*Research Article*

# Seismic Energy Assessment of Buildings with Tuned Vibration Absorbers

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Seismic analysis and energy assessment of building installed with distributed tuned vibration absorbers (d-TVAs) are presented. The performance of d-TVAs is compared with single tuned vibration absorber (STVA) installed at the top of the building. The placements of the d-TVAs are based on the modal properties of the uncontrolled and controlled buildings. The governing equations of motion of the building with the STVA and d-TVAs are solved by employing Newmark's integration method. Various energies under earthquake ground excitations are computed to study the effectiveness of using the STVA and d-TVAs. It is concluded that the use of the d-TVAs is the most competent because it effectively dissipates the seismic energy, and they are convenient to install requiring reduced space, as are placed at various floors.

## 1. Introduction

Earthquake events caused damages in structures especially buildings. The input energy of an earthquake excitation can be dissipated by tuned vibration absorber (TVA), which causes improvement in the performance of the buildings. Several researchers had employed the TVA and reported fairly good results of dynamic response control in structures [1–7]. Later in [8–13], researchers proposed multiple TVAs to overcome the decreased efficiency of the TVA in off-tuning and occupy large floor area with its massive weight. For tall and flexible buildings, Aly et al. [14] and Aly [15, 16] showed that providing the TVAs prove to be useful in mitigating their dynamic response. Dynamic response reduction in different structures achieved by using the tuned vibration absorbers was studied by Lu [17, 18]. They found that the particle movements of the plug flow pattern could yield good vibration attenuation effects. The detailed literature survey on passive TVA is provided by Elias and Matsagar [19]. However, in majority of studies, it is assumed that the structure vibrates in only one direction or in multiple directions independently. In addition, the fundamental modal properties of the structures were used to

design the TVA or the TVAs. In reality, the structures will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. Therefore, many researchers considered the three-dimensional (3D) model installed with TVA or TVAs [10, 20–36]. The common outcome of the studies shows that TVAs are more robust and effective as compared to the TVA in response mitigation of 3D structures.

In most of the studies, absorbers are placed at the top of the structures. The loss of effectiveness of the multiple TVAs is minimal if they are distributed based on mode shape of the main system as concluded by Elias et al. [37]. Elias et al. [38] presented the better performance of the d-TVAs as compared to the STVA/TVAs all placed at the topmost floor. They had conducted study on dynamic response control of buildings wherein both (a) placement and (b) tuning of the TVAs in the main system are made in accordance with the modal proprieties of the uncontrolled and controlled buildings. Gill et al. [39] showed that the d-TVAs were quite effective in seismic response control of structures. However, these studies do not establish energy-based seismic performance assessment. Therefore, the objective of this study is to investigate the efficient positioning of the TVAs based on

the modal properties of the uncontrolled and controlled buildings. The TVAs are placed where the mode shape amplitude of the building is the largest or larger in the particular mode and tuned to the corresponding modal frequency. The number of modes to be controlled is decided depending on total mass participation in the controlled modes.

## 2. Mathematical Modeling and Governing Equations of Motion

Mathematical models of the buildings are developed by making the following assumptions:

(1) The superstructure is considered to remain within the elastic limit during the earthquake excitation.

(2) The system is subjected to a single horizontal (unidirectional) component of the earthquake ground excitation.

(3) The effects of soil-structure-interaction (SSI) are not taken into consideration.

(4) At least ninety percent of total mass is included in the controlled modes.

As shown in Figure 1, the total degrees of freedom (DOF) of the controlled system become $(N + n)$ by summing up the $N$ DOF building with $n$ number of TVAs. For the system under consideration, the governing equations of motion for the earthquake-excited building installed with the TVA at the top and installed with the d-TVAs are obtained by considering the equilibrium of forces at the location of each degree of freedom during earthquake excitation as

$$
\begin{aligned}
& \begin{bmatrix} [M_N]_{N \times N} & [0]_{N \times n} \\ [0]_{n \times N} & [m_n]_{n \times n} \end{bmatrix} \begin{Bmatrix} \{\ddot{X}_N\}_{N \times 1} \\ \{\ddot{x}_n\}_{n \times 1} \end{Bmatrix} \\
& + \begin{bmatrix} [C_N]_{N \times N} + [c_n]_{N \times N} & -[c_n]_{N \times n} \\ -[c_n]_{n \times N} & [c_n]_{n \times n} \end{bmatrix} \begin{Bmatrix} \{\dot{X}_i\}_{N \times 1} \\ \{\dot{x}_i\}_{n \times 1} \end{Bmatrix} \\
& + \begin{bmatrix} [K_N]_{N \times N} + [k_n]_{N \times N} & -[k_n]_{N \times n} \\ -[k_n]_{n \times N} & [k_n]_{n \times n} \end{bmatrix} \begin{Bmatrix} \{X_i\}_{N \times 1} \\ \{x_i\}_{n \times 1} \end{Bmatrix} \\
& = - \begin{bmatrix} [M_N]_{N \times N} & [0]_{N \times n} \\ [0]_{n \times N} & [m_n]_{n \times n} \end{bmatrix} \begin{Bmatrix} \{r\}_{N \times 1} \\ \{r\}_{n \times 1} \end{Bmatrix} \ddot{x}_g,
\end{aligned} \tag{1}
$$

