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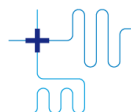
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Shiba States Coupled to a Resonant Cavity

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Abstract. When a quantum dot is embedded into a superconducting environment, it leads to the formations of localized Shiba states inside the gap. If the Coulomb interaction is sufficiently small, the Shiba states consist of a pair of singlet states and a doublet, that compete for the ground state and induce a quantum phase transition. In the presence of an external microwave field, the Shiba energy spectrum is significantly modified. Moreover, the transmission of the cavity inherits features that can pinpoint the exact location of the quantum critical point. In terms of methods used, our analytical calculations are supplemented by state of the art numerical renormalization group calculations.

INTRODUCTION

Quantum information processing in hybrid circuits, which integrate atomic and mesoscopic systems with optical resonators has been a very active area of research in recent years [1]. In particular, microwave cavities could serve as data buses between components of a quantum computer, in which artificial qubits would produce measurable effects on the amplitude and phase of a transmitted or reflected electromagnetic field, as well as a shift and broadening of the resonant frequency [2, 3]. Recent experimental advances [4] have demonstrated the coherent manipulation with microwave fields of a pair of Andreev bound states, which could become a test-ground for quantum information storage, manipulation and readout.

Motivated by these experimental findings, in the present work we investigate the change in the response of a microwave cavity probed by an weak external field, when it is coupled to a mesoscopic device that supports localized Shiba states [5, 6]. The device that we consider is displayed in Fig. 1 and consists of a quantum dot (QD) coupled to a pair of superconducting leads. The Shiba states form inside the superconducting gap, they have a given parity (singlet or doublet) and arise from the hybridization of the dot atomic levels with the superconducting excitations in the external leads. The competition between the superconducting correlations and the Kondo screening drives the system through a quantum phase transition (QPT) (also known as $\pi - 0$ transition). Thus, the symmetry of the ground state is different on the two sides of the quantum critical point (QCP). When $T_K \gg \Delta$, the ground state is a singlet¹, while in the opposite limit, i.e. $T_K \ll \Delta$, the superconducting correlations overcome the screening and the ground state becomes a doublet.

When the device is embedded into a cavity, the Shiba states, which behave as resonant states, become dressed by the photonic field. The presence of multiple states with the same parity leads to the formation of a level repulsion [7] when the resonance condition is fulfilled, i.e. $\hbar\omega_0 \sim \Delta E$, where ΔE is the energy difference between two such states. Such an avoided level crossing can be experimentally confirmed by measuring either the phase or the amplitude of

¹ T_K is defined as the Kondo temperature associated with the formation of the many body Kondo state in the absence of superconducting correlations

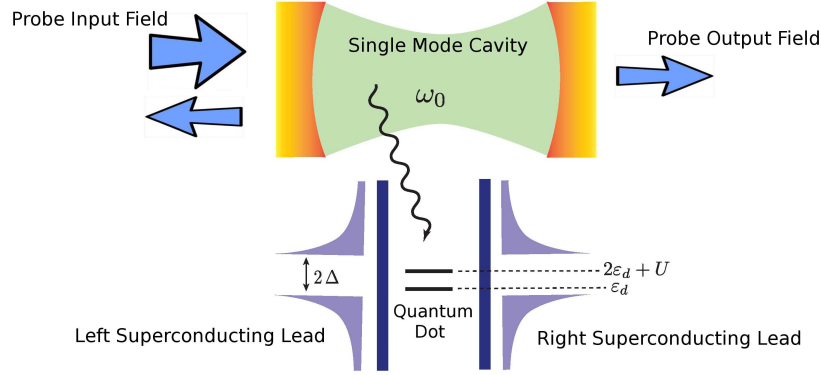


FIGURE 1. Sketch of the setup. A strongly correlated quantum dot with on-site energy ε_d and Coulomb repulsion U , is tunnel-coupled to two superconducting leads. The dot is capacitively coupled to a microwave cavity, with resonant frequency ω_0 . The superconducting gap, Δ is the largest energy scale. The cavity is probed by a weak external microwave field. Here we focus on the transmitted signal.

the transmission of the cavity, when it is probed by a variable microwave field with frequency $\omega_f \sim \omega_0$. Although the physics seems to be complicated due to the presence of the many energy scales involved, the powerful numerical renormalization group (NRG) approach [8] is able to capture it correctly. In the present work we obtain robust analytical estimates for the transmission in the so called atomic limit [9], corresponding to $\Delta \rightarrow \infty$, which are in good agreement with our recent NRG results [10].

