and Q8 on opportunities and challenges that silent video tasks bring to the mathematics classroom (see section 5.4 on p. 83). Furthermore, the above findings counted toward answering Q1 and Q2 (see section 5.1 on p. 65) about expectations and experiences, by providing insights into three tensions created by silent video task practices–tensions regarding the use of technology, the assessment of students' responses to the task, and the prospect of leading group discussions.

For example, the theme "Teachers notice what is missing or unclear" illustrates that teachers attended to what they were used to attend to: the evaluation of correctness of students' responses. As opposed to viewing students' responses as being their participation in an ongoing dialogue (participationist perspective), the view that teachers are holders and evaluators of knowledge demonstrates an acquisitionist perspective where students' responses serve the purpose to inform teachers of students' knowledge for the purposes of evaluation (grading).

To summarize, the contributions of paper II consist of i) the development of a teacher noticing framework that can be used for analysis of data where teachers reflect on students' responses to a task, and ii) the use of that framework and reflexive thematic analysis to identify themes across the data set, which give insights that inform all eight research questions.

4.3. Paper III

The third paper discusses that silent video tasks might be useful for formative assessment practices. First, by going through a list by Wright et al. (2018) of six potentials that technology-based formative assessment have to support learning, checking for which of them could be fulfilled by silent video task practices. Second, by setting focus on the success story of the only teacher from the first phase who managed to "get the discussion going" in her classroom and how her practices differed from those of the other participating teachers. Similar to paper I, paper III is a peer-reviewed conference paper. It is written for the proceedings of ICTMT-14, which took place in Essen in Germany and provided a very fruitful atmosphere for discussion.

The paper findings, list the following five (out of six) practices from the list by Wright et al. (2018) that silent video tasks were considered to support or encompass:

- *Encouraging discussion*. On the basis of the success story it is argued, that silent video tasks have potential to spark discussion.
- *Provide a meaningful way to represent problems and misunderstandings*. Students' responses to the task can cause disturbances, for example by uncovering misunderstandings or commognitive conflicts (a conflict between incommensurable discourses, see Sfard, 2008). An example of such a situation where students got into a heated debate with the teacher is given in the paper.
- *Giving opportunities to use preferred strategies in new ways*. Silent video tasks open up a possibility for students to use spoken language instead of written language in their task response. In classrooms where the norm is to remain silent it might be too much of a hurdle for students to speak up directly, but via recording a response they can add their voice to the discussion without hitting that hurdle.
- *Help raising issues that were previously not transparent for teachers*. All participating teachers gained new insight into their students' ways of thinking and experienced surprise of some kind.
- *Provide different outcomes feedback*. Silent video tasks require students to use screen and/or audio recording technology to record their responses to the task. What these responses bring into the classroom discussion is something different

from what other technology such as responses written on paper would give. Feedback could be given via whole class discussion and written individual feedback.

The reasons *why* the silent video tasks were considered to support the above practices were not presented in paper III due to space limitations. They are therefore outlined briefly above (and also in paper IV in a section on formative assessment). The one (out of six) practice, which silent video tasks in the first implementation phase did not support had to do with the timing of feedback. Namely, silent video tasks did not:

- *Provide immediate feedback*. Since the follow-up lesson could happen more than one day later than the recordings of students' responses to the silent video task, the feedback given in a group discussion was considered to be delayed.

Moreover, the paper discusses the influence of the classroom social norms and socio-mathematical norms (Yackel & Cobb, 1996) on the classroom practices. It connects norms to Brousseau's notion of the *didactical contract* (Brousseau, 1997). However, norms and the didactical contract do not coincide. For example, the didactical contract is not described to be affected by the community.

For two years, I continued working with Brousseau's theory of didactical situations, but because of a conflict between acquisitionist and participationist metaphors of learning, I decided to give it up[1].

The findings of paper III informed the development of the task instructional sequence, thus contributing to Q4. This was done by identifying what practices from the Building Thinking Classroom (BTC) framework (Liljedahl, 2018)[2] were implemented in the classroom by Lilja, the only teacher in the first implementation phase who successfully initiated discussion in her classroom.

Considering that the other three teachers who participated in the first implementation phase used two or three of the BTC practices, Lilja got the discussion going by integrating six of them and managed to withstand tensions created by many of them. The practices that she implemented were:

- *Begin with an open task*. There is no one correct response to the silent video task.
- *Assign students visibly randomly into groups*. Students were not used to this and

[1]Or maybe not completely give it up, because I continued to refer to what Brousseau calls a situation of *institutionalisation* (Brousseau, 1997). I do not refer to this situation in paper III, but in paper IV (the under-review version of it included in this thesis), it is used and I will also briefly mention it as part of the design principles in the next chapter of this thesis. However, the conflict remains such that I might need to define a new term, which fits the participationist view.

[2]In paper III, a reference is made to (Liljedahl, 2016) because even though I knew from other internet resources that the BTC framework had been extended from nine to eleven practices, I had not yet found access to the book chapter and thus could not cite it.

were quite upset to start with, but Lilja managed to stand her ground.

- *Give short oral instructions*. Lilja took care to only give the minimum information needed such that students could work on their own.
- *Answer only keep-thinking questions*. She withstood some students' attempts to fish hints on what to include in the recording. The existence of these "fishing" attempts signals that some students expected that there might be some desired outcome.
- *Build student autonomy*. She transferred the responsibility of tackling potential technological issues (that might come up in the process of recording a response to the task) entirely to students. Also, by not answering the "fishing" questions she made make clear that it is the students' responsibility to make decisions regarding what to focus on in their voice-over, thus emphasizing students' autonomy.
- *Formative assessment*. She played students' responses in a quest to establish the norm that listening to various different responses was a valuable experience. She also addressed some conceptual conflicts and let students debate about them, e.g. when students did not agree whether it was acceptable to refer to a circle as a unit circle before the video revealed units in the coordinate system, indicating the unit radius.

As can be seen above, in paper III I do refer to one of the practices from the BTC framework as *Begin with an open task*, but it actually is intended to be a *problem-solving* task and a silent video task can hardly be categorized in that way. At least it is not similar to problem-solving tasks that we normally think of and are discussed by e.g. Liljedahl and Santos-Trigo (2019) or Schoenfeld (1992), although the preparation of recording a response to a silent video task might engage students in some pattern exploration or conjecture formulation.

Putting this sloppiness of mine aside, it was interesting to see how the implementation of BTC practices supported Lilja in creating a constructivist environment for students in her classroom. Results from paper III added to the puzzle of answering research questions Q3 and Q4 and influenced the design principles of silent video tasks.

4.4. Paper IV

Paper IV has in its present state underwent one major and one minor review cycle in the process of becoming one of the chapters of the second edition of a book called *The Mathematics Teacher in the Digital Era*. Information provided in this section might change in the ongoing review process.

This fourth paper aims to inform research questions Q7 and Q8 on the opportunities and challenges of implementing silent video tasks in the mathematics classroom. It had been established that silent video tasks could be used for formative assessment and therefore, teachers in the second implementation phase had been selected to work in schools that aimed at using formative assessment practices. With these teachers I further developed the instructional sequence of silent video tasks. Findings from paper IV thus also informed research question Q4 and the development of design principles for silent video tasks (see section 5.2 on p. 70).

As mentioned in the previous section, five out of the six practices/strategies for technology-based formative assessment that Wright et al. (2018) listed, were considered to be supported by silent video tasks. At the start of the second implementation phase, this was going to change. The findings section of paper IV describes how teachers influenced the silent video tasks' instructional sequence in order to *make feedback immediate*, in other words, to make the silent video task practices support all six strategies that Wright et al. (2018) describe (see the preceding section for a list of these strategies).

Written descriptions of ways in which teachers implemented silent video tasks are provided in the form of a vignette in the introduction to paper IV, and at the start of the findings of paper IV. The main change–as compared to the implementation in the first phase–was that immediately after teachers received students' responses to the silent video task, they would gather everyone for group discussion. During this group discussion, all students' responses were viewed in random order. They were reacted to and reflected on with the aim deepen and widen students' understanding and to become aware of differences and similarities in how we and our peers understand, describe and explain what they see in the silent video. For example, as described at the end of the vignette in paper IV, we see Orri use tactics described by Stein and Smith (2011) regarding what type of questions can support active thinking and participation in classroom discussion. He uses rephrasing ("So you say that... did I understand you correctly?") and asks learners to repeat or rephrase someone else's reasoning ("Can you repeat or explain to me what they just said?"). Also, he allows for enough wait time for students to think about their answer.

Data from Orri, a teacher who implemented three silent video tasks over the course of one semester helped to see what influenced teachers' decisions regarding the instructional sequence. This was made possible by introducing *think aloud exercises*, where Orri thought aloud about his intentions regarding the task instructional sequence, either before task implementation or in reflection to the last task implementation. Orri developed his practices to accommodate discussion in his classroom. Findings based on these data informed research question Q4 (see section 5.2.2 on p. 75 in the next chapter).

In addition to using the six strategies identified by Wright et al. (2018), paper IV also uses some of the five key strategies identified by Wiliam and Thompson (2008) for formative assessment and the five dimensions of the TRU framework (Schoenfeld, 2018) for the purpose of data analysis. The five dimensions of the TRU framework are the following:

i. *Mathematics*, the richness of the mathematical content,

ii. *Cognitive demand*, an opportunity for students to engage in productive struggle,

iii. *Equitable access to content*, all students participate and are involved in meaningful ways,

iv. *Agency, ownership, and identity*, opportunities for students to develop a sense of agency, i.e. to make mathematics their own, and to develop productive mathematical identities as thinkers and learners, and

v. *Formative assessment*, the degree to which students' ideas and interpretations are made public and responded to in productive ways (Schoenfeld, 2018).

These dimensions were previously in the literature used to inform professional development, but to me they acted as a helpful lens through which I could view data from the second implementation phase. When looking into whether silent video tasks offered opportunities for practices along the five dimensions–practices used by teachers to support students' understanding–I was confronted with tensions that arise when teacher practice goes against persistent previously established classroom social norms or socio-mathematical norms (Yackel & Cobb, 1996). Although I find it likely that I will have to work further (in the next cycle of reviews) on my analysis, some descriptions of opportunities and challenges that silent video tasks bring to the mathematics classroom (based on data) are described in the current version of paper IV.

The opportunities of silent video task practices identified in paper IV are:

V1 Students' task responses reveal information that was previously inaccessible for teachers.

V2 Silent video task practices might support teachers in institutionalising knowledge.

V3 Silent video task practices might provide access for all students to the classroom discussion.

The last opportunity, V3 *Silent video task practices might provide access for all students to the classroom discussion* was also considered to be a challenge and therefore it is listed again in the following list of challenges of silent video task practices:

X1 Silent video task practices might provide access for all students to the classroom discussion.

X2 It is hard to change a prevailing socio-mathematical norm (for example, that there is a single correct answer).

X3 It can be tempting to return to teacher-centred transmission of knowledge.

X4 It is challenging to lead group discussions based on students' ideas.

X5 It is hard to change prevailing social norms on the motivation role of the final grade.

These opportunities and challenges informed research questions Q7 and Q8 and thus will be discussed in the next chapter in section 5.4 on p. 83. The TRU framework was not only used as a lens for the interview data but also to organize the intentions behind the design of silent video tasks, along the framework's five dimensions. Finally, the intentions and the results based on data were discussed and contrasted along the five TRU dimensions.

There is a problem that I encountered in the final stages of writing this thesis. This problem is connected with my reference to Brousseau (1997)'s notion of institutionalising knowledge in paper IV as part of one of the opportunities that silent video tasks bring to the mathematics classroom. I will address this problem in section 5.4 on p. 83.

To summarize, the methodological contribution of paper IV consists firstly of the use of *think aloud exercises* and secondly of the use of the TRU framework as a lens through which to view data. Paper IV informs research questions Q7 and Q8 on opportunities and challenges and Q1 and Q2, by providing insights into tensions created by the assessment of students' responses to the task and the prospect of leading group discussions. It also informs Q5 and Q6, because it gives insights into what teachers pay attention to when listening to students' responses to silent video tasks. Furthermore, it contributes significantly to inform research question Q4 on decisions made by teachers which influenced the silent video tasks' instructional sequence.

5. Findings

This chapter gives an overview of the research project findings, organized by the eight research questions such that each section answers a pair of consecutive research questions:

Q1 What are participating teachers' expectations of using silent video tasks in their classroom?

Q2 What are participating teachers' experiences of using silent video tasks in their classroom?

Q3 What are some characteristics of a video that can be used in a silent video task?

Q4 What are some factors that influence teachers' decisions regarding their implementation of a silent video task, how did these decisions impact the tasks' instructional sequence development, and why?

Q5 What do teachers notice when listening to student responses to the silent video task?

Q6 What do teachers notice about their own abilities to use the silent video task to support student understanding?

Q7 What are some opportunities that silent video tasks can bring to the mathematics classroom?

Q8 What are some challenges that silent video tasks bring to the mathematics classroom?

5.1. Teachers' expectations and experiences of using silent video tasks

This section provides some answers to questions Q1 and Q2. These two questions were fundamental because they influenced the pre-defined questions used in semi-structured interviews. As a consequence, they impacted all papers I, II, III, and IV.

Drawn from all the papers, the answers provided here deal with particular parts of silent video task practices that created teacher tensions and are organized into subsections on the use of technology, on the assessment of students' responses to the task, and on the prospect of leading group discussions.

5.1.1. Use of technology

As described in papers I and II (see p. 99 and p. 109 respectively), based on analysis of data from the first implementation phase, teachers expected students to run into problems with technology. Some teachers expressed worries regarding technology possibly failing and one teacher displayed his worries by preparing detailed written instructions for students. The latter happened despite clear suggestion to give short oral instructions for the task and letting students solve technological issues if these would appear. When students coped well with solving technological problems, teachers were surprised by their students ability to manage the use of technology. This caused me to suspect that teachers might not be aware of their students' technological reality.

Teachers worries and lack of awareness regarding students' technological reality might be explained by the fact that they belong to generations that did not grow up with internet and smart phones like their students did. Many students use social media regularly to share video recordings from their daily lives. When given the responsibility to solve any unexpected technological issues themselves, students were noted to deal with it and feel like winners afterwards. In the second implementation phase, even those few students that at first seemed upset by the announcement that their recording would be played for everyone to listen to later on, were observed explicitly asking their teacher not to forget to play their recording during the classroom discussion.

All in all, it might therefore be that the worries regarding the use of technology were not only related to what technology they would be using but also (and possibly even more) to the fact that teachers would be introducing new ways of working. In other words, teachers' worries were connected to them knowing that they would be going against previously established norms by leaving the solution of potential technical difficulties to the students. It would therefore be likely to cause tension in the classroom. Different from what teachers expected, interviews in both implementation phases and observations during the second implementation phase demonstrated that there was little to be feared when it came to students dealing with technology. A teacher who gave students full responsibility to tackle technological issues themselves created situations of productive struggle for her students.

5.1.2. Assessment of students' responses to the task

Papers II and IV describe how teachers expected students' responses to include certain concepts and/or ways of narrating the video and to "make good use of time". Also, if students handed in two responses to the task, teachers expected the second response to be more detailed and were disappointed when this was not the case. Teachers' focus of attention seemed to point toward what was missing and what was unclear rather than what was there.

This implies that teachers pointed their attention to whether students' responses were correct or not, something which teachers are used to do with other tasks in the past. Stepping back into their previous ways of working in a situation that teachers expected to create tension hindered them from getting discussion started in the classroom. Four teachers described or implied how they experienced a need to rectify information that was incorrect either by asking closed questions such as "How was the formula for the slope of a line, again?" or by giving an example of how they themselves would have narrated the video.

Traditionally, assessment includes teachers evaluating the correctness of their students' task responses. Thus, teachers paying attention to errors or gaps in students responses rather than listening to *what is there*, making connections or coming up with questions that further students in their learning does not mean that the teachers failed or did not pay attention to my suggestions. It rather indicates that I was not explicit enough about what my intention was and/or that their particular classroom situation–the context where the assessment took place–did not allow for or made it hard for them to make a change.

According to Morgan and Watson (2002) some factors that influence teachers acts of assessment are 1. Teachers' professional knowledge of mathematics and the curriculum, including affective aspects of their personal mathematics history, 2. Teachers' beliefs about the nature of mathematics, and how these relate to assessment, 3. Teachers' expectations about how mathematical knowledge can be communicated, 4. Teachers' experience and expectations of students and classrooms in general, 5. Teachers' experience, impressions, and expectations of individual students, and Teachers' linguistic skills and cultural background (Morgan & Watson, 2002). This framework was briefly mentioned in section 2.5 on p. 23.

Thinking in terms of these factors, I noticed

- how teachers seemingly thought of transmissive methods as "the way to go" in order to give feedback to students. This might be connected to their view of mathematics, because "many teachers do not see the value or even the possibility of discussion in mathematics as a consequence of the view of mathematics which they hold" (Pimm, 1987).

- how one teacher expressed his worries that students might be negatively affected by hearing potentially wrong task responses. This might be related to this teachers' expectations about mathematical learning being a process of acquisition where students passively receive knowledge and ideas will be internalized by students without (critical) thinking.

- how teachers in remedial classes seemed to have low expectations of their students' performances, and

- how teachers in non-remedial classes expected their students to "do better" than they did.

Contrasting the view of discussion described by Pimm (1987) above, one teacher described how her questions and contrasting of students' responses led her students into a heated debate based on two different interpretations of the definition of the unit circle. The unit circle links together arithmetic, algebra and geometry. For students to understand these different representations takes time (Lakoff & Núñez, 2011). Therefore, some students did not realize the video's connection to the trigonometric functions until the discussion started. But they could enter the discussion based on their peers' voice-over responses and a recent in-class presentation of what the unit circle is and how it connects different branches of mathematics. The teacher had not anticipated this issue to come up in the discussion but she had previous experiences of using constructivist teaching methods, and realized that students' discussion was important in their process of learning. Another teacher was observed asking students to clarify their own responses as well as asking other students to reflect on their peers' response. After leading this discussion that students participated in, he expressed a sense of empowerment.

5.1.3. The prospect of leading group discussions

In papers II, III, and IV, I discuss how the request of leading group discussions created tensions that in some cases made teachers do unexpected things. Quite different from what we had discussed beforehand, three teachers in the first implementation phase prepared a teacher monologue in the follow-up lesson instead of leading a group discussion. Even during classroom observations in the second implementation phase, I observed teachers suddenly shifting from leading a discussion to giving a monologue. As mentioned in the last subsection, one teacher explained that he had feared that students would be negatively affected by listening to flawed responses, in other words he expected them to potentially learn some nonsense from each other.