where $[M_N]$, $[C_N]$, and $[K_N]$ are the mass, damping, and stiffness matrices of the building, respectively; $\{X_N\}$, $\{\dot{X}_N\}$, and $\{\ddot{X}_N\}$ are the unknown relative floor displacement, velocity, and acceleration vectors, respectively; earthquake ground acceleration is represented by the scalar $\ddot{x}_g$; and $\{r\}$ is the vector of influence coefficients. The mass, damping, and stiffness matrices of the TVAs, respectively, are $[m_n]$, $[c_n]$, and $[k_n]$; whereas, $\{x_n\}$, $\{\dot{x}_n\}$, and $\{\ddot{x}_n\}$, respectively, are the unknown relative displacement, velocity, and acceleration of the TVAs. Modal analysis is conducted to determine the natural frequencies, mode shapes, and modal mass participations of the uncontrolled/controlled

building. Only the first few modes are considered for controlling in this work as they predominantly influence the total dynamic response being sum of their modal mass participations reaches ninety percent. The locations for installation of the TVAs are identified based on the mode shapes of the uncontrolled and controlled buildings. Figure 1 also shows the placement of the five TVAs as follows: TVA-1 at 20th floor, i.e., on the topmost floor; TVA-2 at 7th floor; TMD-3 at 4th floor; TMD-4 at 14th floor; and TMD-5 at 16th floor. The first five natural frequencies of the uncontrolled building are 0.3434, 1.0282, 1.7069, 2.3754, and 3.0305 Hz, which are the tuning frequencies for the TVA-1, TVA-2, TVA-3, TVA-4, and TVA-5, respectively, controlling each corresponding mode.

*2.1. Tuning of TMDs.* Figure 2 shows the procedure followed for (a) placement of the d-TVAs and (b) optimization of parameters of the d-TVAs. The modal analysis is conducted to find the natural frequencies $[\Omega_{i'}, \omega_{j'}]$, mode shapes $\{\phi_{i',j'}\}$, and modal mass contribution $[M_r]$ of the uncontrolled and controlled building using its stiffness $[K_s]$ and mass $[M_s]$ matrices for $(N + n)$ degrees of freedom (DOF). Based on an assumption that 90% of the modal mass participates in the response, the first five modal responses $(n = 5)$ are controlled, and subsequently, the optimum tuning frequency of each TVA [40] is calculated as follows:

$$
f_{\text{opti}} = \frac{1}{1 + \mu_i \phi} \left( 1 - \zeta_N \sqrt{\frac{\mu_i \phi}{1 + \mu_i \phi}} \right), \quad i = 1 \text{ to } n, \tag{2a}
$$

$$
\omega_i = f_{\text{opti}} \Omega_i, \quad i = 1 \text{ to } n, \tag{2b}
$$

where $\mu_i = \mu/n$, $\phi$ and $\zeta_N$, are, respectively, the mode shape and damping ratio of the $N$th mode. Moreover, $\omega_i = \omega_1 ... \omega_5$ are the frequencies of the five TVAs, respectively, from the lowest to the highest, and $\Omega_i = \Omega_1 ... \Omega_5$ are the first five natural frequencies of the building. For the TVAs, while keeping the masses, $m_1 = m_2 = m_3 = ... = m_n$, equal for all the TVAs, the mass $(m_i)$ of each TVA is calculated by

$$
m_i = \frac{m_t}{n}, \quad i = 1 \text{ to } n. \tag{3}
$$

The stiffness is used for frequency tuning of each TVA and the stiffness of the TVAs $(k_i)$ installed on different floors $(i)$ are then determined by

$$
k_i = m_i \omega_i^2, \quad i = 1 \text{ to } n. \tag{4}
$$

The optimum damping ratio $(\zeta_{\text{dopti}} \neq \zeta_1 \neq \zeta_2 \neq \cdots \zeta_n)$ of the TMDs is not constant for all TMDs. The optimum damping ratio is calculated for base excited structure as per the formula given by Sadek et al. [40].