THE HAMILTONIAN AND THE ENERGY SPECTRUM

Our model consists of a strongly correlated QD, tunnel-coupled to two superconducting leads (L and R), and capacitively coupled to a single-mode electromagnetic resonator [11]. The free Hamiltonian of the superconducting leads, H_{leads} is given by:

$$H_{\text{leads}} = \sum_{\mathbf{k}, \sigma} \sum_{l=L,R} \xi_{l\mathbf{k}\sigma} c_{l\mathbf{k}\sigma}^\dagger c_{l\mathbf{k}\sigma} - \sum_{\mathbf{k}} \sum_{l=L,R} (\Delta_l c_{l\uparrow}^\dagger c_{l-\mathbf{k}\downarrow}^\dagger + \text{H.c.}), \quad (1)$$

where Δ_l is the left-right (L-R) superconducting gap, $\xi_{l\mathbf{k}\sigma}$ is the dispersion in the superconducting leads and $c_{l\mathbf{k}\sigma}^\dagger$ are annihilation operators for an electron in lead l , with momentum $\mathbf{k}$ and spin σ . For practical NRG calculations, the L - R operators can be rotated into an even-odd basis, in which the odd channel decouples [12]. We shall also assume that $\Delta_L = \Delta_R = \Delta$ is a real positive number. The QD is described by

$$H_{\text{dot}} = \sum_{\sigma} \varepsilon_d d_{\sigma}^\dagger d_{\sigma} + U d_{\uparrow}^\dagger d_{\uparrow} d_{\downarrow}^\dagger d_{\downarrow}, \quad (2)$$

with d_{σ} the annihilation operator on the QD for spin σ , ε_d the on-site energy and U the Coulomb repulsion. We shall assume the particle-hole symmetric case, $\varepsilon_d = -U/2$, although our main results remain unchanged for the more general, non-symmetric case. The QD hybridizes with the superconduction leads, which we describe by

$$H_{\text{tun}} = \sum_{\mathbf{k}, \sigma} \sum_{l=L,R} V_l c_{l\mathbf{k}\sigma}^\dagger d_{\sigma} + \text{H.c.}, \quad (3)$$

where V_l are the tunneling matrices. We introduce the hybridization strength (which shall be used as a scanning parameter) $\Gamma = 2\pi\rho V^2$, where we assumed $V_L = V_R = V$ and $\rho = 1/(2D)$ is the constant density of states in the superconducting leads, with a bandwidth equal to $2D$. The single-mode cavity has the Hamiltonian

$$H_{\text{phot}} = \hbar\omega_0 a^\dagger a, \quad (4)$$

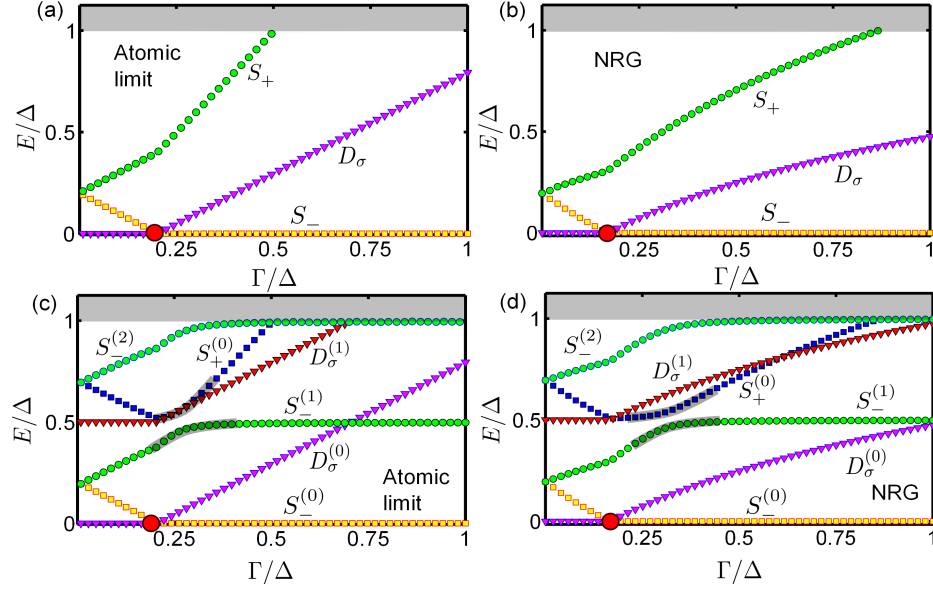


FIGURE 2. Energy spectrum: (a)-(b) Analytical and respectively NRG results in the absence of photons; (c)-(d) Analytical and respectively NRG results in the presence of photons. The other parameters are fixed to: $\omega_0/\Delta = 0.5$, $g/\Delta = 0.05$ and $U/\Delta = 0.4$. The small U value insures that both singlets are visible inside the gap. The shaded lines highlight the avoided level crossing. The shaded area above $E = \Delta$ represents the continuum. The QPT is marked by the red dot.