Teachers giving feedback to students' task responses via a monologue rather than leading classroom discussion does not mean that teachers ignored or failed to follow my suggestions. These teachers did their best to cope with the new situation created by the silent video task. Teachers were rather inexperienced when it came to leading group discussions and some of them seemed to have the expectation that group discussion would not to be a good way of giving feedback to students and that a monologue would be more effective. One teacher explicitly stated that normally when he asked questions it did not spark discussion, but maybe one response–something which might be connected to "give the correct answer"-norms and the type of questions asked (more closed than open).

The efficiency of a monologue was discussed by two teachers and implied by one teacher in the first implementation phase. For the non-remedial courses that these teachers were responsible for, they described considerable time pressure. In such cases, discussions might give the impression of being a waste of time. Understandably, teachers did not want to waste any time when it came to preparing students for the final exams.

It is worth noticing that these final exams were made and designed by the teachers themselves and that there are no standardized exams in Icelandic upper secondary schools. Exams in four out of five participating schools were mostly composed of procedural tasks and teachers for the most part used transmission-oriented teaching methods. Such methods often give the impression of being time-efficient. Teachers who were open toward and already experimenting with socio constructivist teaching practices (e.g. inquiry-based methods) considered it more likely to use silent video tasks again.

Like discussed in the previous subsection, teachers were used to being evaluators of students' work in terms of right and wrong. This is something which is hard to imagine to be done via discussion. These teachers had seldom if ever led group discussions in their mathematics classes and most of them had little or no experience of non-transmissive teaching practices. Some of them were using technology such as GeoGebra but it was seldom used in an explorative way.

At Common Raven, Whooper Swan, Arctic Tern, and Rock Ptarmigan High Schools teachers normally wrote notes on the board and assigned practice problems to work on from the textbooks. Students could copy the notes from the board and in some cases they had time to work on problems in class, generally after seeing an example of how a similar task could be solved. These practices shaped the classroom social norms.

Such teacher-oriented lessons are common not only in Iceland, but also in Germany, the UK, and the Netherlands (Engeln et al., 2013). Watching teachers reactions to the implementation of silent video tasks was therefore quite interesting. They are not meant to be teacher-oriented, but based on my previous experiences from the NGGN teaching experiment, I was hoping for the tasks to be somewhat accessible for all teachers. At least, I continued to refine the ways in which teachers could be supported in the shift to using silent video task practices.

After the first implementation phase, it was clear that teachers had been hesitant toward playing examples of students' responses. They worried that their students might be shy to share their task response with the rest of the class. To address this issue, as mentioned in subsection 3.3.2 on p. 36, I considered involving some pseudo-responses as suggested by Evans and Ayalon (2016). However, in the first interview of the second implementation phase, Andri and Edda pointed out that there was no point in bringing in responses including issues that did not emerge from students' participation. These issues might come up later or they might even not be relevant and thus only create confusion. We decided to concentrate on where students are now and how to reach the next goal.

To summarize, the past three subsections described practices where there was considerable difference between teachers' expectations and experiences. Tensions that arose had to do with practices that clearly went against previously established norms in classrooms. These tensions were created by the use of technology, the assessment of students' task responses, and the way of giving feedback to students via leading a group discussion.

Rather than a conclusion, a new question may emerge from the above considerations: How can teachers be pointed towards looking at what is there rather than what is missing in students responses, and to build on that? This is a question that needs to be tackled in future work.

5.2. Video characteristics and instructional sequence development

To answer Q3 and Q4, this section first gives an introduction to and describes the design principles of silent video tasks (subsection 5.2.1) and then continues to discuss some factors that influenced teachers' decisions regarding in what ways they would implement the silent video task (subsection 5.2.2).

5.2.1. Design principles

This subsection describes the design principles of silent video tasks using a format suggested by van den Akker et al. (2013, p. 67). The design principles are presented in their current state, based on experiences from the research project, and open to further developments in the future. They include the characteristics of a silent video, the silent video task instructional sequence, and some underlying theoretical and empirical arguments. The video characteristics are intended as a state-of-the-art guideline for the design and creation of animated video clips that can be used in a silent video task. The instructional sequence describes the procedures by which a silent video task can be implemented.

The use of silent video tasks was studied in the context of Icelandic upper secondary school classrooms. However, according to i) experiences from a pilot study where silent video tasks (at the first stages of design, in 2014) were used in Estonian, Icelandic, Latvian, Lithuanian, and Swedish lower secondary school settings (Hreinsdóttir & Kristinsdóttir, 2016) and ii) workshops given in 2015–2019, where I used silent video tasks with in-service and pre-service teachers in Austrian, German, Icelandic, Slovakian, and Welsh professional development settings, these design principles can also be used in the context of lower secondary school, pre-service and in-service teacher training settings.

A silent video task is intended to expose students' current thoughts, ideas and understanding of a certain previously studied mathematical topic in recorded speech, such that these can be utilized in a discussion with the aim to deepen and widen our understanding. When designing a silent video, it is advised to give it the following **characteristics**:

C1 Keep it short (less than 2 minutes long).

C2 No words or sound, the silence is intended to start an inner discourse (thought) and thus give a reason for a voice-over to be made (words, outer situated discourse).

C3 Include some change or movement (otherwise there would be no need for it to be a video).

C4 Keep it simple (not too overwhelming) and thus accessible for learners to narrate.

C5 Pay attention to the pacing of movement (timing of events). It should allow for learners to describe, explain or narrate what is happening in the video.

C6 Focus on one mathematical concept, its definition and/or properties thereof.

C7 Do *not* show a one-way-to-describe process (e.g. do not display one certain problem solution procedure) but rather make the video offer possibilities for various (also creative) responses–in other words, make the video possess some sense of openness such that there is no single correct way of describing, explaining or narrating the silent video.

C8 Make it offer learners an opportunity to relate mathematical ideas, possibly deepening their mathematical understanding.

C9 Make it offer an opportunity for learners to express a generality.

The silent video task is implemented via the following **procedures**:

P1 (optional) Teachers might present the silent video at the start of a learning sequence and collect words that come into learners' minds (e.g. in a word cloud) without starting the silent video task itself.

P2 Teachers show the video to the whole group of learners toward the end of a learning sequence.

P3 (optional) Teachers might guide students through a process of closing their eyes and making gestures to indicate what happened in the video.

P4 Teachers make learners aware of that various approaches are possible (many different voice-overs "fit to" the video), that their task response might help other learners to gain access to the mathematics shown in the video, and that it is theirs to decide what to focus on in their response.

P5 Teachers assign learners into groups of two using a visibly random method.

P6 Pairs of learners watch the video as often as they like on their own device while they prepare their voice-over recording.

P7 Learners get an opportunity to acknowledge, express and discuss their own interpretation of what they see in the video.

P8 Teachers refrain from answering proximity questions (asked only because the teacher is nearby) and stop-thinking questions (such as "Should I rather mention intercepts or slopes?" or "Is this right?").

P9 Learners record their voice-over and send it to their teacher.

P10 All learners' responses get listened to (in random order) and discussed in a whole group discussion.

P11 Teachers encourage learners to reflect on and reason about their own and their peers' responses to the task.

P12 Teachers keep their ears open to possible conceptual obstacles that can be addressed in the group discussion.

P13 Teachers facilitate awareness of how precision in language plays a role when it comes to understanding others and making ourselves understood.

P14 (optional) After group discussion learners can write a reflection in their journals ("notes to my future self").

P15 Teachers re-listen to students' responses for reflection and for the planning of further teaching activities.

P16 (optional) Teachers might send individual feedback to students.

It is founded on the following **theoretical arguments**:

T1 We can think of cognition as a process of communication, either intercommunication within the individual or social communication between individuals (Sfard, 2008).

T2 A lesson without the opportunity for learners to express a generality is not, in fact, a mathematics lesson (Mason, 2005, p. 297).

T3 Spoken language is an important learning medium in mathematics classrooms (Pimm, 1987).

T4 When evidence about learner achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction, these steps are likely to be better, or better founded, than decisions they would have made in the absence of that evidence (Wiliam, 2011).

T5 Teachers might be able to create a situation of *institutionalisation* (Brousseau, 1997), by acknowledging what students did (give it status) and linking it to what has been done previously.

T6 When learners record their voice over to the animated video clip they are restricted by not being able to point or touch. According to Pimm (1989) such a situation can help encourage learners to move from predominantly informal spoken language (with which students are more or less fluent) to the formal written language of mathematics (Pimm, 1989, pp. 65-66).

T7 By talking about mathematical phenomena in our own words, we might develop and become aware of our own understanding (Pimm, 1987).

and the following **empirical arguments**:

E1 The discursive practice of explaining (asking students to give oral explanations) in mathematics was identified as too seldom practised, too seldom set as an explicit learning goal, but an important way of learning in mathematics classrooms (Erath, 2017)

E2 When learners report to their peers, they have to choose what to say, take into account what they know, and what they believe their audience knows. Such practice can place sophisticated linguistic demands on learners communicative competence and help teachers gain access to learners' proficiency (Pimm, 1989, p. 66).

E3 Practices from the Building Thinking Classroom framework (Liljedahl, 2016, 2018, 2020) support teachers in creating an environment where students are expected to think and given opportunities to think:

- Form groups in a visibly random way.
- Give short oral instructions, such that groups will discuss what is asked of them rather than potentially get stuck trying to decode written instructions.
- Acknowledge proximity questions and stop-thinking questions but answer only the keep-thinking question.
- Foster student autonomy.
- Students write "notes to their future forgetful selves" based on the discussion that took place.
- Use formative assessment to help students become aware of where they are now and where they are headed in their learning.

E4 Teachers might identify some inconsistencies or obstacles in students' responses and facilitate a discussion aiming for a conceptual reorganisation: a negotiation process between students and teachers including 1) an exposing event where students explore how they use conceptions, 2) a discrepant event which serves to create a cognitive conflict, and 3) a resolution where students make a conceptual shift (Romberg, 1993).

E5 Make students responsible for solving possible technological issues. This relates to the fostering of student autonomy mentioned above.

E6 In a classroom where thinking classroom practices have not been integrated, the selecting and sequencing of students' responses to the task could be interpreted by learners as judgemental ("going from worst to best").

E7 Students put in more effort when they know that their work will get presented to the whole class (Hreinsdóttir & Kristinsdóttir, 2016).

E8 In a silent film, no commentary directs the viewer's attention where to look. It requires a considerable amount of work from viewers to internalise and describe for others what captured their attention (Pimm, 1995, p. 47).

E9 Asking students to close their eyes and imitate what is happening in the video with gestures and later with drawings before they record their voice-over, might make the path toward recording smoother (Sinclair, 2016).

As these design principles demonstrate, empirical findings from this study along

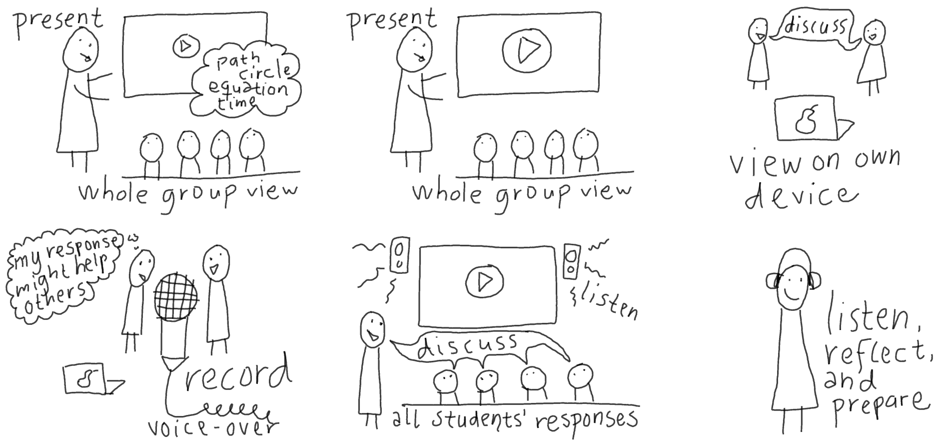

Figure 5.1. Comics that describe the task instructional sequence. Read from top left to bottom right: Teacher presents the video to the whole group. Students can view the video as often as needed on their own device as they work in pairs to prepare their voice-over. Students record their voice-over, keeping in mind that their response might help their peers gain access to the mathematics shown in the video. Teacher and students listen to all students' responses to the silent video task and the teacher leads a group discussion. After the lesson, teachers re-listen to students' responses, reflect, and plan the next steps of instruction.

with empirical and theoretical arguments from other studies influenced the silent video tasks' instructional sequence. To keep track of changes in suggested implementation procedures for the task, I created flow charts and updated them regularly for presentations at local and international conferences and seminars. Viewing these flow charts (see appendix D on p. 213), the development of the task' instructional sequence becomes visible. Also, in 2019 I started drawing comic frames to illustrate steps in the process of implementing a silent video task. Each frame of the comic shows one step in the implementation process. A sequence of comic frames showing an example of the task instructional sequence–as it was toward the end of the research project–is shown in figure 5.1 on p. 75.

5.2.2. Teachers' decisions regarding the instructional sequence

Based on findings from papers II, III, and IV, this section discusses some factors that influenced teachers' decisions regarding in what ways they would implement the silent video task. First, some examples of a mismatch between teacher intentions and actions in the task implementation are given. Then, the importance of involving teachers in task design is demonstrated by referring to inputs from teachers that changed the tasks' instructional sequence. Finally, ways in which classroom norms and teachers' knowledge influenced teachers' decisions are discussed.

Intentions and actions do not necessarily coincide

Readers who have seen the same play in a theatre for a few times will know that the performance is never exactly the same. Conditions could have changed such that the stage is different or the artistic emphasis is different between directors, but even with conditions not changed (same stage, same director, same actors) different performances will be brought forward by the very same actors in the same settings at different nights. This does not mean that the playwright did not articulate explicitly enough what they wanted to get through. The very same feelings can be brought about differently and the whole feeling will also depend on the audience and the atmosphere or "vibe" that they bring.

In general with workers, according to Dejours and Deranty (2010, p. 168) "there is always an inescapable and irreducible gap between assigned work (the task) and actual work (the activity)" due to a number of possible incidents, abnormalities, breakdowns or similar that change the foreseen organization. The very same thing can happen in school settings. When developing mathematical tasks, researchers and designers discuss with teachers in what ways the task can be implemented and either give instructions or suggestions of ways they would like to see it implemented (the task), then teachers decide what to do with that information (their enactment, the teacher activity), and finally students do something based on teachers' actions (the student activity). What happens is dependent upon the tools available, on the classroom social norms and the individuals involved in and shaped by these norms.

So, even if I had described as carefully as I could what my intention was with silent video tasks, it was not unlikely that classroom norms and teacher beliefs might alter what practices were implemented in the classrooms. As discussed in paper II and III, teachers in the first implementation phase made some minor suggestions for change of the instructional sequence verbally and some major changes with their actions in the classroom. The minor suggestions were technical. They included adding more details to the video (making the task less open) and using smart phones to record voice-overs in noisy classroom settings.

The suggestion regarding the use of smart phones was made early in fall 2017 and was communicated to other teachers. The major suggestions made by teachers in the first implementation phase were all presented indirectly by actions. They included giving *detailed written instructions* (instead of the suggested short oral instructions) for the task, performing a *teacher monologue* (instead of leading a group discussion) in the follow-up lesson, and sometimes even *telling students in what ways the teacher would have narrated the video* (instead of using students' responses). All of these major suggestions illustrated the tension which teachers faced when they changed their ways of working and went against the prevailing classroom norms. They did not support a constructivist learning environment and were therefore not considered in the making of the design principles.

Another suggestion which was made indirectly by actions and not included in the design principles was discussed in paper IV. One teacher shifted from leading a group discussion to giving a lecture on line slope. She directed some questions to students but these questions were closed (e.g. "What was the formula for line slope again?") and she did not wait long for an answer. Her shift from discussion to monologue might have been partly caused by unexpected problems with the sound system in the classroom, which made it hard to hear students' responses when they were played for the class. Afterwards, she explained that an important part exam would be handed out to students in the week after, and she wanted students to do well. In other words, her reaction to students' responses was to rectify information by giving a lecture.

The importance of involving teachers in design

As discussed in paper IV, teachers in the second implementation phase made some major suggestions for change of the instructional sequence verbally and via their actions. They took on a more active role already in the first interview and acted as co-designers. The first major suggestion–made before their first task implementation–had to do with feedback to students. Feedback, in their opinion, should take place directly after students handed in their task responses instead of waiting for a follow-up lesson. This suggestion was in accordance with formative assessment teaching strategies presented by Wright et al. (2018) and was accepted immediately. It might seem strange, but I did not realize this possibility until after my discussion with teachers in the first interviews.

Teachers in the second implementation phase also experimented with written peer and self assessment. After trying it out, Orri decided to put more emphasis on the classroom discussion. He would have liked to see that students put in effort when writing their peer or self assessment, but what he received from them was mainly in the form of very short comments such as "Nothing that I notice". A few added evaluative comments such as "It sounded like a description of a football match, not mathematics" or "They don't know the difference between horizontal and vertical" but Orri did not find these helpful for learning. It might be that students comments were affected by the lack of structure for the written assessment, which consisted of open comment fields (see appendix C on p. 209) and no rubric was given. In Orri's next implementation, students did not participate much in the classroom discussion. Still, Orri expressed that this form of feedback seemed to have potential to provide richer feedback for students and more information for himself as compared to the written peer and self assessment.

Before Orri's implementation of SVT2 (see section 3.4.3 on p. 40), at the start of a learning sequence, Orri played the silent video–before students got acquainted with a new topic–to collect students pre-ideas about the topic. To clarify, students did not work on the silent video task until the end of the learning sequence, they only watched the video and sent words to be included in a word cloud via the LMS. Some weeks later, at the end of the learning sequence, Orri then presented the same video again for students to work on a silent video task. This suggestion was included in the design principles (see P1 in the previous section) as an optional step for the instructional sequence of silent video tasks.

Practices that orchestrate productive mathematics discussions were suggested by Stein and Smith (2011) to involve teachers anticipation of possible task responses. In that way, teachers could prepare themselves for monitoring in-class work in the search for task responses that could be sequenced purposefully for presentation. The group discussion during presentation of these sequenced task responses could connect between different approaches and their underlying mathematics (Stein & Smith, 2011). The act of sequencing purposefully for presentation is also addressed by the suggestion to "consolidate from the bottom" in a thinking classroom, i.e. to start with a response that all students will relate to and continue with responses that are harder to unfold, asking students to participate in the *reification* process–to make something that is intangible (or abstract) concrete and real (Liljedahl, 2020). However, as described in paper IV, none of the four teachers participating in the first implementation phase felt confident to select and sequence their students' responses. Instead, these teachers either discussed their own ideas for a voice-over or played a few student responses after asking for volunteers.