$$
\zeta_{\text{dopti}} = \phi \left( \frac{\zeta_N}{1 + \mu_i} + \sqrt{\frac{\mu_i}{1 + \mu_i}} \right), \quad i = 1 \text{ to } n, \tag{5}
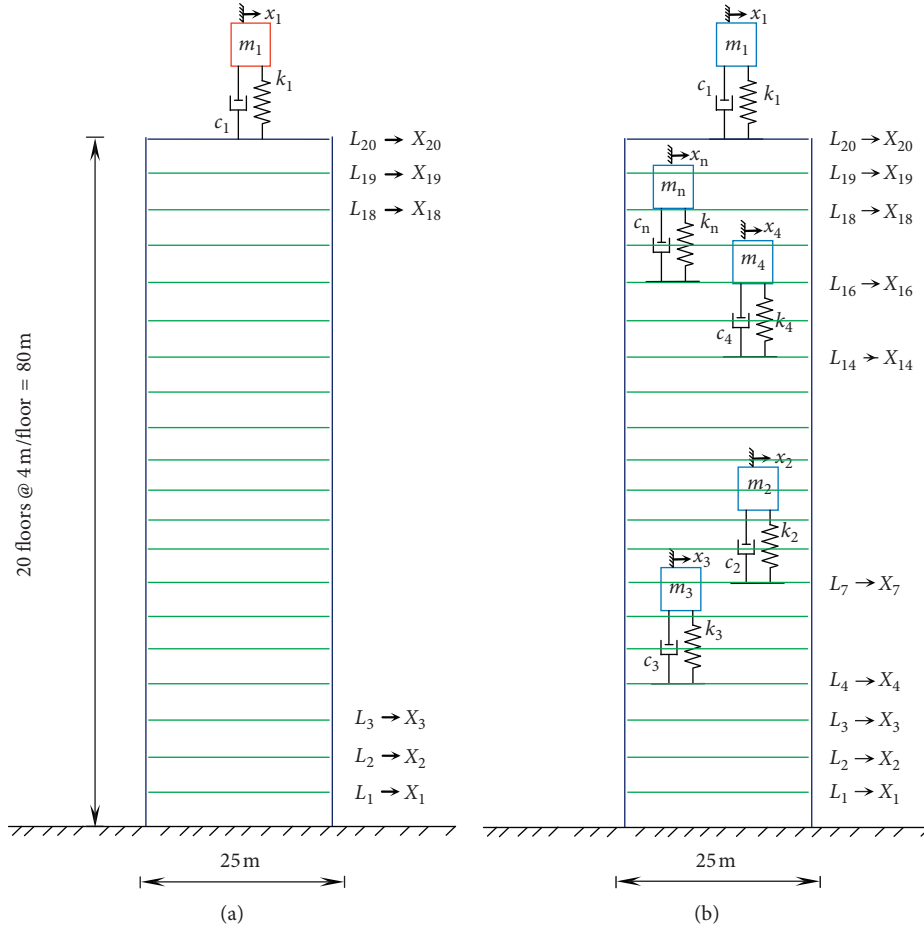$$

Figure 1: Model of 20-storey building installed with (a) STVA and (b) d-TVAs.

and the damping coefficients ($c_i$) of the TMDs are calculated by

$$c_i = 2\zeta_{\mathrm{dopti}} m_i \omega_i, \quad i = 1 \text{ to } n. \tag{6}$$

### 2.2. Energy Formulation.

Energy is input to the building by the ground motion during an earthquake excitation. Nevertheless, entire input energy is not dissipated at any time during the earthquake excitation, and remainder of the energy is stored in the structure in the form of interchangeable kinetic and strain energies. The input energy ($E_i$) is the work done by the ground motion on the building.

$$E_i = -\frac{1}{(M_t)} \sum_0^{t_k} \left\{ \begin{array}{c} \{dX\}_{N\times1} \\ \{dx\}_{n\times1} \end{array} \right\}^T \left[ \begin{array}{cc} [M_N]_{N\times N} & [0]_{N\times n} \\ [0]_{n\times N} & [m_n]_{n\times n} \end{array} \right]$$
$$\cdot \left\{ \begin{array}{c} \{r\}_{N\times1} \\ \{r\}_{n\times1} \end{array} \right\} \ddot{x}_g, \tag{7}$$

where $\{dX_i\}_{t_d} = \{X_i\}_{t_d} - \{X_i\}_{t_d-1}$, $\{dx_i\}_{t_d} = \{x_i\}_{t_d} - \{x_i\}_{t_d-1}$, $t_k$ is the duration of the earthquake excitation, and $t_d$ is the small time increment. It is desirable that the chimney

installed with the TVAs be able to dissipate more damping energy than without installation of the TVAs, as the tuned mass dampers help in maximizing the damping energy dissipation. In this study, $E_d$ is defined as the damping energy per mass of the building and dampers. The damping energy per mass for uncontrolled (NC) and controlled building is calculated by