where a is the photon annihilation operator. The coupling between the cavity photons and the QD is assumed to be capacitive and of the form:

$$H_{\text{int}} = g \hat{n} (a + a^\dagger), \quad (5)$$

where g is the coupling strength and $\hat{n}$ the occupation of the QD ($\hat{n} = \sum_\sigma d_\sigma^\dagger d_\sigma$). The Hamiltonian of the system is

$$H_{\text{sys}} = H_{\text{leads}} + H_{\text{dot}} + H_{\text{tun}} + H_{\text{phot}} + H_{\text{int}}. \quad (6)$$

Our main purpose is to investigate the effect of the mesoscopic device on the transmission $t(\omega)$ of the cavity in the subgap frequency domain $\omega < \Delta$. As we shall demonstrate in the next section, it is related to the full Green's function for the photons in the cavity. To find it we solve the full Hamiltonian (6) with the NRG. However, it is more intuitive to investigate at first analytically the electronic part of Hamiltonian: $H_{\text{el}} = H_{\text{leads}} + H_{\text{dot}} + H_{\text{tun}}$ in the atomic limit $\Delta \rightarrow \infty$. For that we integrate the superconducting leads to obtain a quadratic Hamiltonian of the form

$$H_{\text{el}}^{(\Delta \rightarrow \infty)} = \sum_\sigma \xi_d d_\sigma^\dagger d_\sigma - \Gamma_\varphi (d_\uparrow^\dagger d_\downarrow^\dagger + \text{H.c.}) + \frac{U}{2} (\hat{n} - 1)^2, \quad (7)$$

in which $\xi_d = \varepsilon_d + U/2$ and $\Gamma_\varphi = 2\Gamma/\pi \arctan(D/\Delta)$. Thus, the presence of the superconducting leads induces superconducting correlations in the QD, which are represented by the second term in Eq. (7). Hamiltonian (7) can be diagonalized exactly. By denoting the empty dot state by $|0\rangle$, the four eigenstates can be grouped into a pair of singlets $|S_\pm\rangle$ and a doublet $|\mathcal{D}_\sigma\rangle$:

$$|S_\pm\rangle = \frac{1}{\sqrt{2}} (1 \pm d_\uparrow^\dagger d_\downarrow^\dagger) |0\rangle, \quad (8)$$

$$|\mathcal{D}_\sigma\rangle = d_\sigma^\dagger |0\rangle, \quad (9)$$

with the corresponding eigenenergies $E_\pm = U/2 \pm \Gamma_\varphi$ and $E_\sigma = 0$. Notice that the two singlet states are a mixtures of zero and two-particle states. When $\Delta \ll T_K$, the ground state is the singlet $|S_-\rangle$, while in the opposite limit,

$\Delta \gg T_K$, the ground state changes to the doublet $|\mathcal{D}_\sigma\rangle$. Their evolution as a function of the broadening parameter Γ is represented in Fig. 2 (a). When $E_- = E_\sigma$, (corresponding to $T_K \sim \Delta$) the system experiences a quantum phase transition (QPT), indicated by a red dot. In Fig. 2(b) we represent the same evolution, but computed exactly with the NRG. The good agreement between the two approaches guarantees that the simplistic atomic limit captures all the important features of the model.

In the presence of the cavity, the Shiba states hybridize with the photons. By adding the photons, in the atomic limit, the system Hamiltonian $H_{\text{sys}}^{(\Delta \rightarrow \infty)} = H_{\text{el}}^{(\Delta \rightarrow \infty)} + H_{\text{phot}} + H_{\text{int}}$ can be diagonalized exactly, by keeping only a small number of photonic states. The corresponding energy spectrum is shown in Fig. 2(c). Here we have fixed $\hbar\omega_0 = \Delta/2$ and we represent again only the subgap states. When the photonic states are hybridized with the electronic ones, the result is the formation of ladder-like states, labeled as in Fig. 2(c): $D_\sigma^{(0)}$, $D_\sigma^{(1)}$, $S_-^{(0)}$, $S_-^{(1)}$, $S_+^{(0)}$, etc. The labeling of a hybridized state illustrates both to which electronic state it belongs, and the number of photons used to build the state in the singlet sector, far away from the resonance. In the $\Delta \ll T_K$ region, where the ground state is a singlet, the system behaves effectively as a two-level system, resulting in an avoided crossing (represented by the shaded lines in Fig. 2(c)) due to H_{int} , at the point where $\omega_0 \simeq E_{S_+}$. The presence of the photons does not alter the position of the QCP. For comparison, we represent in Fig. 2(d) the exact energy spectrum computed with the NRG. Again, the agreement between the simple atomic model and the NRG calculations is very good.