Teachers in the second implementation phase explained that the selecting and sequencing of students' responses might be interpreted by learners as judgemental (see E6) and they therefore preferred to listen to all responses in a visibly random order. Nevertheless, according to (Erath et al., 2018), the practice of allowing "any contribution", might produce a superficial broad participation of all students. Students that lack fluency in academic language (e.g. describing something in a general or abstract way using mathematical terms) might experience difficulties in joining the conversation and thus their only contribution would be their recorded response.

Some influential factors

This section describes some of the factors that influenced teachers' decisions when implementing silent video tasks. There were indications of factors at play which included classroom social norms and socio-mathematical norms (Yackel & Cobb, 1996), teachers' situated knowledge (Manizade & Mason, 2011), and teachers' goals, aims, and beliefs. Here, I focus on the factors discussed in papers II and IV, i.e. classroom norms and teachers' knowledge.

The reason for either skipping listening to students' responses, listening only to volunteer responses or listening to all students' responses in random order seemed to be connected to classroom norms, ways of working which constitute what is accepted social behaviour in the classroom context and that teachers and students are aware of (Yackel & Cobb, 1996). The negotiation process of norms seems to happen early in the school year (Cobb et al., 1991). Among the classroom norms that I noticed were the following:

- Teachers did not put emphasis on discussion (or at least had not done so hitherto),
- Teachers normally saw themselves as evaluators of the correctness of students' responses to mathematical tasks,
- The ultimate or main goal seemed to be that students could practice for and pass summative evaluation consisting of mainly procedural tasks,

It was clear from my interviews with teachers that they were aware of constructivist ideas about mathematics teaching and learning. Nevertheless, as can be seen above, their normal practice did not necessarily reflect that and their normal practice seemed to be based on behaviouristic ideas. It would have been unrealistic to expect that teachers would make a fundamental shift from the existing classroom norms all of a sudden.

Students are also resistant to changing their routines. If we consider the above list and some excerpts from teacher interviews collected in the first implementation phase, it seems likely that students were used to traditional teacher-oriented lessons: Getting presented with a procedure to tackle a certain problem, to copy notes from the teacher whiteboard, and work on textbook problems similar to those presented to them earlier in order to practice for exams. Students who are used to such routines will not necessarily be happy about anyone disturbing that pattern. They might, for example, consider it a waste of time to work on a single task for the whole lesson instead of completing twenty procedural problems from their textbook, knowing that similar problems will be on the final exam.

Orri frequently expressed his longing for making mathematics fun and interesting for the students. He wanted to raise his students' enthusiasm such that they would *put in more effort*. Students might have been aware of this. By complaining and putting little effort into working on new types of tasks, they might even have tried to influence Orri's practices such that they probably would not need to deal with that type of tasks again. However, when it came to the silent video task Orri acted as if he did not hear when two students asked if they could rather work on tasks from their topic booklets. He was interested in participating in our task design collaboration and seeing where it would lead. Generally, when Orri was asked to reflect on his work environment, he mentioned the following two limiting factors:

- *Final grade*. The ministry requires him to produce a final grade at the end of the semester,
  - pushing him toward summative assessment practices to produce sub-grades as valid argumentation when parents or students complain about

the final grade, and

  - making it hard to use formative assessment as students' motivation is influenced by whether the task they are confronted with counts toward the final grade or not.

- *Collaboration between mathematics teachers is sparse*, because

  - even though he is encouraged to use new teaching methods and experiment in his teaching, he mostly works on his own, and

  - although he probably would receive support for such practices in the form of productive, reflective, and critical discussion, if he asked colleagues of his to join him in such discussions, he did not find the drive to initiate such discussions thus making is easy to give up when implementing new practices.

Due to these constraints, Orri was very willing to try out silent video tasks in collaboration with a researcher, hoping to see some positive effects on students' engagement. He regularly got disappointed by students' low motivation and sparse effort, but never gave up and continued trying out silent video tasks. He was continuously looking out for new methods that could help him support students' understanding of mathematical concepts. After all, the curriculum expected him to equip his students with understanding of mathematical concepts, although it did not give any suggestions or examples of *how* that goal might be reached. Orri also described how he repeatedly experienced a return to old habits of answering stop-thinking questions and going through lists of things to cover, in reaction to having a hard time with doing teaching experiments on his own. He knew it was not effective, but it was the easy way out.

During Orri's reflection of his second silent video task implementation, he addressed this tension between using what he called the *easy way out* and the *hard way out* in certain situations. He gave the following three examples of such situations:

1. A student asks "Is this right?" and the teacher answers

   a) "No it is not done this way, it is done *this* way."

   b) "Interesting. Can you explain to me your thinking behind this result?"

2. The teacher asks a question and when a student answers, the teacher

   a) accepts surface or shallow explanations without follow-up questions.

   b) follows up the question to push the student toward thinking the original question better through.

3. The teacher prepares learning material for a mathematical topic such that it

    a) cuts the topic down into easily consumable units that students might be expected to slide easily through on their own.

    b) requires the teacher to teach for understanding.

All these situations are composed of a pair of two reactions where the first reaction is the seemingly easy *what seems to work* and the second reaction is the seemingly more time consuming *what he knew would work*. Orri commented that in situations where many students are *stuck* he would rather go for the easy reaction. Also, he explained that his expectations of students to *make connections* often were set high and if the experience was that students–despite the time taken to teach for understanding–did not make these connections, he would be tempted to give up and turn back to the easy teaching methods. To me, all these situations described a selection between *keeping the old behaviourist norms* and *introducing new constructivist norms* following what the teacher knew would work due to his studies of pedagogy and reading about new teaching practices. On a side note, it also seemed to me like BTC practices described by Liljedahl (2020) might be a solution to these often stressful situations[1].

Like previously mentioned, Lilja was the only teacher who facilitated a group discussion in the first phase of the research project in which more than two students took part and where students debated among themselves. She recognized a situation that provided various meanings and managed to to move toward making the unpacking of mathematics become a shared responsibility, allowing students to gain access to thought processes and ideas that the mathematical concept of the unit circle presents. Scheiner et al. (2019) call this an *anthropological-sociocultural approach* to teacher knowledge.

5.3. What teachers noticed when using silent video tasks

To answer Q5 and Q6, I will refer to findings from paper IIand IV. In paper II, I identified the following three themes:

- When listening to student responses to silent video tasks,
    - *teachers notice what is missing or unclear*. This theme captured that teachers pointed their attention mainly in the direction that they were used to, to evaluate the correctness of students' responses.

[1]The reader might remember that I met with Orri to discuss BTC practices as compensation for his time taking part in my research project. However, he did not really start implementing BTC practices until after the research project ended.

  - *teachers have the best intentions for their students*. This theme captured when teachers expected students to "know better" than their responses to the silent video task showed, often drawing upon previous evaluation of students' work on other (procedural) tasks in class.

- The biggest challenge that teachers encountered when it came to implement silent video tasks in their classroom was that

  - *discussing student responses with the whole group is a delicate matter*. This theme addressed what teachers noticed about their own practices. Three teachers explicitly or indirectly expressed stress or anxiety when it came to orchestrating a group discussion. This seemed to have to do with the possibility of losing control of the learning process. Their experiences were shaped by the classroom culture because normally discussions were seldom or never practised in their classrooms.

The above themes were not only encountered in the first phase. As can be seen in paper IV, teachers in the second phase also pointed their attention toward what was missing or unclear, i.e. toward correctness of students' responses–something that they were used to attend to. Silent video tasks were intended to be part of practices where students' responses are part of their participation in an ongoing dialogue. This shift from acquisitionist to participationist perspective is hard when teachers–intentionally or unintentionally–view themselves as holders and evaluators of knowledge and students' responses have the role to inform teachers of students' knowledge for the purposes of assessment and grading.

Similarly, in the second phase teachers also had best intentions for their students. Based on their knowledge of students' responses to mathematical problems in previous situations, they were surprised by the same students' responses to the silent video task and sure that they "knew better" or had "meant it right". In this context, one teacher from the first phase described how he "knew what the students know" as if he expected his students to "learn what he taught". Still, this same teacher was fully aware of and explicitly stated later in the same interview that this feeling of students absorbing what he said (and even "getting it", as in understanding it) was illusionary. There this teacher made what van Es (2011) call a *connection between specific events and broader principles of teaching and learning*, in this case to positivist learning theories. However, he did not transfer this knowledge to the situation. If putting this discrepancy into words awoke this teacher's awareness thereof, remains unclear, but I agree with Mason (2017) that it is important to become aware of such previously overlooked or downplayed aspects of our teaching practice.

When reflecting regularly on their own practice, they might become aware of or notice several things about their own practice that were previously overlooked. This was for example visible in the second phase when Orri, during our third interview[2],

[2]Note that, as promised, I also regularly met with Orri to discuss BTC practices. When the third interview took place, we had already met six times alongside our silent video task design collaboration

gave an account of limiting factors in his work environment and some situations that caused tensions between selecting a constructivist or behaviourist approach to teaching practice (see the preceding subsection). This noticing of his was, however, not necessarily made due to the silent video task practices, and is therefore included in the previous subsection on factors that influence teachers' decisions when they introduce new practices in their classroom.

The reader by now for sure has discovered a reoccurring theme–also in this thesis– when it comes to discussing how difficult and scary and what a big hurdle the orchestration of group discussions seems to be for teachers who are inexperienced in such practices. In both phases of the research project, teachers addressed this hurdle in their interviews connected to either their expectations (before task implementation) or experiences (after task implementation) of a silent video task. Although teachers in the second phase all decided to "give it a go" they all mentioned that they would require more training and practice in the future. It might not be easy to start the practice of engaging students in discussion in the mathematics classroom, but it seems that establishing a *safe space*[3] for students, such that they for example do not fear *being judged* plays an important role (Fraivillig et al., 1999) along with the conscious training of practices that have been suggested by Stein et al. (2008) and Stein and Smith (2011) to encourage discussion.

5.4. Opportunities and challenges of using silent video tasks

This section provides some answers to questions Q7 and Q8. As described shortly in section 2.7 on p. 28 and in paper IV on p. 141, The TRU Conversation Guide by Baldinger et al. (2018) lists a set of questions for planning and reflection organized according to five dimensions of teaching identified by research as critical for students' mathematical learning:

i. *Mathematics*,
ii. *Cognitive demand*,
iii. *Equitable access to content*,
iv. *Agency, ownership, and identity*, and
v. *Formative assessment* (Schoenfeld, 2018).

within the research project.

[3]By safe space, I mean that the classroom as a learning environment is characterized by respect and safety.

In paper IV, questions from each dimension of the TRU framework (see figure 2.2 on p. 29) are answered based on intentions behind the design of silent video tasks (see table 3 in paper IVon p. 141) and based on data and experiences from the second phase of the research project. Some overlap between dimensions of the TRU framework is unavoidable because the categories discussed in each dimension are not completely distinct.

The following are the opportunities identified in paper IV to arise from silent video task practices:

V1 Students' task responses reveal information that was previously inaccessible for teachers.

V2 Silent video task practices might support teachers in institutionalising knowledge.

V3 Silent video task practices might provide access for all students to the classroom discussion.

The last opportunity, V3 *Silent video task practices provide access for all students to the classroom discussion* was also considered to be a challenge, especially in the case when task adaptations were needed, and therefore it is listed again in the following list of challenges of silent video task practices:

X1 Silent video task practices might provide access for all students to the classroom discussion.

X2 It is hard to change a prevailing socio-mathematical norm (for example, that there is a single correct answer).

X3 It can be tempting to return to teacher-centred transmission of knowledge.

X4 It is challenging to lead group discussions based on students' ideas.

X5 It is hard to change prevailing social norms on the motivation role of the final grade.

As has been discussed earlier, previously inaccessible information was revealed by students' task responses. In other words, silent video tasks have the potential to make previously unnoticed inconsistencies or problems regarding students' mathematical ideas (understanding) or ways in which they express their ideas (precision in word use) visible to teachers. This means that for the teacher, silent video tasks are a tool that can be used to reveal students' conceptual or commognitive conflicts, providing situations that are key to learning–that is if the situation gets recognized and resolved. Commognitive conflicts appear when students face a discourse that clashes with their own (Sfard, 2008, pp. 254-260). Since silent video tasks can create such situations where a conceptual conflict or a commognitive conflict, they can provide opportunities for teachers to address them and resolve them.

In the last chapter, when presenting an overview of the results from paper IV, I mentioned that its reference to the notion of institutionalising knowledge might be problematic. This is something that I realized in the final stages of writing this thesis and I will for sure continue to work on in future versions of paper IV and/or in other papers to come. Namely, the way in which I view teaching and learning mathematics as a constant process of becoming–becoming part of a certain community, participating (according to certain norms that are agreed upon explicitly or indirectly) within that community, in some social and cultural context (see e.g. Sfard, 1998) might not coincide with the ideas behind Brousseau (1997)'s notion. I will therefore, either need to find someone who already addressed this or develop a concept myself that addresses *how in the process of learning teachers might bridge between students' discourse and mathematical discourse*, defining relationships between students' "free" ways of describing, explaining, or narrating the video, and the more "formal" or "mathematical" ways of describing, explaining, or narrating the video that are defined by the culture of mathematics as an academic discipline.

This might be connected to what Sfard (2008) calls an *agreement on the leading discourse* in a section where she discusses some implications of commognition for the practice of teaching and learning. In that section she, among other things, concludes that the idea of a *learning-teaching agreement*, would be necessary–now I am surely not the only one to see the analogue here to the *didactic contract*. It might also be connected to some wider sense of what Sfard (2008) calls the *challenge of saming* where two hitherto unrelated objects come to be counted equivalent. According to Sfard (2008) "saming may be most problematic when it is supposed to bridge between colloquial and literate discourses", i.e. the challenge of saming is at its maximum when it comes to bridging between conversational and formal discourse (Sfard, 2008, p. 188). However, saming is also described to be what happens when we have a number of things previously not considered or thought of as being "the same" and then give one name to all those things (Sfard, 2008, p. 302). We therefore might need a wider term than saming, as saming seems more limited than the notion of institutionalisation. In the current version of paper IV, I thus started looking at what Sfard calls the process of *reification*–a process involving a transition from describing processes towards talking about objects (Sfard, 2008, p. 44). In the case of silent video tasks, this process might take place during the whole group discussion if the teacher supports students in connecting their ideas (in the form of descriptions of processes) to the mathematical objects that appear in the video. What remains as a finding is that situations created by silent video tasks might enable teachers to connect between their students' and their discipline's discourses.

In the second phase of the research project teachers made adaptations to generate access to the task for all students. This was considered valuable because the adaptations created access. However, this was also considered to be challenging for the teacher because ways in which tasks can be adapted are not always obvious. The adaptations were made by teachers "on-the-go" and included in one situation allowing an autistic student to hand in a written response to the task and in another situation making sure that second language learners–who in addition to not speaking Icelandic fluently, also did not speak English fluently–would work in a group of three to ease the communication. In the first place, by offering students a new way of communicating in the classroom (by using technology) via the implementation of a silent video task, teachers already created access. This was demonstrated by students who participated in the silent video task but according to teachers, under normal circumstances, these students would have refused to take part in group activities or to say anything out loud in the classroom.

Other four challenges that were identified above are all connected to classroom norms. Challenge X2 addresses how hard it is to change a prevailing socio-mathematical norm, for example that of a single correct answer. This norm of expecting a single correct answer to exist manifested itself repeatedly and might be country-specific. Challenge X5 indicated that it is hard to change prevailing social norms on the 'trading' role of the final grade, meaning that it can be cumbersome for teachers to enhance students' motivation in formative assessment practices when students are mainly driven by final summative assessment. Due to these two norms of students assuming 'one correct answer' to exist and the motivation role of the final grade, teachers can encounter tensions when implementing silent video tasks. These tensions are due to the openness of the task and its formative assessment role, respectively.

Another norm-connected challenge, X3, is connected to X2 and X5 and describes how tempting it can be for teachers to return to teacher-centred transmission of knowledge. When teachers are required to shift to working in a socio-constructivist way–like happens in the implementation of silent video tasks–and then encounter tensions in the situation, the easiest way out of that situation can simply seem to be taking control back and using transmissive methods. This is nothing new. In most situations it is understandable to turn back to previous practices when problems are encountered with the new practices, especially in complex situations such as that of integrating classroom discourse into the classroom culture (Walshaw & Anthony, 2008). That brings us to X4, namely the challenge to lead group discussions based on students' silent video task responses.

My findings indicate that silent video tasks can create discourse opportunities in the classroom–something which, according to Walshaw and Anthony (2008, p. 542), can be quite complex. However, it cannot be taken for granted that teachers *can make use of the learning opportunities* which might be present in this shared experience of discussion based on students' task responses. Challenges for teachers are present both in terms of time management and in terms of in-the-moment awareness of opportunities to deepen or widen the discussion, e.g. based on some kind of a disturbance that can spark development. This could for example be a situation of conceptual or commognitive conflict. If the teacher recognizes and makes use of such opportunities, development or learning can find place (Mason, 2002). In turn it might lead to the process of *conceptual reorganisation*, which was mentioned in section 2.1 on p. 13 where students–after being confronted with a conceptual conflict–reach a resolution by making a conceptual shift (Romberg, 1993), or the resolving of a commognitive conflict.

5.5. Summary

This chapter provided answers or part of answers to eight research questions. These answers addressed tensions that were created by silent video task practices, provided some insight into what teachers notice when using silent video tasks, and discussed some opportunities and challenges that arise from using silent video task. Also, one of the answers was presented in the form of a framework for silent video task practices, as presented by design principles. These design principles include silent video tasks' instructional sequence, based on findings from the research project and both theoretical and empirical arguments that were adapted and taken from underlying constructivist theories. To name a few of these underpinnings provided by existing theoretical and empirical arguments, I used some strategies used for formative assessment practices, the five dimensions defined by the TRU framework, and some BTC practices were among the underlying theoretical and empirical arguments listed in the design principles. In his commentary chapter to the book on task design edited by Watson and Ohtani (2015), Ruthven (2015) summarizes four elements, one or more of which any framework used for task design encompasses: 1) a template for phasing task activity, 2) criteria for devising a productive task, 3) organization of the task environment, and 4) management of crucial task variables. With the help of different frameworks, my own bricolage framework comprises of at least three of these elements.

The overview of findings demonstrated how important it is to inform teachers of students' technological reality and to provide them with professional development courses that offer support when they implement new practices in their teaching. For example, the fact that teachers–even teachers who are willing to change and enthusiastic about using constructivist practices–repeatedly get pushed back to see themselves as evaluators rather than facilitators of students' learning, must imply that they need support of some kind. Knowing from the findings of this study that teachers do not necessarily take the initiative themselves to seek colleagues to collaborate with, to discuss with, and to train their awareness with, it seems that this initiative must be supported by other professionals.