$$E_d = \frac{1}{(M_t)} \sum_0^{t_k} \left\{ \begin{array}{c} \{dX\}_{N\times1} \\ \{dx\}_{n\times1} \end{array} \right\}^T$$
$$\cdot \left[ \begin{array}{cc} [C_N]_{N\times N} + [c_n]_{N\times N} & -[c_n]_{N\times n} \\ -[c_n]_{n\times N} & [c_n]_{n\times n} \end{array} \right] \tag{8}$$
$$\cdot \left\{ \begin{array}{c} \{\dot{X}_i\}_{N\times1} \\ \{\dot{x}_i\}_{n\times1} \end{array} \right\}.$$

In these expressions, at any point of time, the input energy and damping energy dissipated by the building and TVAs is the summation of the total damping energy dissipated till that time [41]. The energy stored in the
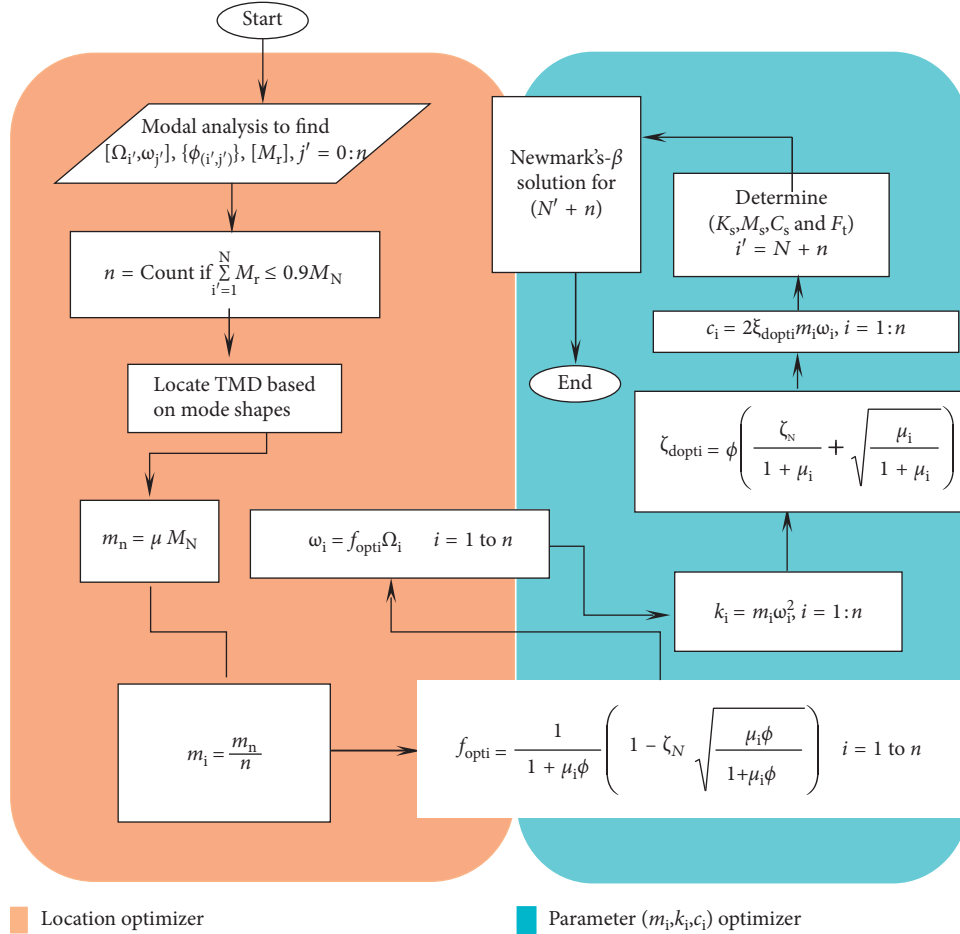
FIGURE 2: Flowchart for location and design parameters of d-TVAs for seismic response control of buildings.

building due to its deformations is strain energy ($E_s$). In this study, it is assumed to model shear building; therefore, $E_s$ can be

$$E_s = \frac{1}{2}\frac{1}{(M_t)}\left\{\begin{array}{c}\{X_i\}_{N\times 1}\\ \{x_i\}_{n\times 1}\end{array}\right\}^T$$

$$\cdot\left[\begin{array}{cc}[K_N]_{N\times N}+[k_n]_{N\times N} & -[k_n]_{N\times n}\\ -[k_n]_{n\times N} & [k_n]_{n\times n}\end{array}\right] \quad (9)$$

$$\cdot\left\{\begin{array}{c}\{X_i\}_{N\times 1}\\ \{x_i\}_{n\times 1}\end{array}\right\}.$$