TRANSMISSION THROUGH THE CAVITY

The main focus of the present work is to find distinctive feature in the transmission of the cavity coming from the hybridization with the mesoscopic device. In this section we present how these effects are captured, following closely the approach in Ref. [13, 10]. We shall assume that the resonator is weakly coupled to two bosonic baths (external transmission lines), so that the total Hamiltonian takes the form:

$$H = H_{\text{sys}} + \sum_{\beta=\{L,R\}} \sum_{\mathbf{q}} \left[\omega_{\mathbf{q}} b_{\beta\mathbf{q}}^\dagger b_{\beta\mathbf{q}} + \lambda_{\mathbf{q}} (b_{\beta\mathbf{q}}^\dagger + b_{\beta\mathbf{q}}) (a + a^\dagger) \right], \quad (10)$$

where $b_{\beta\mathbf{q}}^\dagger$ is the creation operator of a photonic mode $\mathbf{q}$ in the transmission line $\beta = \{L, R\}$ with photon energy $\omega_{\mathbf{q}}$ and coupling $\lambda_{\mathbf{q}}$ to the cavity. The photons leak out of the cavity at a rate [13]

$$\mathcal{J}(\omega) = \pi \sum_{\mathbf{q}} \lambda_{\mathbf{q}}^2 \delta(\omega - \omega_{\mathbf{q}}). \quad (11)$$

Close to the resonant frequency of the cavity, the escape rate of the cavity κ is approximately constant and given by $\kappa = 2\pi \alpha \omega_0$, with α a dimensionless constant. In order to characterize the transmission through the cavity, we need to calculate the retarded Green's function for the cavity photons [13], which is defined as

$$\mathcal{D}^R(t) = -i\Theta(t) \left\langle [a(t) + a^\dagger(t), a(0) + a^\dagger(0)] \right\rangle. \quad (12)$$

In the absence of any sources of dissipation, the non-interacting correlator is

$$\mathcal{D}_0^R(\omega) = \frac{2\omega_0}{\omega^2 - \omega_0^2 + i0^+}. \quad (13)$$

The effects of the external transmission lines and the coupling to the electronic states are captured by including the self-energy corrections in the full Green's function (12), using Dyson's equation [13, 14]

$$\mathcal{D}^R(\omega)^{-1} = \mathcal{D}_0^R(\omega)^{-1} - \Sigma_{\text{el}}(\omega) - \Sigma_{\text{ph}}(\omega), \quad (14)$$

where $\mathcal{D}^R(\omega)$ is the Fourier transform of the temporal Green's function appearing in Eq. (12). Here, $\Sigma_{\text{ph}}(\omega) = -i\kappa/2$ incorporates the dissipation due to the coupling of the cavity to the external modes. The self-energy contribution coming from the light-matter interaction can be obtained using an equation of motion approach, and is given by [14]:

$$\Sigma_{\text{el}}(\omega) = g \frac{\mathcal{F}^R(\omega)}{\mathcal{D}^R(\omega)}, \quad (15)$$