Finally, and maybe most importantly, the findings indicate that silent video tasks allow teachers to get insight into what students are thinking, and that students' responses can form a base for conversations between students and teacher and among students that aim to deepen and widen our understanding of mathematical phenomena. Furthermore, teachers can utilize the information gained in the process of implementing a silent video task to make decisions on where to head next with their students.

6. Conclusions

6.1. How this research has contributed to the bigger aims

In this thesis, I have discussed a research problem concerning the identification of opportunities and challenges that arise from using silent video tasks in the mathematics classroom. In order to tackle that problem, I had to define what characteristics a silent video must possess and iteratively describe and develop the silent video tasks' instructional sequence. In preparation thereof and throughout the process I read papers and books from various fields of mathematics education and communicated with other researchers. However, what was most important, was to work with teachers in their classrooms and follow their expectations and experiences of the task. Based on work with teachers, tensions that were created by silent video task practices were identified and some insight into what teachers notice when using silent video tasks was gained.

The idea of silent video tasks is grounded in social constructivist theories that consider it important that interaction happens between teacher and learners and among learners themselves, who work together (support each other) toward richer understanding of mathematical content. The learner is seen as an active participant in the teaching and learning process and in the case of silent video tasks, learners get an opportunity become aware of their own and their peers' current knowledge of a mathematical topic.

With my work, I hope to have managed to give sufficient detail about silent video task practices, especially their instructional sequence such that they can be used by any interested teachers, teacher educators, and education researchers. Thus, giving learners a chance to reflect on various explanations, descriptions and narratives and possibly–by recognizing their own present knowledge state–develop their metacognition.

Teachers were challenged by experiences that they made with silent video tasks, as was demonstrated in the preceding chapter, where opportunities and challenges were identified. The challenges listed there are quite substantial. Teachers might, for example, view it as too tedious of a task to embark on the journey of orchestrating classroom discussions or building a thinking classroom. Still, since every participating teacher experienced surprise, curiosity might make teachers want to try these tasks out.

6.2. Toward a more student centred classroom: What I learnt

> Teachers need support and resources to experience new ways of working (Swan, 2006).

Now, at this doctoral thesis writing-up milestone on my journey of becoming an active participant in mathematics education research, I look through my notes and excerpts from literature and realize that in many ways I have re-invented the wheel. My feeling that students' voices matter–the discovery I made when listening to my students' responses to a silent video task back in 2014–was already made by many before me and for example phrased by a teacher participating in a study by Paul Cobb who said:

> I have become a better listener. Teachers are basically talkers who feel a strong desire to share their knowledge with other people. Children are no different. *If we really make an effort to listen to our students, we will become richer for it* (Cobb et al., 1990, p. 135).

If teachers were doctors, it is rather likely that it would cause an uproar that learners are being "treated" with ideas of learning that have been shown not to be helpful. How come that even though the same topics are taught year and year again with little modification, students mostly memorize and forget facts and procedures? Might it be because these make little sense to them? Because they have been sitting at their desks starting on page eleven, problem three, mimicking some method that had been shown to them at the start of class and continuing mindlessly from there, parroting in silence, struggling in isolation and never entered a mathematical debate? Never were confronted with a problem of a type that they had "never seen before", seldom had to reason about or discuss with others what they did, never experienced the classroom as a safe space for mathematical discussion, where their points of view were valued?

Most if not all mathematics teachers in Iceland and their learners' parents of my generation and above went through elementary and secondary level–some also university level–education where mathematics was presented as a set of procedures to be learnt. In Iceland, teachers are given the power to suggest their course curricula–which is a wonderful idea in a way–but the environment is generally not supportive of them changing their practices toward creating a constructivist environment in their classrooms and the few mathematics textbooks that are available in Icelandic do not support such practices. Providing funding for the creation or translation of teaching resources is something that can be changed.

Another hurdle when it comes to changing practices is that the change can make parents puzzled and teachers feel insecure and out of control. In the context of a professional development course, teachers might be provided with a safe space where they can experience on their own what students will experience: active engagement, exploration, creating and testing hypotheses, reasoning. They might come to learn–like I did–that implementing a few new activities that build on constructivist ideas solely will not change anything. It will only create the same illusion as I had back in the days when starting carefully to integrate a few ideas from Mason and Swan–the illusion of having made "quite some change" but actually having only added a little constructivist drop into the ocean.

In the preface to this thesis, I mentioned my surprise upon receiving my students' responses to the first silent video task that I implemented in my classroom. One of the things I learnt from this research is that we often forget to listen to students. In a way this implies that we forget to approach our students as "math persons". And students claiming that they are "not a math person" or "not good at math", they develop math anxiety, avoiding the situation of the teacher intervening and therefore–at least in a situation where students are working individually–act as if they are learning instead of actively thinking. They may even act helpless to receive assistance that will get them through the task–because they do not believe they themselves are able to handle it–and there starts the vicious cycle of spoon feeding. Is it strange that teachers are surprised by students' responses to a silent video task if we in general do not train our awareness and pay attention to what our students are saying? For my parts, I have started paying more attention to stopping myself and thinking "Wait a minute, should I answer this? Do I have enough information in this situation? Should I not rather be *asking* some questions here?"

6.3. My message to you: Communication and collaboration is key

To summarize, what we can learn from the work presented in this thesis is mainly the importance of discussion and communication in the classroom. By providing students with ways of participating in the classroom discourse–both in terms of how they participate and what they bring to the discussion–we are contributing to their sense-making and the development of their understanding. After all, that is one of the various important goals that we have as educators.

One might think, when reading this thesis, that my view of procedural exercises were that they are all bad and should be thrown out of curricula, into the trash bin. This is certainly not the case. It just so happens, that the silent video tasks are not about procedural exercises and, more importantly, it very much matters–when procedural exercises are used–*how they are used*. Students' personal construction of mathematics and mathematical meaning is mostly ignored in classrooms *where transmissive teaching* *methods are practised*, but not necessarily *where there are procedural tasks present*. For example, Watson and Mason (2006) illustrated how a set of procedural exercises (viewed as one object) can build the basis for conceptual understanding.

On basis of what has been presented in this thesis, I claim that silent video tasks allow i) students to make sense of things and articulate their mathematical ideas, ii) us teachers to catch a glimpse of what students are thinking, iii) all of us to attempt to understand each other, and iv) based on information received in the process we teachers can make decisions regarding how to move forward from where we are at.

This thesis reported on research that showed that silent video tasks are one way of providing students with ways of participating in the discourse of the mathematics classroom. In a silent video task, teachers prompt students to explain, describe or narrate a video in collaboration with a peer, allowing for sufficient time to prepare a voice-over response to the task. This situation offers an opportunity for thinking. Furthermore, if students refer in their voice-over to special cases or examples, and generalize about the situation shown in the video, trying to convince the viewer that what they are saying is true, then they are using processes which are identified by Mason et al. (2010) and Stacey (2007) as being fundamental processes in *thinking mathematically*. Moreover, in the group discussion based on students' responses to the task, teachers position students as mathematical thinkers worth listening to because their responses are results of students' thinking.

The shift toward assigning open tasks, utilizing whole-class discussions, and making use of formative assessment all imply changes regarding what is valued as important in mathematics classrooms. It changes the classroom social norms (Yackel & Cobb, 1996). In an upper secondary school classroom in Iceland where lessons normally follow the *traditional* form where teachers introduce a new procedure, assign some similar problems for students to work on individually and wander around the classroom answering some reassuring proximity questions and stop-thinking questions, a task like a silent video task is certainly like an alien visiting, going against all norms. Because of tensions created by the (alien) silent video task practices, it becomes tempting for teachers to–rather than orchestrate a group discussion–make a short-cut, present their own knowledge directly via monologue and "save the time" of letting students think for themselves. This is as understandable as it is unfortunate.

Nevertheless, as the findings indicated, teachers "knew" that such practices do not support students' understanding. One of the things that broke my heart during data collection was when Snorri told me about his multiple teaching experiments made throughout the years where he received little or no support, was in doubt about himself and resided back to previous practice, sad and defeated. Not only Snorri, but all of "my teachers" had tried out and given up on practices that are intended to support student learning and understanding, and attempted to create situations that allow students to think. Seemingly the reason why they gave up was mainly twofold: It was hard to go against the prevailing classroom social norms and socio-mathematical norms and they had no chance of taking part in collaboration of the type that Wenger (2011) calls a *community of practice*, a group of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly (Wenger, 2011). They expressed how they themselves (alone) had a hard time finding the drive to initiate such collaborations. Teachers are–more or less all of them, at least all I know–dedicated professionals. They can lead the way in implementing new ways of working if they get the support needed. It is therefore important that school principals, teacher educators, and education institutes become aware of–if they are not aware of it already–that support is needed for teachers to build communities of practice, within which teachers can collaborate on developing their practices.

6.4. Some remaining or freshly generated, unanswered questions

As mentioned in the design principles' list of procedures for implementing silent video tasks, practices developed by Nathalie Sinclair (see section 1.4 on p. 5) using gestures and imitation could in future work be connected and integrated into the process of assigning a silent video task. Students could be asked to close their eyes and retell the story via gestures (draw in the air), to watch again and draw on paper, and to draft a script on the whiteboard. In this way, the task introduction would take a more meditative approach and students might become better aware of what is expected of them. Within the frame of this research project, no experiments with such meditative approaches were made. It remains as something for us to try out in the future.

In a study by O'Connor et al. (2017) they come to the conclusion that if the classroom norms include a culture of active participation, it does not matter *which* students take part in the classroom discussion as long as *some* of them do. However, as observed in both phases of the research project, teachers who are inexperienced with leading class discussion–be it based on student contributions or not–might feel strongly inclined toward making moves to correct inaccurate or erroneous responses instead of engaging students in discussion based on their ideas. Thus, (sadly) the attention shifts toward how faulty students' thinking can be instead of seeing student thinking as useful material to develop our thoughts. It is hard to change the habits of teachers seeing themselves as mainly in the role of evaluators of students' work toward making it a shared responsibility of the whole class to evaluate and reflect upon emerging ideas (Doerr, 2006).

Rather than a conclusion, a new question may emerge from the above considerations: How can teachers be pointed towards looking at what is there rather than what is missing in students responses, and to build on that? This is a question that needs to be tackled in future work.

Questions that arise from my work are partly already being worked on. In his PhD project, Chris Kooloos at the Radboud University is exploring ways in which teachers can orchestrate classroom discourse about variation in solution methods, building on suggestions by Stein et al. (2008) and Stein and Smith (2011). His teacher community of practice built within a professional development course had visible positive effect on teachers previously inexperienced with using group discussion in their mathematics classroom (Kooloos et al., 2020).

Chris is working on identifying some ways to make such effects sustainable, although he underlines that it is harder than expected (personal communication, December 2, 2020) and of course, it cannot be viewed as a task for one researcher to tackle. Hopefully, there will be a chance for many others–me among them–to contribute to this research in the future.

Fernanda Martins da Silva is at the start of her PhD research under supervision of Marcello Borba and plans to look deeper into how silent video tasks can be used with pre-service and in-service teachers. Maybe she will look into how silent video tasks can support pre-service and in-service teachers in developing awareness of the multitude of student interpretations, or whether and how silent video tasks evoke the need for developing clear definitions of mathematical concepts.

Another interesting path would be to define in what ways silent video tasks can become a mediator between different types of discourses, and if they might shorten the so-called discourse-gap described by Leung and Bolite-Frant (2015). This gap between the phenomenal world and the conceptual world exists because students rather use their own presumed mathematical ideas and discourses in learning activities that promote discussion instead of the more formal mathematical concepts defined by teachers or textbooks. In that work, focusing on the learner, Sfard's commognitive (Sfard, 2008) view might be helpful in the process of analysing of data.

6.5. Final reflection

> The transformation that takes place in educational researchers while undertaking a study is the most significant product of their research, for it is *their* questions that change, *their* sensitivities that develop, *their* attention that is restructured, *their* awarenesses that are educated, *their* perspectives that alter. In short, it is their *being* that develops (Mason, 1998, p. 358).

Looking back on these five years of research on silent video tasks, my ideas have sharpened and developed, my brain has made new connections that almost feel physically sensible as I am writing this now, and I have become a participant in a discourse that I–at the beginning of this journey–had very mixed feelings about. I wanted to stay on the ground, aimed to return to Hamrahlíð Junior College for teaching after my studies, and above all I wanted to avoid becoming one of those academics who were flying high in the air with ideas practically impossible to implement in the classroom and using fancy words that we teachers do not understand.

In 2016 at ICME-13, after attending the early career researchers day, I joined thematic working group (TWG) number 38 on task design and analysis. It was hard to choose between groups, but I thought this topic sounded just about what I would be doing. We were all warmly welcomed by Anne Watson with a very friendly and inviting talk where she shared with us that if anyone was having difficulties accessing the freshly published book on *Task Design* edited by herself and Minoru Ohtani, we should not hesitate to contact her, and she would personally make sure that we would get a copy.

Next up was a talk by Gravemeijer. In the flood of abbreviations and complicated words that followed, I got so frustrated that I could not sit still. What if I was missing something in other groups? Why not check out my list of possibly fitting ones? The room on Hamburg University campus was packed with people and it would be rude to squeeze between people all the way to the front door exit. Lucky, me (I thought, back then), it was a sunny July day, the windows were open and I quickly–with only my seat neighbours noticing–sneaked out by the window at the back. Pheuw! On I went to check out TWG 8 on the teaching and learning of geometry at upper secondary school level. There, I found two gentlemen almost fighting because one of them was so upset that GeoGebra was available for free, and thus ruining his business of a commercial software. This TWG was placed in a room with no windows and the atmosphere seemed tense and unproductive, so I continued my wander around the campus until I reached a TWG on small scale assessment (since then, the small scale and large scale assessment groups have been joined), located in a beautiful room with large windows and a friendly atmosphere. Fine! I will stay here, I thought, and so I did.

Since this first official day of ICME-13, very much has changed. As a teacher, I now have gained more experience with leading group discussions, using open tasks, the use of formative assessment. As a researcher, I am now able to read papers by Gravemeijer and enjoy it (!), understanding almost every word of it and being able to relate to what he is writing about. Of course I always encounter texts within our field of study that are tough to read, but the threshold is much lower for me to enter now. This winter, I partly returned to teaching at Hamrahlíð Junior College, and even though I was lucky to be familiar with the Twitter MTBoS[1], various online resources and methods in which one can engage students in the remote learning setting of Covid-19, it was a surprisingly big challenge for me when schools opened again (with a two meter distance rule and masks) to enter the classroom again with the big backpack of good practices that I have studied about and repeatedly noticing myself falling back into practices that I am not fond of, in order to tackle the situation. It was a great exercise!

[1]MTBoS is an acronym for *Math Twitter Blog-o-Sphere*. It is a community of mathematics teachers who blog and write tweets about their own practices, sharing their experiences, resources and ideas.

The original reason why I started this journey was very much connected to what I talked about in the Preface of this thesis: The fact that I only experienced a constructivist learning environment in other subjects than mathematics in school. This is something that I want to change. After this winter, I realized that working at the university alongside teaching in upper secondary school with different academic years in different countries is a bit too much of a juggle for me. Therefore, I decided to continue working with my university colleagues in Iceland and Austria with particular focus on triggering change in Iceland, because that is what got me on this journey to begin with. Of course, I cannot stop the flow of ideas and possible collaborations and will continue to work on task design as well[2].

To conclude this section, I feel like I have become a part of the mathematics education research community. This feeling is reflected in me being able to take part in the planning and organization of conferences within our field, to take part in writing collaborative papers, to enter the discussion and give constructive feedback at conferences, and to review papers for education journals. It is also reflected in the big network of colleagues that I have been so fortunate to build up. It takes a village to raise a child and it takes a community to raise a doctoral student. I am beyond grateful for having been able to embark on this journey, thankful for everyone involved in it, and happy to be able to present and discuss with you the result of my work. Thank you.

[2]Currently, I am working with colleagues in Austria, France, Germany, Slovakia, and the Netherlands on designing tasks for computational thinking (see https://colette-project.eu/).

Scientific Publications

Author Contributions

Paper I: Realizing students’ ability to use technology with silent video tasks

I prepared the data collection, analysed the data, and wrote the paper. I discussed the findings with Freyja and Zsolt, as well as in a doctoral seminar in Linz, Austria to receive useful feedback.

Paper II: Teachers’ noticing and interpretations of students’ responses to silent video tasks

I prepared data collection, analysed the data, and wrote the paper. Charlotte pointed out to me to use van Es and Sherin’s framework and we discussed the findings. I also received useful feedback from discussing my findings with Freyja and Zsolt, in a doctoral seminar in Linz, Austria, and in a thematic analysis course in Laugarvatn, Iceland.

Paper III: Using silent video tasks for formative assessment

Same as described for Paper I.

Paper I

Paper I

Kristinsdóttir, B., Hreinsdóttir, F., & Lavicza, Z. (2018).
Realizing students' ability to use technology with silent video tasks
In Weigand, H.G., Clark-Wilson, A., Donevska-Todorova, A., Faggiano, E., Grønbæk, N. & Trgalova, J. (Eds.), *Proceedings of the 5th ERME Topic Conference MEDA 2018* (pp. 163—170). University of Copenhagen.

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Paper II

Paper II

Kristinsdóttir, B., Hreinsdóttir, F., Lavicza, Z., & Wolff, C. (2020).
Teachers' noticing and interpretations of students' responses to silent video tasks
Research in Mathematics Education, 22(2), 135–153.

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Paper III

Paper III

Kristinsdóttir, B., Hreinsdóttir, F., & Lavicza, Z. (2020).
Using silent video tasks for formative assessment
In B. Barzel, R. Bebernik, L. Göbel, M. Pohl, H. Ruchniewicz, F. Schacht, & D. Thurm (Eds.), *Proceedings of the 14th International Conference on Technology in Mathematics Teaching* (pp. 189—196). Universiy of Duisburg-Essen. doi.org/10.17185/duepublico/70763

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Paper IV

Paper IV

Kristinsdóttir, B. (under review).
Opportunities and challenges that silent video tasks bring to the mathematical classroom
In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The Matehmatics Teacher in the Digital Era (2nd ed.)*. Springer Nature.

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Appendices

A. Consent and Information Letters

Consent letters and information letters for teachers, students, principals, and guardians.

A.1. Information for principals

School principals received information about the research project in Icelandic. What follows are translations of the emails sent to school principals at the start of the first and second implementation phase respectively.

A.1.1. First implementation phase

Dear [principal name],

Before the start of this semester, I contacted [teacher name] regarding a project connected to my doctoral studies in mathematics education at the University of Iceland.