Similarly, the kinetic energy ($E_k$) is stored in the structure in the form of its motion.

$$E_k = \frac{1}{2}\frac{1}{(M_t)}\left\{\begin{array}{c}\{\dot{X}_i\}_{N\times 1}\\ \{\dot{x}_i\}_{n\times 1}\end{array}\right\}^T\left[\begin{array}{cc}[M_N]_{N\times N} & [0]_{N\times n}\\ [0]_{n\times N} & [m_n]_{n\times n}\end{array}\right]$$

$$\cdot\left\{\begin{array}{c}\{\dot{X}_i\}_{N\times 1}\\ \{\dot{x}_i\}_{n\times 1}\end{array}\right\}. \quad (10)$$

## 3. Numerical Study

The seismic energy assessment of the 20-storey concrete building installed with the STVA and d-TVAs is investigated. The 20-storey building is modeled as shear-type lumped masses. The mass each floor is 252 tonnes, and stiffness of each floor is calculated from the columns. In this study, the beam and column sections are assumed to be $0.6 \times 0.6\,m^2$ and $0.7 \times 0.7\,m^2$, respectively. The modulus of elasticity ($E_c$) of the concrete is assumed to be $2.5 \times 10^{13}\,N/m^2$ and $2.9 \times 10^{10}\,N/m^2$, respectively, for the beam and the column, and density of the concrete is considered $2,400\,kg/m^3$. The floor-to-floor height is 4 m, and the floors are assumed to be rigid in the horizontal plane because they provide diaphragm action in its own plane. The damping matrix is not explicitly known; hence, it is defined with the help of the Rayleigh's approach using damping ratio ($\zeta_s = 5\%$) in all modes of vibration. The mass participation factors ($\Gamma_i$) for the first, second, third, fourth, and fifth vibration modes are about 0.7300, 0.0829, 0.0502, 0.0306, and 0.0294, respectively. The present energy study contains three parts as follows: (i) the uncontrolled building (NC); (ii) the STVA is installed at the topmost floor of the building; (iii) in case of the d-TVAs, the TVAs are placed at the locations where the mode shape amplitude of the
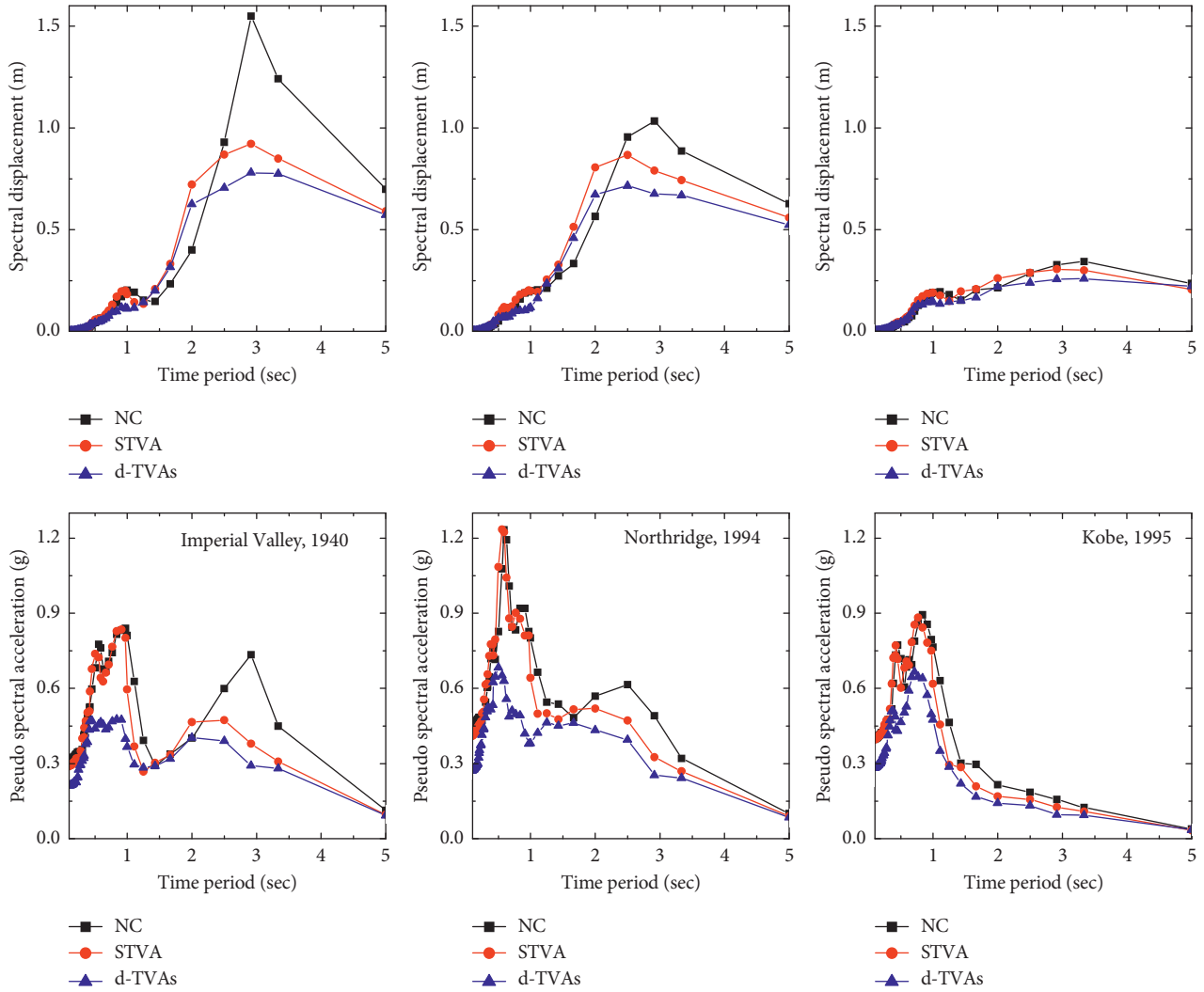
FIGURE 3: Spectral displacement and pseudospectral acceleration for the NC, STVA, and d-TVAs subject to different ground excitations.