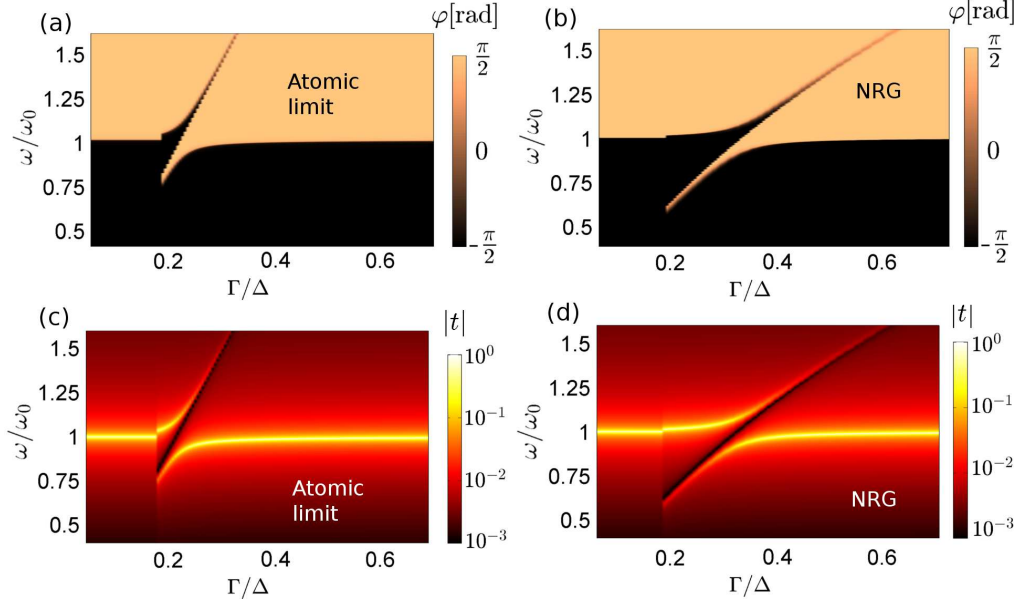


FIGURE 3. Density plots for: (a)-(b) the phase and (c)-(d) the absolute value of the transmission through the cavity, as calculated using the atomic limit and the NRG method at zero temperature. We assumed a dissipation $\alpha = 5 \times 10^{-4}$ through the contacts to the external transmission lines. The other parameters used correspond to those in Fig. 2. The Shiba bound states were broadened using gaussians with a standard deviation $\sigma/\Delta = 5 \times 10^{-3}$.

where $\mathcal{F}^R(\omega)$ is the Fourier transform of the retarded bosonic Green's function: $\mathcal{F}^R(t) = -i\Theta(t)\langle[\hat{n}(t), a(0) + a^\dagger(0)]\rangle$, which can be calculated directly from the NRG. Within the NRG framework, the spectral function for the photons can be computed directly, however, it was shown [14, 15] that by using the self-energy correction, the quality of the spectral function is improved. Using standard input-output theory [13, 16], in the limit of a weak coupling between the cavity and the external photonic lines, the transmission through the cavity is given by:

$$t(\omega) = i\mathcal{J}(\omega)\mathcal{D}^R(\omega). \quad (16)$$

We define the amplitude $|t(\omega)|$ and phase $\varphi(\omega)$ of the transmission through the relation:

$$t(\omega) = |t(\omega)|e^{i\varphi(\omega)}. \quad (17)$$

Using both approaches, we have calculated the phase and the amplitude of the transmission through the cavity. They are represented in Fig. 3. There is an excellent qualitative agreement between the two methods. When the system is off-resonance, the transmission amplitude has a Lorentzian shape centered at $\omega \simeq \omega_0$ and a width κ (see Fig. 3 (c)-(d)). The shape of the transmission corresponds to the case of a cavity coupled only to the transmission line, without any dissipation in the mesoscopic device. In the phase component of the transmission, a π phase jump is expected in this case (see Eq. (13) and compare with Fig. 3 (a)-(b)). These features are preserved in the whole region where the ground state for the H_{el} is a doublet, and the transmission is basically unaltered. However, in the singlet sector, close to the resonance ($\omega_0 \simeq E_{S_+}$), the transmission peak splits into two, corresponding to the transitions $S_-^{(0)} \rightarrow S_-^{(1)}$ and $S_-^{(0)} \rightarrow S_+^{(0)}$ in Fig. 2. Correspondingly, the self-energy contribution of the light-matter interaction, $\Sigma_{\text{el}}(\omega)$, introduces extra $\pm\pi$ -jumps in the phase, each time an excited singlet Shiba state is encountered, and the transmission is drastically affected. Thus, both the phase and the amplitude of the transmitted probing microwave field can be used to pinpoint the position of the QCP in a non-invasive manner.

CONCLUSIONS

In conclusion, we have studied a hybrid system that consists of a quantum dot, tunnel-coupled to superconducting leads and also capacitively coupled to a microwave cavity. In the absence of photons, such an electronic system

develops multiple Shiba states inside the superconducting gap. We demonstrated that photonic measurements can be used to probe such localized Shiba states, as they leave a distinct fingerprint on both the amplitude and phase of the probing field. The analytical method employed in the atomic limit successfully reproduces the exact results obtained with the NRG.

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