[School name] was one of the schools that came up in a random selection for the project and [teacher name] is ready to participate with her / his students as well as possibly [names of other teachers if applicable].

Here is a link to a website with information about the research project: https://beamia.wordpress.com

Among other things you will find an information sheet for teachers and principals: https://goo.gl/Aery7K

As stated there, no personal identifiable information will be collected but rather the research aim is to gain insights into teachers' expectations and experiences of using silent video tasks in their teaching and to improve teaching instructions.

I consider it to be important to inform principals and students as well as teachers who participate in this project and to get from them an informed consent.

Here you will find an informed consent for school principals: https://goo.gl/ouvKH3

Let me know if anything is unclear or must be discussed further.

Best regards, Bjarnheiður Kristinsdóttir

A.1.2. Second implementation phase

Before the second implementation phase, the new GDPR made principals and teachers increasingly insecure regarding what data collection to allow and what not. I first approached principals by talking to them directly in person and adjusted information sheets for students and guardians in accordance with advice that I received from the IDPA lawyer. What follows is a translation of an information email to principals:

Dear [principal name]

In the research project that we discussed last week, the aim is to develop silent video tasks and explore how they might be used as part of formative assessment. No personal information about students will be collected.

The following link contains an information sheet for parents, guardians and students: tiny.cc/InfoSVT

Teachers can send the link to guardians and present its contents to students enrolled in courses that are invited to participate. As you can see, it is assumed that students give intended consent for the researcher to listen to their voice-over. It is also made clear that students (and/or guardians) can request that their voice-overs will not be made accessible to the researcher. This is done in compliance with the UNCRC regarding youth's rights to express themselves - something that the IDPA lawyer considered more important as students' responses to a mathematical task hardly will include any personal information. Thus, students and guardians will not need to sign a consent.

Everyone can reject participation any time and no personal information about students will be collected.

Feel free to contact me with any questions that you might have.

Best regards, Bjarnheiður

A.2. Information for teachers

Teachers received information about the research project via phone and email. What follows are translations of the emails sent to teachers at the start of the first and second implementation phase respectively.

A.2.1. First implementation phase

Dear [teacher name],

thank you so much for being willing to take part in this project with me.

Here is a link to a website with information regarding the research project: https://beamia.wordpress.com

Where you will among other things find an information sheet: https://goo.gl/Aery7K

and instructions: https://goo.gl/L1gybb

Feel free to contact me if anything is unclear or needs further discussion, e.g. regarding the technical side of the implementation.

We can change the interview to a group interview if [names of coo-teachers] are willing to try this out as well.

Best regards, Bjarnheiður

The above email links to the following information sheet:

Silent video can be found here https://youtu.be/eK3-8VTml-Q and here https://www.geogebra.org/m/BfRqGSKq and a recording in mp4-format can also be provided.

Form groups of two and send out a link to view a short video. Students can watch the video as often as they like.

Students get the following two tasks:

1. Describe what you see.
2. Again, describe what you see and now try to use as mathematical wording as you can.

Students preferably use a) but possibly use b) to hand in their response:

a) screen recording (e.g. by using Screencast-o-matic or the add-on Screencastify for the Chrome browser) and then answer the first questionnaire

b) sound recording (using a laptop or a phone) and then answer the first questionnaire

Refrain from giving students words or concepts to use. You can assist students if they encounter technical difficulties (e.g. if the video does not buffer). Students can hand in up to two responses to the task (general voice-over and mathematical voice-over).

If many students are located in a small classroom it does not fit well for everyone to record at the same time. The task can be introduced at the start of the lesson, then students can work on other exercises while 2-3 groups of students exit the classroom for recording at the library or in the school corridor. This will eliminate recording background noise.

When evaluating students' responses to the task, you can for example use the SOLO taxonomy in discussion with the researcher. It is not necessary to give students a grade or feedback but if you give them feedback, then use general words and hints (do not use the SOLO taxonomy with students).

The SOLO taxonomy has five levels of understanding:

Pre-structural	The task is not tackled in an appropriate way; the student does not understand the task or uses too simple way to work on it.
Uni-structural	Students' answer only touches on one of the aspects that are relevant when solving the task.
Multi-structural	Students' answer touches on several of the aspects that are relevant when solving the task but they are listed without connecting them to each other.
Relational	The aspects in students' answer are connected into a coherent whole. The student shows adequate understanding of the topic.
Extended abstract	In addition to understanding the topic, the student draws general conclusions that extend to other areas or related topics.

It is recommended to view the video together in a follow-up lesson, discuss the findings in general (e.g. possible misunderstanding or imprecision) and add to the information provided in students' answers in addition to giving students new tasks to think about. If students answers allow, they can be sequenced from daily language to mathematical language and viewed at the start of the class.

In the end, students are asked to answer the second questionnaire.

Links that will be sent to students

For the lesson where students record their voice-over:

- GeoGebbra link to the video https://www.geogebra.org/m/BfRqGSKq
- YouTube link to the video https://youtu.be/eK3-8VTml-Q
- Link to the first student questionnaire: https://goo.gl/Hd8b95

For the follow-up lesson with discussions:

- Link to second student questionnaire: https://goo.gl/JkSfSn

A.2.2. Second implementation phase

The following information letter is a translation of an email that was sent to Orri in Blackbird High School before school started in fall 2019.

Dear Orri.

Thank you so much for taking part in this project!

What time is best suitable for you to meet, plan, and discuss?

Shortly speaking, the agenda will be:

- We select one of your groups/courses and plan for three topics that might be a silent-video-worthy material.
- I will prepare the videos (they are usually 30 seconds to 2 minutes long) and adjust them in collaboration with you
- We will meet to discuss (recorded interviews) before and after each task implementation (we will time these according to the course schedule and adjust our plans along the way)
- If it suits you and the group (you can decide on this when you get to know the group better), then I would appreciate to be able to visit your classroom during implementation and create some field notes (observation notes).

The aim is to develop and explore silent video tasks' potentials as a tool that might support teachers and students when it comes to formative assessment.

In addition to this research project, I can support you as a critical friend and conversation partner regarding building thinking classrooms. We should both gain something from this collaboration: me gathering data for the project, you having someone to discuss your ideas with, and students experiencing something new that hopefully can be helpful on their learning path.

Best regards, Bjarnheiður

After initial discussions with Andri, Edda, and Orri, the following email was sent as a follow-up:

Dear [teacher name].

Here is a link to instructions for teachers regarding the task implementation: http://tiny.cc/leidbeiningar

Do not hesitate to contact me if you have any questions.

The file should contain everything we discussed and in accordance with what you suggested I have updated it regarding peer- and self-assessment as well as providing immediate feedback.

Best regards, Bjarnheiður

The above email includes a link to the following instructions for teachers:

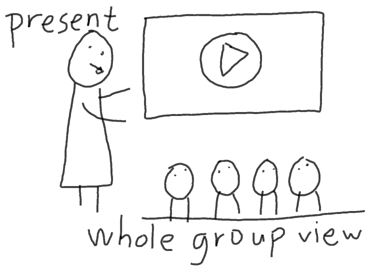
Silent videos are located on YouTube and mp4-versions can be provided if you prefer them for the LMS.

https://youtu.be/8cLrbJM4F-I

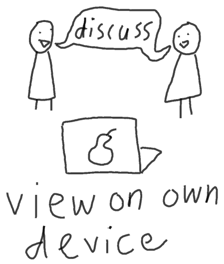
https://youtu.be/-snC4JLe63g

https://youtu.be/aBtlIVTcs8M

Show the video to students and introduce the task, for example: "Describe what you see. Record a voice-over to the video. Use mathematical concepts as you find appropriate. Hand in one response or up to two responses if you wish. Note that other students will listen to your response".



Divide students into groups of two (visibly randomly, e.g. by drawing cards) and give them access to the link to view the video as often as they like on their own device.



Students can record a voice-over by e.g. recording on their smart phone (apps such as audio recorder) or laptop (freeware such as the screencastify addition to the Chrome browser). Let students decide what software they want to use: "You can use a phone app or your laptop to record either a sound file or a screen recording with sound".

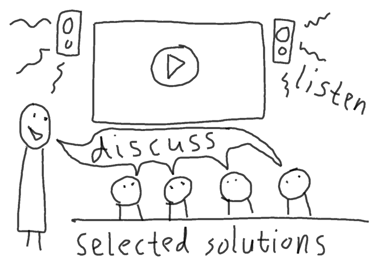
Students can act as if they are radio/tv producers preparing instructional videos and their recording might be useful to other students. Refrain from giving students a list of words or concepts to cover in their task response. Students themselves will select the words and concepts they would like to use. Allow students to work independently and support them by acknowledging but not answering "Is this right?" questions (or other questions of similar kind).

Deleted text starts: Students can hand in two responses, e.g. if they would like to make one mathematical response and another less mathematical. Deleted text ends. (This option was deleted because it was considered confusing).

Students send their recording to the teacher via email or save it to the LMS. The teacher can listen to students responses at the end of the day and the researcher will take part in listening. Let students know that they can refuse participation: "If you do not want your voice-over to be included in the research project then let [teacher name] know and your voice-over will immediately be removed from the collection."

Preparing next class. What follows are three examples of different planning for a group discussion class. Example 1 utilizes peer- and self-assessment. Example 2 and 3 include group discussions lead by the teacher and example 3 includes one additional recording.

Example 1 of Group discussion class: Students work in groups, each group listens to 2-3 randomly chosen student responses and discuss the differences between them, clarity and if anything surprises them. Self- and peer-assessment can be utilized. The teacher can also select student responses for students to listen to.



Example 2 of Group discussion class: Group view of the video and students' responses get discussed in general (e.g. possible misunderstandings and how precision in word use can increase the quality of the voice-over). Additional information can be gathered and teachers can give students new ideas to think about. If students' responses allow, a few of them can be sequenced from daily language to more formal mathematical language.

Example 3 of Group discussion class: Immediately after students hand in their responses to the task, they all get played for the whole group. Questions such as "What do you think they mean by...?" and "Can you explain ... in more detail?" can be used during the discussion in order to get students to explain what they mean (be it the response authors or other students). Finally, students are asked to create a new recording. One could check what students catch from the discussion–if they see a reason to change it, and then what they change.

Note this suggestion from September 17. Possibly, the task could be assigned at the start of class, recordings collected via the LMS and distributed to the pairs of students for immediate peer- and self-assessment. This would happen something like this: In a group of 22 students there are 11 pairs. Group 1 listens to group 2 and 3, group 2 listens to groups 3 and 4, ... group 10 listens to groups 11 and 1, and group 11 listens to groups 1 and 2.

In this way, students who attend the class on the day of task assignment get to evaluate their own task response and the task response of two pairs of peers. They could answer questions such as "Was there anything that surprised you?", "What did you learn from listening to your peer's responses to the task?", "Is there anything that you would like to change in your own voice-over after having listened to the other two voice-overs?", "Is there anything that you noticed everyone could pay more attention to when creating a voice-over to the video?" and continue from there (possibly students will notice that if they have not understood something, it is hard for them to explain it, also they might see a reason why mathematics often asks for clarity when describing something in words). What is different from examples 1-3 above is that the teacher cannot listen to students' responses to the task until after class is over–students receive information about the variety of responses earlier than the teacher.

A collection of links that will be sent to students:

1. To create a voice-over, send a link to **one** of the following videos:

    a) https://youtu.be/8cLrbJM4F-I

    b) https://youtu.be/-snC4JLe63g

    c) https://youtu.be/aBtlIVTcs8M

2. For group discussion class send either both or only the second of these two links:

a) Peer-assessment and self-assessment similar to what is shown here (only the teacher will have access to the answers, as they contain students' names) http://tiny.cc/Mat

b) Send students a link to this survey: http://tiny.cc/Konnun

Recordings

Recording via a phone - Students find an app on their phone that suffices to record sound (audio recorder). A 1 minute long sound file is approximately 10 MB if highest quality recording is selected but 1 MB if standard quality is selected. The standard quality is sufficient for the task. Sending such an audio file via email takes around 1 minute.

Upload the recordings to GDrive and rename the files as recording 1, 2, ... in accordance with the group names. Students can open the drive and listen in the Chrome browser. They can download the file but it is unlikely that they will do so. To prevent students from downloading the sound recordings, they can be uploaded to a private playlist on SoundCloud or similar.

A.3. Information for students and their guardians

Students received information via a questionnaire in the first phase and via email in the second phase of the research project. Students' guardians also received access to the information provided to students.

A.3.1. First implementation phase

Students received information at the start of the first questionnaire that the answered. Their guardians had access to the same text in a shared document via the link: https://goo.gl/1C1Z1q.

Information placed at the start of the first student questionnaire:

Now you have tried creating a voice-over to a silent video task in mathematics class.

The following questionnaire is a part of a doctoral research project lead by Bjarnheiður Kristinsdóttir (bjarnhek@hi.is) in the field of mathematics education on upper secondary school level, called The use of silent video tasks in mathematics teaching. Creating voice-over for silent videos is an innovative task and the project among other things aims to improve instructions for teachers regarding the implementation of the tasks. Teachers will be interviewed about their experiences with implementing the tasks and students answer this questionnaire.

The aim of this questionnaire is to gather information regarding students' experiences of adding a voice-over to a silent video in mathematics class. The questionnaire is anonymous and by handing in your answers, your intended consent for participating in the research project is assumed.

To evaluate silent video tasks as a tool for mathematics teaching, students' voice-over recordings will be collected anonymously. If you do not agree with your voice-over to be collected by the researcher you can notify your teacher about that and your voice-over will immediately be removed from the collection.

Note that the research project is not intended to involve any examination of teachers' or students' performance. Participants can resign from participation at any time. Information that will be collected will be used anonymously and without any personally identifiable information in preparation for presentations at conferences, articles in journals or proceedings, and a doctoral thesis.

Information placed at the end of the first student questionnaire:

By answering these questions your intended consent for participating in the research project is assumed. If you do not agree with your voice-over to be collected by the researcher you can notify your teacher about that and your voice-over will immediately be removed from the collection.

Information placed at the start of the second student questionnaire:

Now, the voice-over recordings for the silent video have been discussed briefly in mathematics class.

The following questionnaire is a part of the same research project as the last questionnaire on The use of silent video tasks in mathematics teaching. (*)

The aim of this questionnaire is to gather information regarding students' experiences of discussing voice-over recordings to a silent video in mathematics class. The questionnaire is anonymous and by handing in your answers, your intended consent for participating in the research project is assumed.

(*) In case you forgot: The doctoral research project is lead by Bjarnheiður Kristinsdóttir (bjarnhek@hi.is) in the field of mathematics education on upper secondary school level, called The use of silent video tasks in mathematics teaching. Creating voice-over for silent videos is an innovative task and the project among other things aims to improve instructions for teachers regarding the implementation of the tasks. Teachers will be interviewed about their experiences of implementing the tasks and students answer two questionnaires.

To evaluate silent video tasks as a tool for mathematics teaching, students' voice-over recordings will be collected anonymously. If you do not agree with your voice-over to be collected by the researcher you can notify your teacher about that and your voice-over will immediately be removed from the collection.

Note that the research project is not intended to involve any examination of teachers' or students' performance. Participants can resign from participation at any time. Information that will be collected will be used anonymously and without any personally identifiable information in preparation for presentations at conferences, articles in journals or proceedings, and a doctoral thesis.

Information placed at the end of the second student questionnaire:

By answering these questions your intended consent for participating in the research project is assumed. If you do not agree with your voice-over to be collected by the researcher you can notify your teacher about that and your voice-over will immediately be removed from the collection.

About the researcher

Bjarnheiður Kristinsdóttir (bjarnhek@hi.is) is a doctoral student in mathematics education at the University of Iceland School of Education. Her supervisors are Freyja Hreinsdóttir (freyjah@hi.is) associate professor at the School of Education and Gunnar Stefánsson (gunnar@hi.is) professor at the School of Engineering and Sciences.

A.3.2. Second implementation phase

Students and guardians received information via email, linking to an online information sheet: tiny.cc/InfoSVT.

The contents of the information sheet are as follows:

Information for students and guardians regarding participation in a research project

In fall 2019 teachers and students of selected upper secondary schools will take part in testing and developing further some innovative assessment tasks called silent video tasks. Participating schools were selected to have teachers that have experience with using formative assessment.

Responsible for this research project Bjarnheiður Kristinsdóttir, doctoral student at the University of Iceland School of Education (bjarnhek@hi.is)

Advisors Freyja Hreinsdóttir, associate professor at the University of Iceland School of Education (freyjah@hi.is) Zsolt Lavicza, professor at the Johannes Kepler University in Linz, Austria (lavicza@jku.at)

Research aims in a nutshell This study is part of the doctoral project of Bjarnheiður Kristinsdóttir in mathematics education on upper secondary school level. The research project aims to develop silent video tasks as part of formative assessment in mathematics classrooms. Information regarding teachers experiences with using silent video tasks will be collected. Silent videos are short animated films that show mathematics dynamically without words or text. Students get the task to add their voice-over to the video. An example of a silent video task tiny.cc/Myndverk

What does teacher and student participation entail? Teacher interviews will be conducted with participating teachers. The interview topics are: the implementation of silent video tasks, students' voice-overs, and group discussion on the topic of students' responses to the task. The researcher will get to listen to students' responses to the task unless the student or a guardian explicitly requests that only the teacher will be allowed to listen to their response. The researcher will not receive any information on which students created each of the responses that she gets to listen to. Students will be able to give their suggestions, opinions, and comments regarding the tasks via an online system. If the teachers allow and considers it to be helpful, they will ask for students' oral permission to allow the researcher to visit their lessons and write observation notes. These notes will be restricted to the topic of the ways in which silent video tasks can be used for formative assessment. Like mentioned previously, the research aim is to develop in collaboration with teachers, tasks that can be used for formative assessment in upper secondary school mathematics classrooms.

No evaluation or examination of teachers' or students' performance will be included in the research project. No personal information about students will be collected.

Risks and values of participation Little or no risk will be involved in participating in the study. The value for participants is to get to know the innovative tasks called silent video tasks. These tasks are designed to support students' learning. The tasks' mathematical topics are selected with respect to the national curriculum and course syllabus.

Right to refuse and withdraw participation Everyone has the right to refuse participation in the study. Participants can withdraw from the study at any time during the research process. Participating teachers may choose not to answer individual questions posed in interviews. Students can request from their teacher that only the teacher (not the researcher) can listen to their voice-over recording.

Data handling and privacy Raw data such as audio files and text files containing teacher interviews, students' voice-over recordings, and field observations will be stored on an encrypted Dropbox drive accessible only to the researcher. The students' suggestions, opinions ,and comments regarding the study will be collected anonymously through an encrypted GDrive drive that is only accessible to the researcher. The data collected for the study will be used under a pseudonym and without personal identifiable information in the preparation of presentations, journal articles and a doctoral thesis. The study was reported to the Data Protection Authority.