buildings is the largest/larger in the particular mode, and each TVA is tuned to the corresponding modal frequency, while controlling first five modes having total mass participation of greater than 90%. The earthquake motions selected for the study are as follows: (i) 1940 Imperial Valley earthquake recorded at Elcentro; (ii) 1994 Northridge earthquake recorded at Sylmar station; and (iii) 1995 Kobe earthquake recorded at Japan meteorological agency (JMA). The peak ground acceleration (PGA) of the Imperial Valley, Northridge, and Kobe earthquakes are 0.34 $g$, 0.6 $g$, and 0.86 $g$, respectively; where, $g$ denotes the gravitational acceleration.

### 3.1. Variation of Responses for STVA and d-TVAs.

In this section, a comparison is made between the seismic responses of the building controlled with the STVA and d-TVAs. For STVA, the damping ratio ($\zeta_{\text{dopt}}$) is calculated by Equation (5) and mass ratio ($\mu$) is assumed to be 0.05. The fair comparison requirement is to install the d-TVA system having same total mass ratio ($\mu$) of 0.05. Equations (2a) through (6) are used to calculate the optimum parameters for TVA system. The

focus of the present is not optimization of the parameters of the TVA or TVA schemes. In this section, spectral displacements and pseudospectral accelerations are calculated for the NC, STVA, and d-TVAs. The 20-storey building is considered while increasing the rigidity that the fundamental time period reaches 0.03 sec. Similarly, the rigidity of the building is reduced that the fundamental time period reaches 5 sec. The variation in both spectral displacements and pseudospectral accelerations by varying the rigidity is shown in Figure 3. Generally, it is seen that, by making the structure more flexible, the spectral displacement increases. However, the pseudospectral acceleration is decreased significantly. In addition, it is observed that both controller schemes are quite effective in controlling the spectral displacements and pseudospectral accelerations. In case spectral displacement response is concerned, it is seen that up to 40% and 50% reductions achieved, respectively, for the STVA and d-TVAs. Further, it is seen that both schemes are less effective for rigid structures. Best performances are achieved for both schemes while the time period is between 2 sec and 4 sec. Therefore, it is concluded
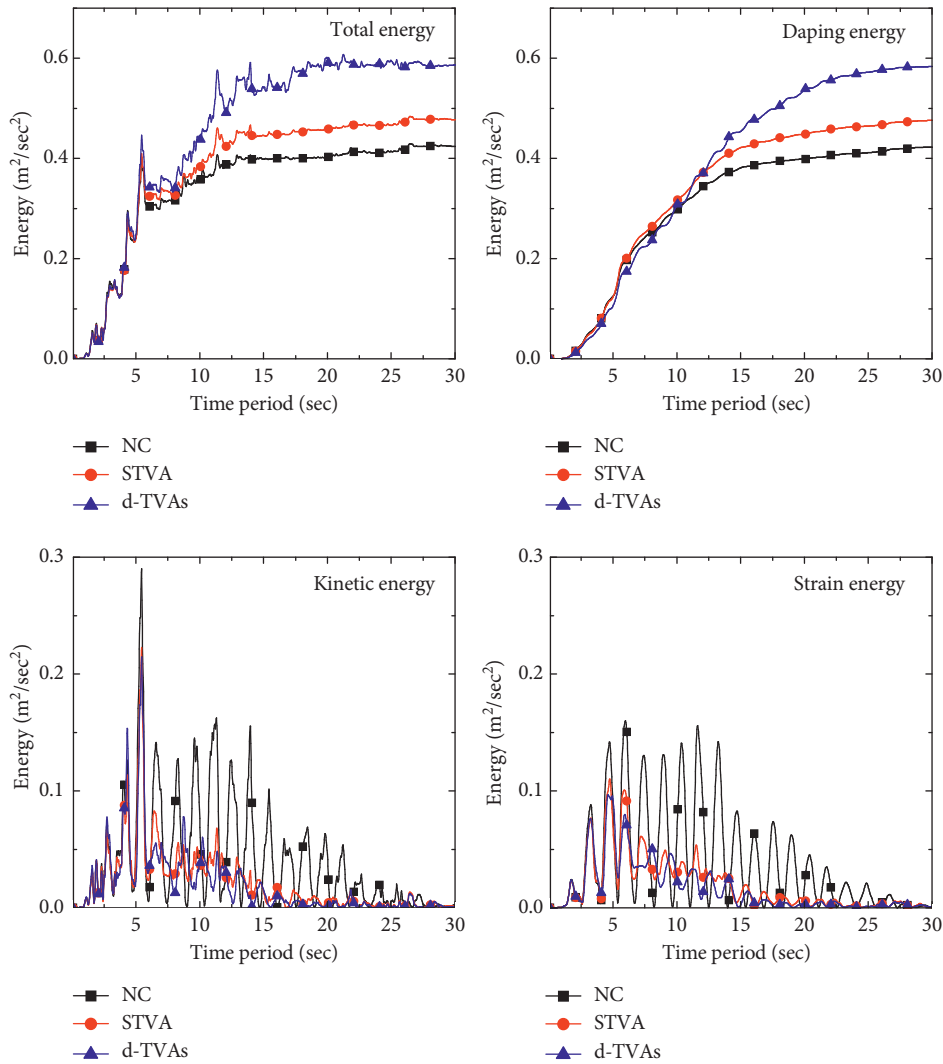