A.4. Consent letters for teachers

School principals and participating teachers signed the following consent letters in the first and second implementation phase respectively.

A.4.1. First implementation phase

Teachers' informed consent for participation in the study Use of silent video tasks in mathematics classrooms at the upper secondary level.

Responsible person Bjarnheiður Kristinsdóttir, PhD student at the School of Education, University of Iceland (bjarnhek@hi.is)

Supervisors of the doctoral student Freyja Hreinsdóttir, Associate Professor at the School of Education, University of Iceland (freyjah@hi.is) Gunnar Sigurðsson, Professor at the Faculty of Engineering and Science, University of Iceland (gunnar@hi.is)

Research aims in a nutshell This study is part of Bjarnheiður Kristinsdóttir's doctoral project in mathematics education at the upper secondary school level. The study addresses teachers' expectations and experience of using silent video tasks in their mathematics classroom (i.e. a short animated film that shows mathematics dynamically without words or text).

Participants Participants were randomly selected. A numbered list including all the 30 upper secondary schools in Iceland that offer matriculation examination was used. A computer randomly selected twenty numbers in the range [1, 30] and the same number could occur frequently. If a course on trigonometry was taught in the fall semester of 2017 and the teacher of the relevant course in the randomly selected school was ready to participate in the research, an attempt was made to work with the teacher in question and his / her students with the approval of the school principal.

What is involved in participation? Teacher a) participates in an interview before voice-over recording finds place, b) uses one lesson for students to work on their voice-over recordings and answer a questionnaire, c) participates in a second interview after the lesson, d) uses the first 10-20 minutes of the follow-up lesson to discuss students' voice-over recordings, and e) participates in a third interview after the follow-up lesson. Also f) the teacher provides the researcher with anonymous copies of students' voice-over recordings. Each of the three interviews takes about 10-60 minutes and will be recorded as an audio file. The study does not involve an examination of teachers' or students' performance.

Risks and benefits of participation Little or no risk is involved in participating in the study. It is considered to be beneficial for participants to get to experience the use of silent video tasks in their mathematics classroom.

Right to refuse and withdraw participation Everyone has the right to refuse participation in the study. Participants who agree to participate can withdraw from the study at any time during the research process. Participating teachers can choose not to answer individual questions posed in the three interviews.

Data handling and privacy Raw data such as audio files and text files containing teacher interviews will be stored on an encrypted Dropbox drive that is only accessible to the researcher. Data collected in the study will be used under a pseudonym and without personal identifiable information in the preparation of conference presentations, journal articles and a doctoral thesis. No sensitive personal information will be gathered in the study and only information that can not be traced to specific individuals is processed. The study has been reported to the Data Protection Authority.

I have been informed about the purpose of this study and what my participation entails. I agree to participate.

Date, Signature

A.4.2. Second implementation phase

Teachers' informed consent for participation in the study Silent video tasks – their definition, development, and implementation in upper secondary school mathematics classrooms

Responsible for this research project Bjarnheiður Kristinsdóttir, doctoral student at the University of Iceland School of Education (bjarnhek@hi.is)

Advisors Freyja Hreinsdóttir, associate professor at the University of Iceland School of Education (freyjah@hi.is) Zsolt Lavicza, professor at the Johannes Kepler University in Linz, Austria (lavicza@jku.at)

Research aims in a nutshell This study is part of the doctoral project of Bjarnheiður Kristinsdóttir in mathematics education on upper secondary school level. The research project aims to develop silent video tasks as part of formative assessment in mathematics classrooms. Information regarding teachers' experiences with using silent video tasks will be collected. Silent videos are short animated films that show mathematics dynamically without words or text.

Participants Participants were selected such that they had experience with formative assessment.

What does participation entail? Teacher a) participates in an interview before implementing silent video tasks to a group of students, b) uses three lessons spread over the course of one semester for students to work on recording a voice-over to a silent video and comment on the process if needed, c) participates in another interview during which the next lesson will be jointly prepared, d) leads a group discussion at the start of the follow-up lesson on the topic of students' responses to the silent video task and possibly allows the researcher to observe the group discussions e) participates in an interview after the group discussion. Also f) the teacher provides the researcher with an anonymous copy of students' voice-over recordings, from students who have allowed the teacher to do so. All interviews take around 10-60 minutes and will be audio recorded.

Students g) record their voice-over to a silent mathematical video and hand it in to their teacher (who discusses it along with other students' voice-over recordings anonymously with the researcher), h) answer an anonymous questionnaire on their experience of recording a voice-over to a silent video in mathematics class and i) answer an anonymous questionnaire on their experiences of group discussions in mathematics class.

No evaluation or examination of teachers' or students' performance will be included in the research project. No personal information about students will be collected.

Risks and values of participation Little or no risk will be involved in participating in the study. The value for participants is to get to know the innovative tasks called silent video tasks.

Right to refuse and withdraw participation Everyone has the right to refuse participation in the study. Participants can withdraw from the study at any time during the research process. Participating teachers may choose not to answer individual questions posed in interviews.

Data handling and privacy Raw data such as audio files and text files containing teacher interviews, students' voice-over recordings, and field observations will be stored on an encrypted Dropbox drive accessible only to the researcher. The students' suggestions, opinions ,and comments regarding the study will be collected anonymously through an encrypted GDrive drive that is only accessible to the researcher. The data collected for the study will be used under a pseudonym and without personal identifiable information in the preparation of presentations, journal articles and a doctoral thesis. The study was reported to the Data Protection Authority.

I have been informed about the purpose of this study and what my participation entails. I agree to participate.

Date, Signature

A.5. Consent letters for principals

A.5.1. First implementation phase

Principals' informed consent for participation in the study Use of silent video tasks in mathematics classrooms at the upper secondary level.

Responsible person Bjarnheiður Kristinsdóttir, PhD student at the School of Education, University of Iceland (bjarnhek@hi.is)

Supervisors of the doctoral student Freyja Hreinsdóttir, Associate Professor at the School of Education, University of Iceland (freyjah@hi.is) Gunnar Sigurðsson, Professor at the Faculty of Engineering and Science, University of Iceland (gunnar@hi.is)

Research aims in a nutshell This study is part of Bjarnheiður Kristinsdóttir's doctoral project in mathematics education at the upper secondary school level. The study addresses teachers' expectations and experience of using silent video tasks in their mathematics classroom (i.e. a short animated film that shows mathematics dynamically without words or text).

Participants Participants were randomly selected. A numbered list including all the 30 upper secondary schools in Iceland that offer matriculation examination was used. A computer randomly selected twenty numbers in the range [1, 30] and the same number could occur frequently. If a course on trigonometry was taught in the fall semester of 2017 and the teacher of the relevant course in the randomly selected school was ready to participate in the research, an attempt was made to work with the teacher in question and his / her students with the approval of the school principal.

What is involved in participation? Teacher a) participates in an interview before voice-over recording finds place, b) uses one lesson for students to work on their voice-over recordings and answer a questionnaire, c) participates in a second interview after the lesson, d) uses the first 10-20 minutes of the follow-up lesson to discuss students' voice-over recordings, and e) participates in a third interview after the follow-up lesson. Also f) the teacher provides the researcher with anonymous copies of students' voice-over recordings. Each of the three interviews takes about 10-60 minutes and will be recorded as an audio file.

Students g) record their voice-over to a silent mathematical video and hand it in to their teacher (who discusses it along with other students' voice-over recordings anonymously with the researcher), h) answer an anonymous questionnaire on their experience of recording a voice-over to a silent video in mathematics class and i) answer an anonymous questionnaire on their experiences of group discussions in mathematics class.

The study does not involve any examination of teachers' or students' performance.

Risks and benefits of participation Little or no risk is involved in participating in the study. It is considered to be beneficial for participants to get experience of using silent video tasks in their mathematics classroom.

Right to refuse and withdraw participation Everyone has the right to refuse participation in the study. Participants who agree to participate can withdraw from the study at any time during the research process. Participating teachers can choose not to answer individual questions posed in the three interviews.

Data handling and privacy Raw data such as audio files and text files containing teacher interviews will be stored on an encrypted Dropbox drive that is only accessible to the researcher. Data collected in the study will be used under a pseudonym and without personal identifiable information in the preparation of conference presentations, journal articles and a doctoral thesis. No sensitive personal information will be gathered in the study and only information that can not be traced to specific individuals is processed. The study has been reported to the Data Protection Authority.

I have been informed about the purpose of this study and what the teacher and students' participation entails. I agree to participate.

Date, Signature

A.5.2. Second implementation phase

Principals' informed consent for participation in the study Silent video tasks – their definition, development, and implementation in upper secondary school mathematics classrooms

Responsible for this research project Bjarnheiður Kristinsdóttir, doctoral student at the University of Iceland School of Education (bjarnhek@hi.is)

Advisors Freyja Hreinsdóttir, associate professor at the University of Iceland School of Education (freyjah@hi.is) Zsolt Lavicza, professor at the Johannes Kepler University in Linz, Austria (lavicza@jku.at)

Research aims in a nutshell This study is part of the doctoral project of Bjarnheiður Kristinsdóttir in mathematics education on upper secondary school level. The research project aims to develop silent video tasks as part of formative assessment in mathematics classrooms. Information regarding teachers' experiences of using silent video tasks will be collected. Silent videos are short animated films that show mathematics dynamically without words or text.

Participants Participants were selected such that they had experience with formative assessment.

What does participation entail? Teacher a) participates in an interview before implementing silent video tasks to a group of students, b) uses three lessons spread over the course of one semester for students to work on recording a voice-over to a silent video and comment on the process if needed, c) participates in another interview during which the next lesson will be jointly prepared, d) leads a group discussion at the start of the follow-up lesson on the topic of students' responses to the silent video task and possibly allows the researcher to observe the group discussions e) participates in an interview after the group discussion. Also f) the teacher provides the researcher with an anonymous copy of students' voice-over recordings, from students who have allowed the teacher to do so. All interviews take around 10-60 minutes and will be audio recorded.

Students g) record their voice-over to a silent mathematical video and hand it in to their teacher (who discusses it along with other students' voice-over recordings anonymously with the researcher), h) answer an anonymous questionnaire on their experience of recording a voice-over to a silent video in mathematics class and i) answer an anonymous questionnaire on their experiences of group discussions in mathematics class.

No evaluation or examination of teachers' or students' performance will be included in the research project. No personal information about students will be collected.

Risks and values of participation Little or no risk will be involved in participating in the study. The value for participants is to get to know the innovative tasks called silent video tasks.

Right to refuse and withdraw participation Everyone has the right to refuse participation in the study. Participants can withdraw from the study at any time during the research process. Participating teachers may choose not to answer individual questions posed in interviews. Participating students can choose not to answer individual questions posed in questionnaires.

Data handling and privacy Raw data such as audio files and text files containing teacher interviews, students' voice-over recordings, and field observations will be stored on an encrypted Dropbox drive accessible only to the researcher. The students' answers to questionnaires, their suggestions, opinions ,and comments regarding the study will be collected anonymously through an encrypted GDrive drive that is only accessible to the researcher. The data collected for the study will be used under a pseudonym and without personal identifiable information in the preparation of presentations, journal articles and a doctoral thesis. The study was reported to the Data Protection Authority.

The researcher will only use the data for the purposes of the research project. The researcher will not try to identify the person behind each student's voice-over recording, as this is not part of the purpose of the research project. Neither interviews with teachers nor student voice-over recordings will be played in the audience of anyone other than the researcher, students and the participating teacher. All data, be it audio recordings, text files or field notes are stored in a safe place. The researcher confirms this intention with her signature

Date, researcher signature

I have been informed about the purpose of this study and what the teacher and students' participation entails. I agree to allowing teachers and students to participate in this study.

Date, principal signature

B. Teacher Interview Questions

Interview guides for the semi-structured teacher interviews with pre-defined questions listed. All interview guides started with reminders regarding turning off the phone and checking if the recording devices were running normally.

B.1. First implementation phase

Interview guides in the first implementation phase started with the following introduction:

Thank you for taking part in this research project. Like we already discussed, I am obliged to inform you that you can choose not to answer questions and you can quit participation whenever you like. The research aim is to check teachers' expectations and experiences of using a silent video task in their mathematics classroom. The information you provide will be utilized to develop the instructional sequence of silent video tasks. On the basis of the research findings, instructions and/or professional development courses for upper secondary school mathematics teachers will be developed. The interview will be recorded and all data gathered will be coded under a pseudonym.

B.1.1. First interview

- When did you start teaching at this school?
- Have you taught in other schools?
- How do you like working at this school?
- Do you currently work in collaboration with other teachers or more on your own?
- Which courses of mathematics do you teach?
- Do teachers at this school generally get freedom to take part in experimental

projects?

- What about the pressure to keep up with the course schedule?
- How do you feel about using a silent video task in your class?
- Regarding the task implementation, is there any particular part of it that you feel anxious about or are looking forward to?
- Is there anything that you would like to get more information about or would like to ask about before you try out the silent video task in your classroom?
- Anything you would like to add?

B.1.2. Second interview

- What was your perception regarding how students reacted to the task?
- Did students bring in many questions or remarks?
- Did you feel particular stress or strain at any point during the task implementation?
- Is there anything that characterizes your lessons in particular?
- What about this lesson, was it similar or different to your normal lessons?
- What do you think about students' responses to the task?
- When listening to students' responses, did you feel like the SOLO taxonomy model was helpful for you?
- Was there anything that surprised you in particular?
- Were there any students that showed you different/new side during their work on the silent video task?
- Did you notice any misunderstanding or impreciseness in students' descriptions of what happened in the video?
- Do you think that some of students' responses could be sequenced for group viewing in the follow-up lesson?
- Are your students experienced with participating in group discussions during

mathematics lessons?

- Do you have any opinion regarding whether learning mathematics is an individual task or a collaborative task?
- How do you feel about discussing students' responses in the follow-up lesson?
- Is there anything you feel anxious about or look forward to regarding the follow-up lesson?
- Is there anything you would like to ask about or get more information on regarding the follow-up lesson?
- If you would repeat today's lesson, is there anything you would have liked to change or do differently?
- Anything you would like to add?

B.1.3. Third interview

- How did students react?
- Was there anything that surprised you?
- Can you name anything that you noticed and was different from a normal mathematics lesson?
- If you think about the task implementation from start to finish, do you feel like it was worth the time to work on this task?
- What instructions or advice would you give to a teacher who was planning to use a silent video task in their classroom?
- Do you think you might ever try using a silent video task again?
- Is there anything you would have liked to change or do in a different way?
- Anything you would like to add?

B.2. Second implementation phase

Interview guides in the second implementation phase started with the following introduction:

Thank you for taking part in this research project. Like we already discussed, I am obliged to inform you that you can choose not to answer questions and you can quit participation whenever you like. The research aim is to develop silent videos as a tool for formative assessment in the mathematics classroom. The information you provide will be utilized to develop the instructional sequence of silent video tasks. On the basis of the research findings, instructions and/or professional development courses for upper secondary school mathematics teachers will be developed. The interview will be recorded and all data gathered will be coded under a pseudonym.

B.2.1. First interview

- When did you start teaching at this school?
- Have you taught in other schools?
- How do you like working at this school?
- Do you currently work in collaboration with other teachers or more on your own?
- Which courses of mathematics do you teach?
- Can you name anything that characterizes your ways of teaching?
- What would a typical lesson with you look like?
- Do you have any opinion regarding whether learning mathematics in terms of it rather being an individual task or a collaborative task? If we had a scale ranging from individual task to collaborative task, where would you place the learning of mathematics on such a scale? Why?
- Do you get freedom for experimenting in your teaching?
- Do teachers at this school generally get freedom to take part in experimental projects?
- What about the pressure to keep up with the course schedule?

- Soon you will be using a silent video task in your teaching. Can you describe to me how you picture that the task implementation and class in general will be? Let us do a think aloud exercise where you describe what you do and possible student reactions.

- Regarding the task implementation, is there any particular part of it that you feel anxious about or are looking forward to?

- Is there anything that you would like to get more information about or would like to ask about before you try out the silent video task in your classroom?

- When it comes to assessment, is there anything specific that you would like to know about students' learning, how they think or what they understand?

- Do your current assessment methods give you a feeling for how the teaching is going?

- Thinking about the assessment methods that you currently use, is there anything that you find hard to measure using these methods?

- Changes in technology and pedagogy are continuous (never end) - do you have any ideas on how teachers might be supported when it comes to these rapid changes?

- Anything you would like to add?

B.2.2. Interview after task implementation

- How do you feel right now after the class?

- What was your perception regarding how students reacted to the task?

- Tasks of this kind are among other things created to make the students' voice be heard.

  - Do you think that the tasks sufficed in making students' voice be heard?

  - In your opinion, does it matter to make the students' voice be heard?

- Did you feel particular stress or strain at any point during the task implementation?

- Was there anything that surprised you in particular?

- Were there any students that showed you different/new side during their work on the silent video task?
- Tasks of this kind are among other things created for the teacher to realize if something was misunderstood or needs further discussion regarding certain mathematical concepts.
  - Do you think that the tasks sufficed in doing so?
  - In your opinion, does it matter to receive such information?
- Was there anything particular that was different from your lessons hitherto?
- What was similar to your lessons hitherto?
- Was there anything particular that you noticed about students' responses to the task?
- Did your students' responses surprise you in some way?
- Do your students have previous experiences with group discussions in mathematics lessons?
- What was your experience with the group discussions?
- How do you feel about discussing their responses in that way?
- Is there anything that you felt anxious about or looked forward to regarding the lesson you just had?
- Is there anything you would like to change or would have liked to do in a different way?
- Is there anything you would like to add?

B.2.3. Interview before the next task implementation

- How are you doing?
- We are going to do a think aloud exercise. Can you please describe how you picture the next implementation of a silent video task and the class in general (students' reactions included).
- Is there anything you would like to change or do differently form the last time

you implemented a silent video task?

- Is there anything that you feel anxious about or look forward to regarding next lesson?
- Is there anything that you would like to get more information about or would like to ask about before you try out the silent video task in your classroom?
- When it comes to assessment, is there anything specific that you would like to know about students' learning? Or regarding how they think or what they understand?
- Do your current assessment methods provide you with the information that you are hoping for?
- Do silent video tasks assist you in any way when it comes to collecting information regarding students' learning? In what ways?
- Do yo think silent video tasks could be made more useful for you when it comes to collecting information regarding students' learning? In what ways?
- Regarding whether learning mathematics is rather considered an individual task or a collaborative task, where do you place yourself on a scale from individual to collaborative? What about this school, where would you place the school in general (its emphasis)? Why?
- What would you tell a teacher who never used silent video tasks and would like to try them out? Any specific advice or something you would like to draw their attention to?
- Anything you would like to add?