FIGURE 4: Different energies for the NC, STVA, and d-TVAs subject to Imperial Valley, 1940, ground excitation.

that the STVA and d-TVAs are effective in displacement response control of flexible structures as compared to rigid structures. Also, better performance of d-TVAs as compared to STVA is evident based on results presented. Further, the installation of d-TVAs is practically easier as compared to STVA because of its reduced size.

In case pseudospectral acceleration response is concerned, the response is reduced for flexible structures for NC, STVA, and d-TVAs. The STVA is generally more effective in structure with time period higher than 1 sec. It is seen that up to 35% reduction is achieved after installation of the STVA. It is also clearly found that the d-TVAs are better controllers as compared to the STVA for the range of the structure from rigid to flexible. The reduction in pseudospectral acceleration is up to around 50% for the case where the d-TVAs are installed. It is concluded that the d-TVAs is more effective in controlling the pseudospectral acceleration response of the structures.

3.2. *Variation of Energies for NC/STVA/d-TVAs.* Based on the results presented in Figure 3, the building with fundamental time period of 2.91 sec is selected as most vulnerable. Therefore, for the energy assessment, the building with fundamental time period of 2.91 sec is selected. Figure 4 shows the time histories of the total energy, damping energy, kinetic energy, and strain energy for the building without (NC) and with the TVAs used in different schemes defined earlier; STVA and d-TVAs and corresponding peak values are given in Table 1. The mass ratio of 5%, tuning frequency ratio by Equation (2a), and damping ratio by Equation (5) are considered for both the schemes. It is generally seen from the energy plots that, as compared to the NC case, higher damping energies are dissipated when the TVAs are installed. The relative increase in the damping energy is significantly more in the d-TVAs than the STVA and NC. Increase in total energy and damping energy are, respectively, 13% and 12.5% by installing STVA, whereas it is 30% and 28% by installing the d-TVAs. Thus, the effectiveness of the multimode control strategy using the d-TVAs in enhancing damping energy in the system is confirmed.

TABLE 1: Peak energy for the uncontrolled building and controlled with STVA and d-TVAs having mass ratio of 0.05.