C. Informal Questionnaires

This appendix includes questions from short informal online surveys that students answered anonymously.

C.1. Student questionnaires

The questions asked in the questionnaires are provided in this appendix. All questionnaires were made with Google Forms and started with information for students that was already provided in appendix A (see p. 190). Students could decide whether or not to answer individual questions.

C.1.1. First implementation phase

In the first questionnaire, which focused on the silent video and recording of a voice-over, the following five questions were answered on a Likert-scale: agree, rather agree, neutral, rather disagree, disagree, cannot answer / does not apply.

In my opinion:

- the task was interesting
- it was rather easy to understand the task
- recording the voice-over went well
- it was interesting to have to "speak mathematics"
- it mattered to know that my voice-over recording might be helpful for other students

and the following question was provided with an open text field:

- Do you have any comments regarding the task or anything else that you would like to add?

In the second questionnaire, which focused on the discussion in a follow-up lesson, the following five questions were answered on a Likert-scale: agree, rather agree, neutral, rather disagree, disagree, cannot answer / does not apply.

In my opinion:

- the discussion about the voice-over recordings interesting
- it was useful to listen to the voice-over recordings made by other students
- it was important that the teacher told us about common mistakes or instances of imprecision if they occurred
- it was interesting to hear about common mistakes or instances of imprecision
- the task helped to consolidate the mathematical concepts that appeared in the video

and the following question was provided with an open text field:

- Do you have any comments regarding the group discussion about the task or anything else that you would like to add?

C.1.2. Second implementation phase

One questionnaire and one reflection sheet were prepared for use in the second implementation phase. The questionnaire was prepared at the start and for all teachers to use, but in the end it was only used one time with Orri's students–after Orri's implementation of SVT1. The reflection sheet was prepared in collaboration with Andri and Edda during an interview prior to their implementation of SVT2.

The questionnaire that was used once by Orri included the following five statements with options on a Likert-scale (agree, rather agree, neutral, rather disagree, disagree, cannot answer / does not apply).

In my opinion:

- the task was interesting
- it was rather easy to understand the task
- recording the voice-over went well
- it was interesting to have to "speak mathematics"

- it mattered to know that my voice-over recording might be helpful for other students

Furthermore, it included the following three open questions where students could write an answer in an open text field:

- Was there anything that surprised you regarding this task, the voice-over recording, or the group discussion?
- Did the completion of the task give you any information about your learning of mathematics? For example, where you are at, ways of working, or where you are headed next?
- Do you have any comments regarding the task or anything else that you would like to add?

The reflection sheet that Andri and Edda's students handed in (after working on SVT2) included the following three open questions:

After listening to all students' voice-over recordings:

- Is there anything that you would have liked to change in your voice-over? Why?
- Was there anything that surprised you?
- Would you like to add any comments or questions regarding the silent video task?

D. Instructional Sequence Flow-Charts

The development of silent video tasks' instructional sequence was documented in flow-charts. They describe the process of assigning a silent video task and show how this process changed over time within the research project.

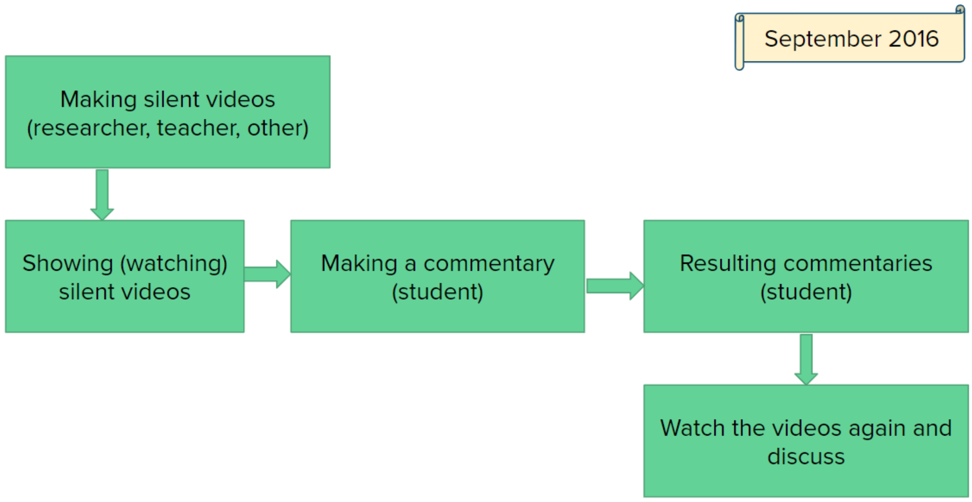


Figure D.1. A flow-chart showing the process of assigning a silent video task as it was visualized in September 2016 at the start of the research project.

At the start of the research project the silent video task instructional sequence included the following:

- a silent video clip that shows mathematics dynamically,
- instructions that a teacher gives students to record a voice-over for the video collaborating in pairs,
- student actions and preparations leading to their recording of a voice-over, and
- a group discussion in a follow-up lesson.

The group discussion in a follow-up lesson was considered to be an important part of the instructional sequence. The reasons for this are that possible previously hidden conceptual obstacles might become visible to teachers and the discussion might also support teachers in reaching the learning goal: That learners become aware of how they themselves, their peers, and the teacher talk about the mathematics shown in the video–a process in which learners might deepen and widen their understanding.

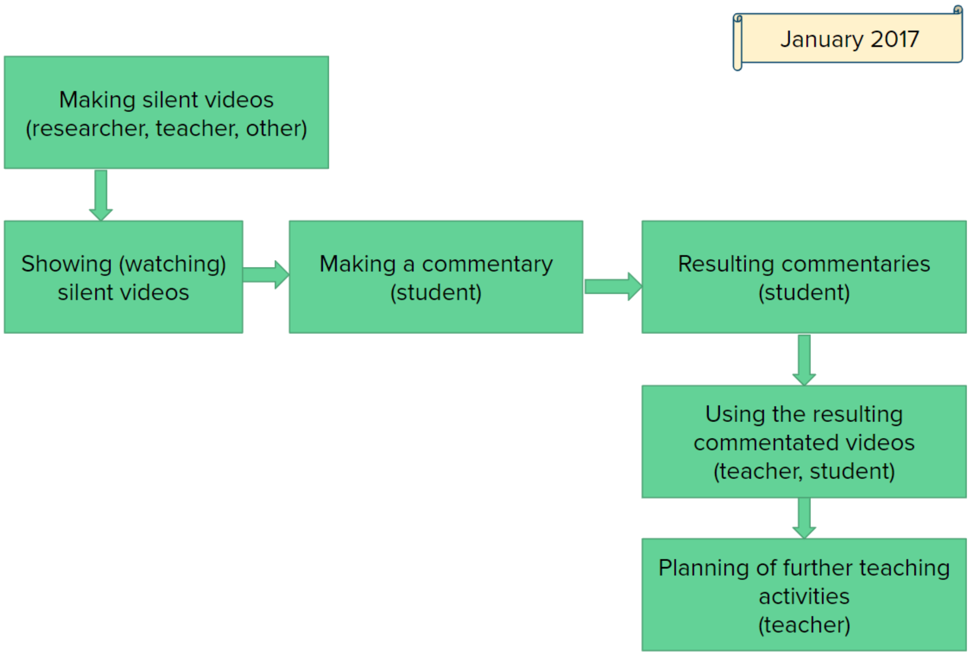


Figure D.2. A flow-chart showing the process of assigning a silent video task as it was visualized in January 2017 during preparation for the first implementation phase of the research project.

Figures D.1 (see p. 213) and D.2 show my initial experiments with setting up a flow-chart to describe the process of implementing a silent video task. Even though figure D.3 (see p.215) is labelled as being from January 2018, it does give a clearer picture (visualization) than figure D.2 of the instructions that teachers received at the start of the first implementation phase in 2017. It includes the teacher listening to students' responses to the task before giving a follow-up lesson and the role of the teacher in selecting some student responses to listen to in a sequence, building a base for group discussion in the follow-up lesson.

Originally, students had been expected to record their responses with screen recording software. This required the use of a laptop or a computer with a microphone. To eliminate background noise, teachers often planned to send some student groups to the school library or corridors. During Gauti's implementation of the task, he soon discovered that students could use their smart phones to record sound. The phone microphones were not as sensitive to background noise and the video and sound recording could be connected to each other.

As discussed in chapter 2, none of the teachers who participated in the first implementation phase felt like they could sequence any of their students' responses to the silent video task. Instead, they either discussed their own ideas for a voice-over or played a few student responses after asking for volunteers. Figure D.4 (see p. 215) shows a

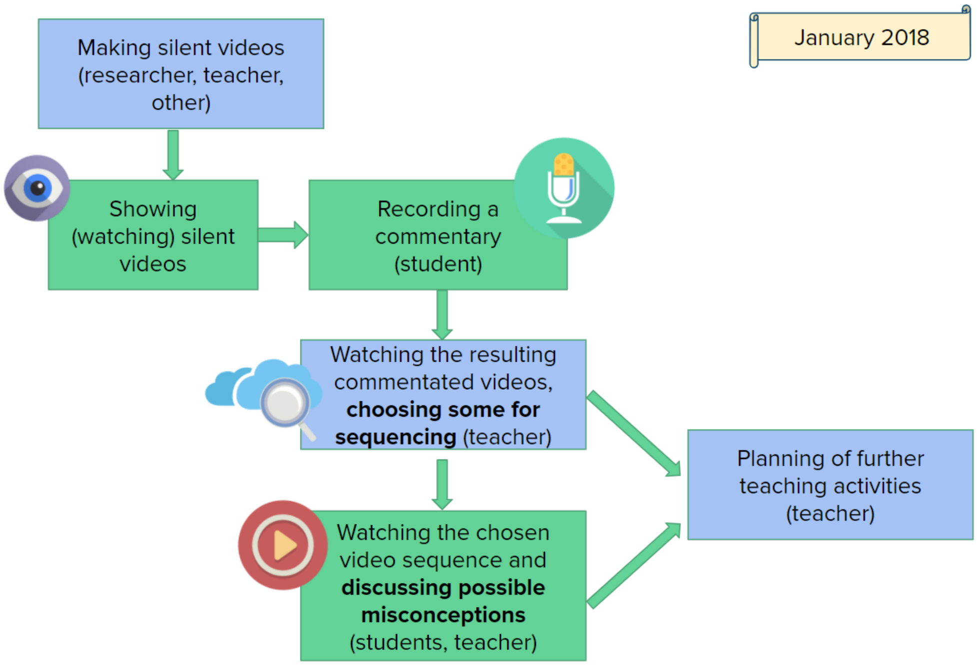


Figure D.3. A flow-chart showing the process of assigning a silent video task as it was visualized in January 2018, when analysis of data from the first implementation phase started.

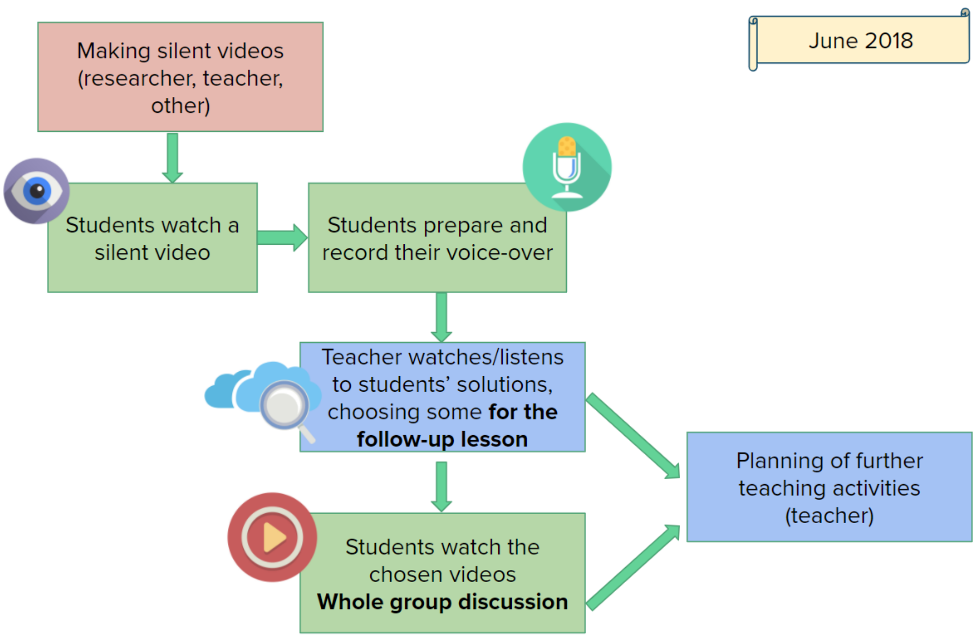


Figure D.4. A flow-chart showing the process of assigning a silent video task as it was visualized in June 2018, when I presented some pre-liminary results from the first implementation phase at international conferences.

slightly different procedure: The focus on sequencing students' responses has been removed. In figure D.5 (see p. 216) the idea of selecting some student responses for the follow-up lesson has been replaced by either *selecting* student responses or *asking for volunteers* among students who would like the group to view their task response.

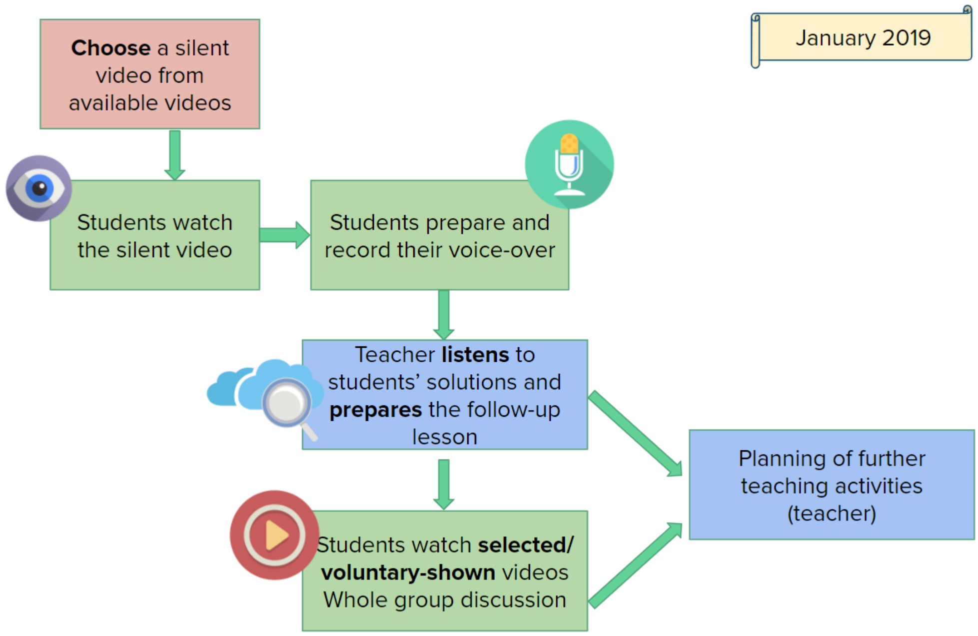


Figure D.5. A flow-chart showing the process of assigning a silent video task as it was visualized in January 2019 during preparation for the second implementation phase of the research project.

The most rapid changes in the instructional sequence were made during the second implementation phase in fall 2019. Orri's plan for his first silent video task implementation (using SVT1) is shown in figure D.6 (see p.217). It included peer assessment where each pair of learners reflected on two or three of their peer's responses to the task. The initial idea was that Orri would listen to students' responses during his organization of the peer review.

In practice, however, there was no time for Orri to listen to students' responses. This is shown in figure D.7 on p. 217. Orri barely managed to quickly assign to each pair of students some two or three of their peers' responses to reflect on. Since class was almost over, students wrote short notes that rarely could have served to push their peers toward deeper understanding and there was no time for classroom discussion. This way of working was considered unhelpful. A part of the reason why students did not write helpful peer reviews might have been students' inexperience with writing reflections to student dialogue. The teachers' input seemed to be needed in order to further the discussion about the mathematical topic.

Therefore, Orri decided that next time he would rather get everyone in the group to listen to all students' responses together. His next version of the task implementation (see figure D.8 on p. 218) furthermore included viewing the silent video twice: before

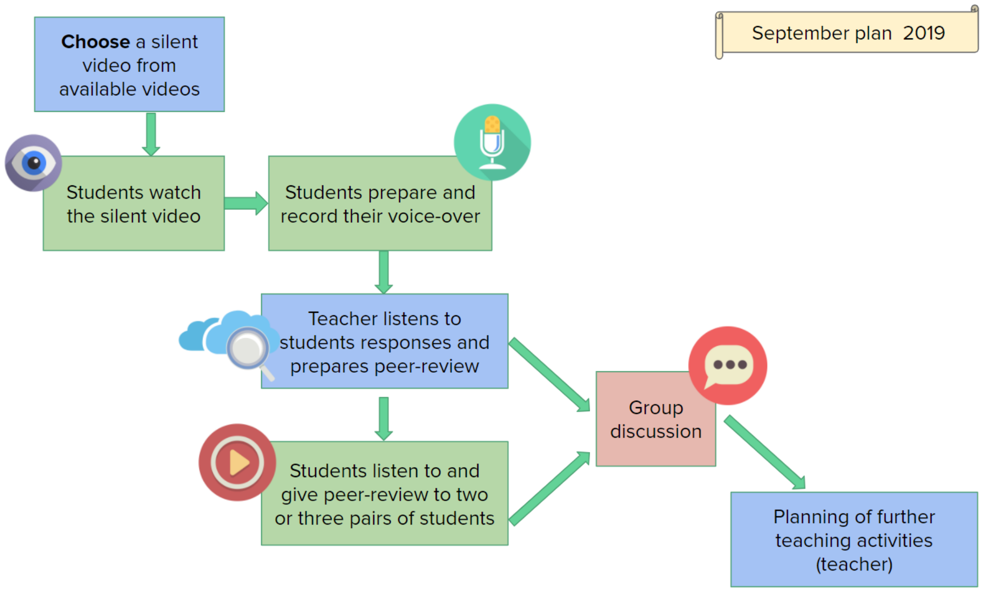


Figure D.6. A flow-chart showing the process of assigning a silent video task as it was planned by Orri in September 2019, at the start of the second implementation phase of the research project.

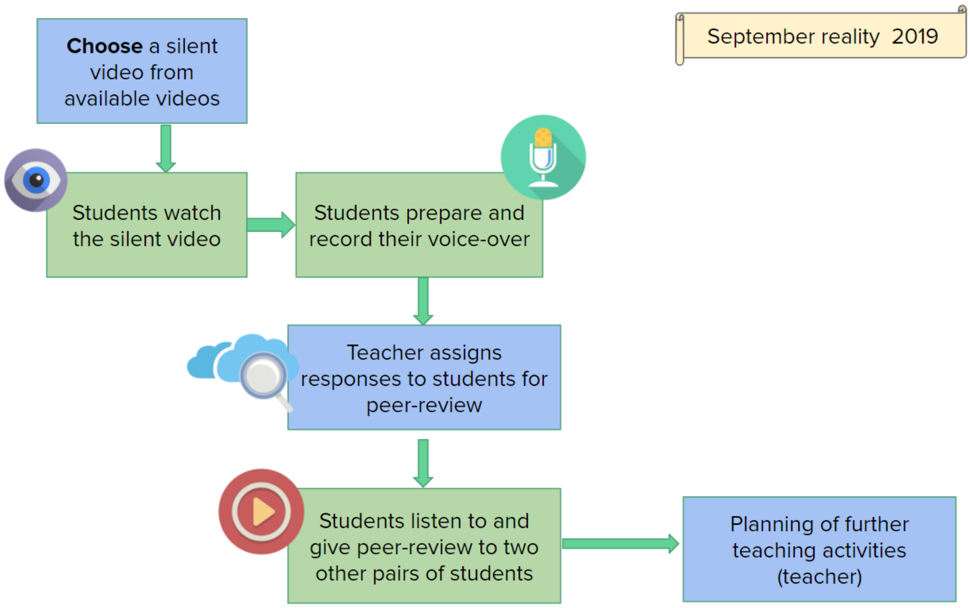


Figure D.7. A flow-chart showing the process of assigning a silent video task as it was enacted by Orri for SVT1 at the start of the second implementation phase of the research project.

and after the learning sequence. At the start of the learning sequence, the video clip was shown to the whole group and students' initial ideas about the topic were collected via the LMS. These were then presented in a word cloud. At the end of the learning sequence, the silent video was viewed again during the introduction of the silent video task. Instead of students peer reviewing two other groups' voice-over responses, now the whole group listened to all student responses, the teacher discussed them and asked students to record a new voice-over. The way in which Orri used the silent video to collect students ideas at the start of the learning sequence was a spontaneous idea that he did not discuss beforehand, but told me about afterwards[1]. He also provided a copy of the word cloud that they had created.

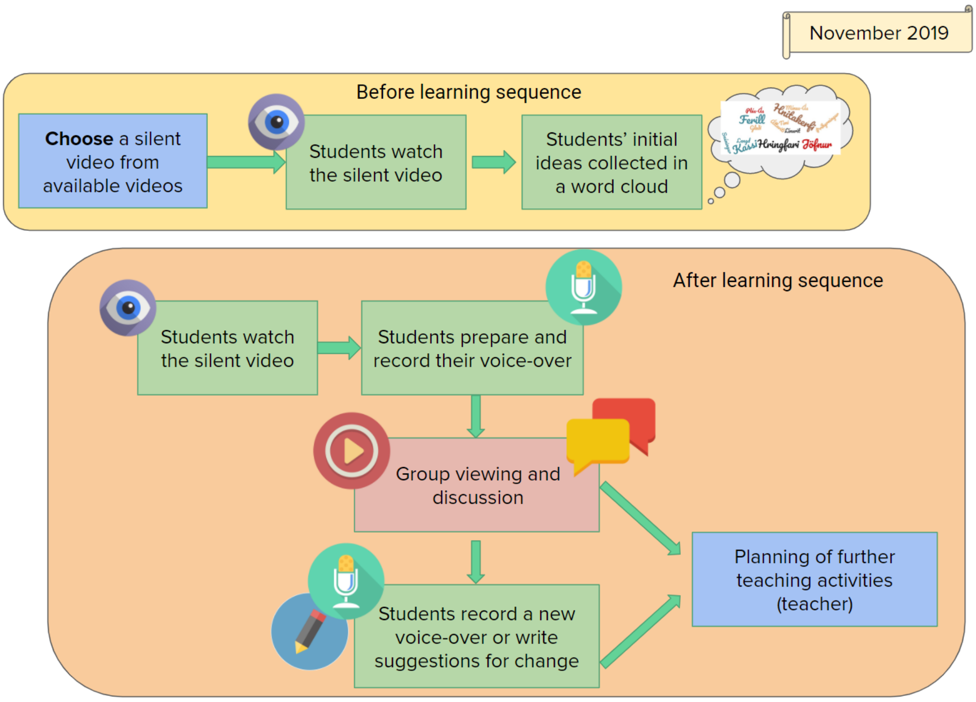


Figure D.8. A flow-chart showing the process of assigning a silent video task as it was enacted by Orri in his implementation of SVT2 in the third week of November 2019, during the second implementation phase of the research project.

Orri had the expectation that students' new voice-overs would be improved versions of their previous responses. Reality was different. Some students did not bother to record a new response and other students created a new voice-over that–contrary to what Orri had expected–put focus on point coordinates instead of line slope (they were working on SVT2 which showed a line rotating around a point, pausing regularly, see section 3.4.3 on p. 40). This was something that Orri found interesting. To him, it was obvious to focus on line slope, but to students it was not obvious at all. They could be focusing on point coordinates, on intercepts with the axes of the coordinate system or even mention for how long time the moving point would pause each time.

[1]During the third interview, before a lesson at the end of the learning sequence where his second implementation of the silent video task took place.

Andri and Edda decided to use yet another procedure of implementing SVT2. It involved written self-assessment that consisted of i) what students would change if they had the chance to repeat the task, and ii) what possible changes could be made to the task itself, for it to be improved. Andri and Edda's implementation procedure is visualized in figure D.9 on p. 219. Their process of implementing the silent video task is influenced by my discussion with them about Orri's first two versions of implementing the task. There was little time between implementations of SVT2 and therefore experiences from Andri and Edda could not be communicated to Orri before his implementation of SVT3. However, this was not considered to be a problem. In total, Orri used three slightly different versions of implementing the silent video tasks SVT1, SVT2, and SVT3. These versions are shown in figures D.7 (see p.217), D.8 (see p. 218), and D.10 (see p. 220), respectively. Orri's last implementation (see figure D.10 on p. 220) involved a group discussion where students were asked to reflect on and respond to each others' responses.

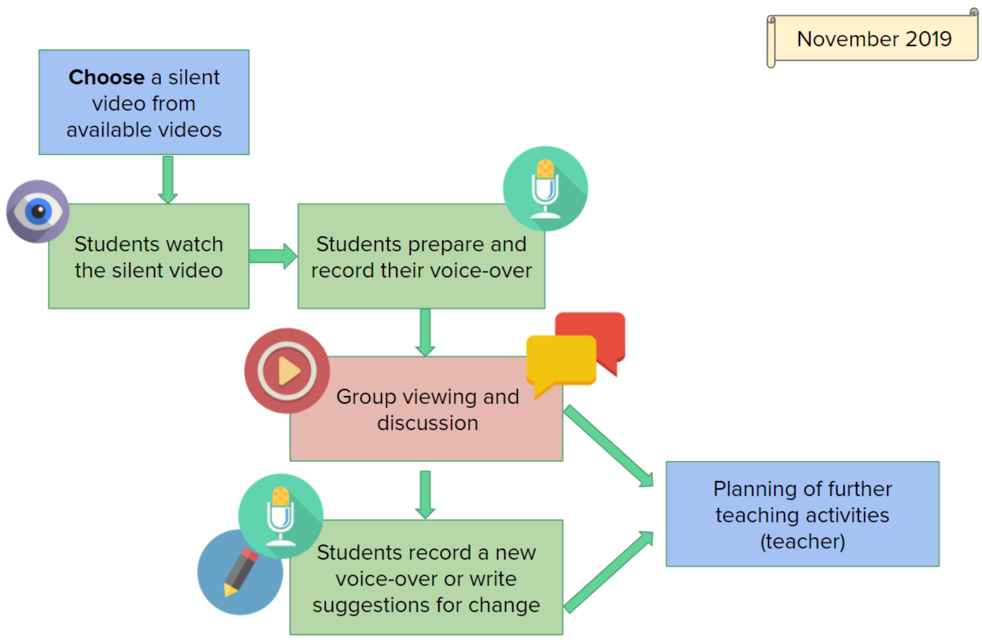


Figure D.9. A flow-chart showing the process of assigning a silent video task as it was enacted by Andri and Edda in their implementation of SVT2 at the end of November 2019, during the second implementation phase of the research project.

Figure D.11 (on p. 221) shows the instructional sequence which is presented in the design principles in section 5.2.1 on p. 70. Its top part is considered to be optional. Teachers can show the silent video before starting a mathematical topic learning sequence and collect students' initial reactions to the video in a word cloud. This is because–based on experiences made–Orri decided to keep the word cloud activity and skip the recording of new voice-overs. It had not shown to be too helpful to make students create a new voice-over. Teachers–on the other hand–might want to re-listen after class, reflect and possibly write some individual feedback to students. At least, Orri noticed that in the process of listening again, he paid attention to new or different things in students' responses. Therefore teachers' re-listening, reflection and possible individual feedback is included in the final flow chart in figure D.11 on p. 221.

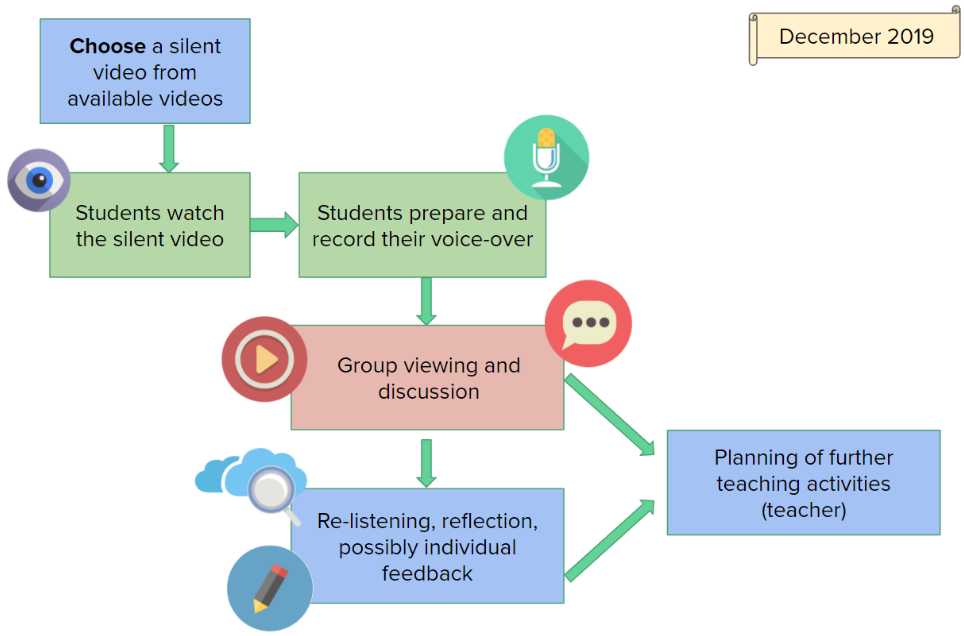


Figure D.10. A flow-chart showing the process of assigning a silent video task as it was enacted by Orri in his implementation of SVT3 at the beginning of December 2019, during the second implementation phase of the research project.

Orri also came up with the new idea of letting students listen to a voice-over to a silent video task without seeing the video. Their task would be to draw according to the instructions that they heard, e.g. using GeoGebra or on a whiteboard. After students handed in their task responses, the roles would be reversed and the teacher would attempt to draw on the whiteboard relying only on the sound from students' voice-over recordings. This idea is not included in the last flow chart, but it might be nice to try out at some point.

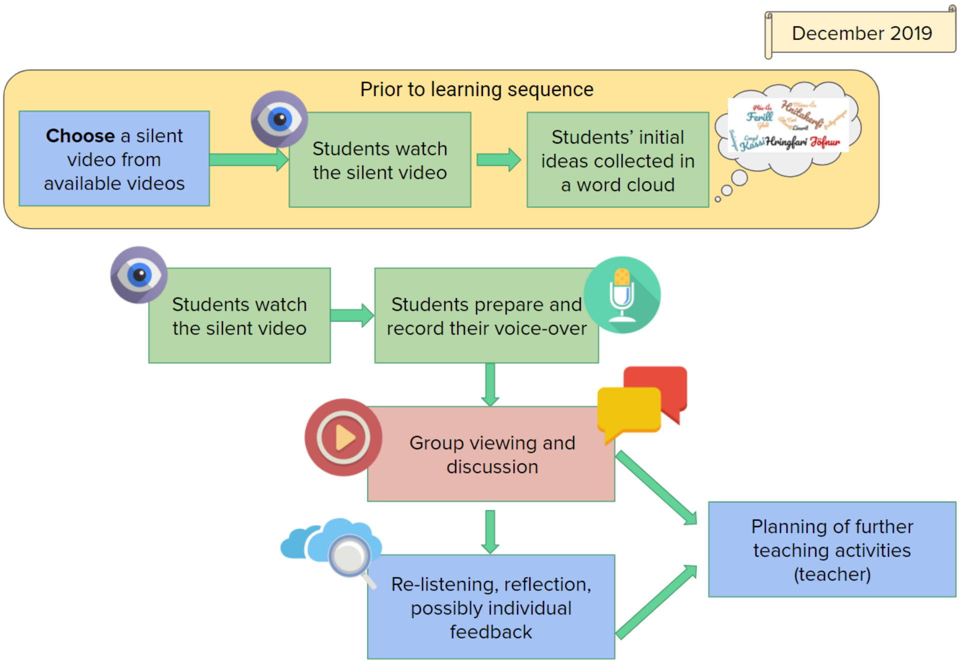


Figure D.11. A flow-chart showing the process of assigning a silent video task as it is presented in the design principles in section 5.2.1 on p. 70.

E. Thematic Maps

Thematic maps from the thematic analysis of data from the first implementation phase.

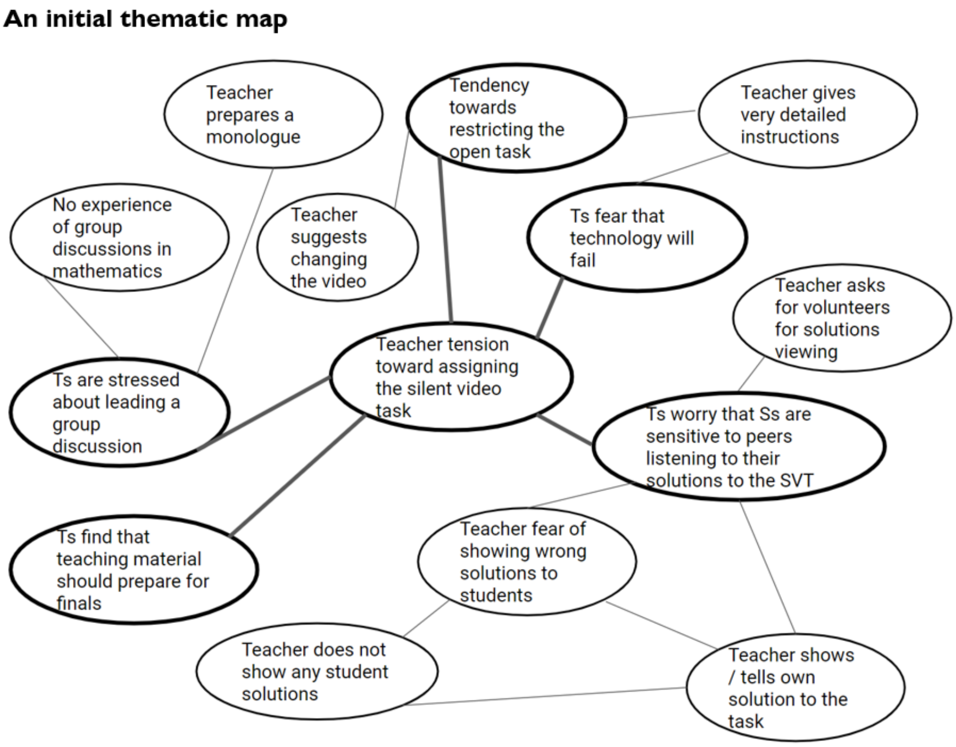


Figure E.1. An initial thematic map created during data analysis after the first implementation phase.

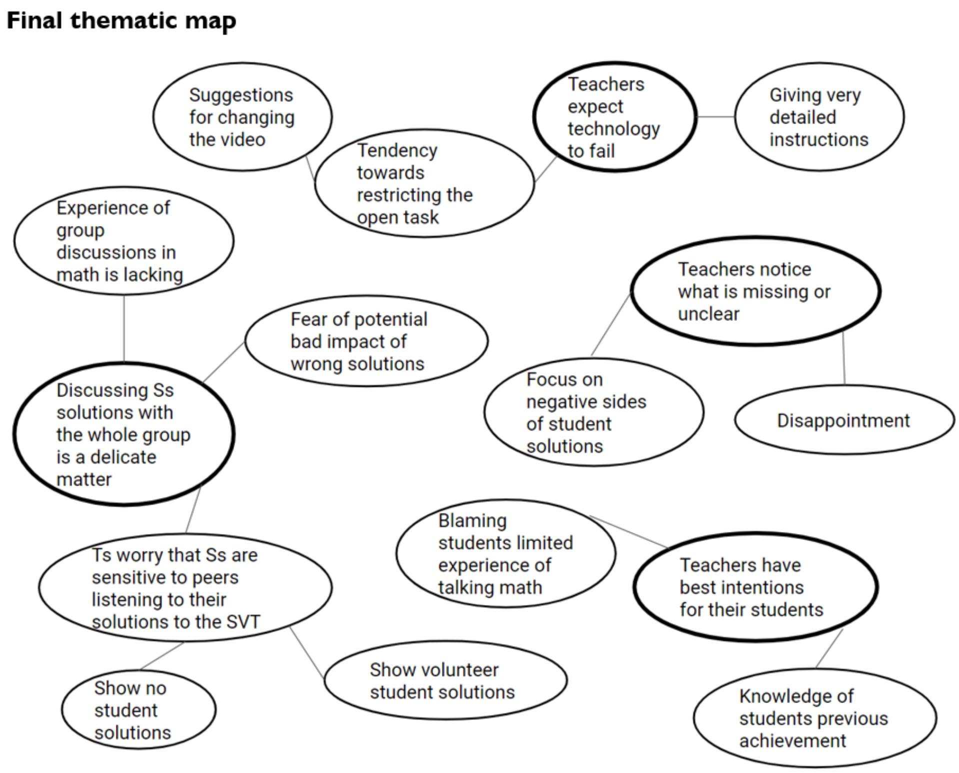


Figure E.2. The final thematic map created during re-analysis of the first phase data and used in paper II on p. 109.