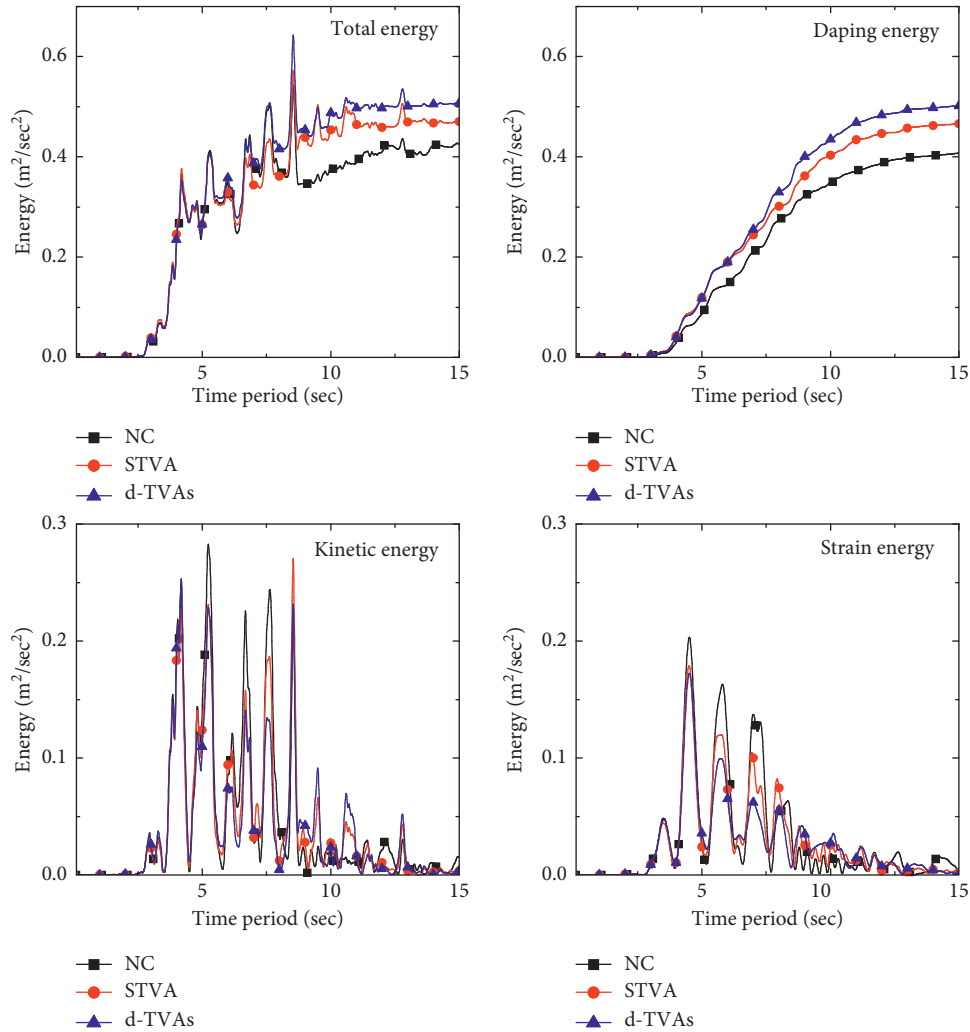| Energy ($m^2/sec^2$) | Imperial Valley, 1940 | | | Northridge, 1994 | | | Kobe, 1995 | | |
|---|---|---|---|---|---|---|---|---|---|
| | NC | STVA | d-TVAs | NC | STVA | d-TVAs | NC | STVA | d-TVAs |
| Total | 0.429 | 0.484 | 0.608 | 0.544 | 0.572 | 0.643 | 0.155 | 0.158 | 0.165 |
| Damping | 0.423 | 0.476 | 0.584 | 0.408 | 0.519 | 0.559 | 0.143 | 0.151 | 0.152 |
| Kinetic | 0.290 | 0.223 | 0.215 | 0.283 | 0.200 | 0.117 | 0.074 | 0.072 | 0.070 |
| Strain | 0.160 | 0.110 | 0.097 | 0.203 | 0.179 | 0.172 | 0.045 | 0.041 | 0.032 |



FIGURE 5: Different energies for the NC, STVA, and d-TVAs subject to Northridge, 1994, ground excitation.

Further, it is evident that the kinetic energy and strain energy are reduced by installing the STVA and d-TVAs as compared to NC. It is generally due to reduction of the displacement and acceleration responses of the structure. The improved performance of d-TVAs is observed as compared to the STVA. Similar trend of response control and observations can be made from Figures 5 and 6 for different earthquake excitations. It is concluded that increase in the total energy and damping energy is significantly more in the d-TVAs than the STVA and NC. It is also concluded that the decrease in the kinetic energy and strain energy is

significantly less in the d-TVAs than the STVA and NC. Generally, it is concluded that the use of the d-TVAs is the most competent because it effectively dissipates the seismic energy, and they are convenient to install requiring reduced space, as are placed at various floors. The procedure adopted here is based on ignoring the torsional degrees of freedom; whereas practically structures will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. The TVAs for seismic response control of the irregular structures using the multimode control approach may require separate complex analysis under other
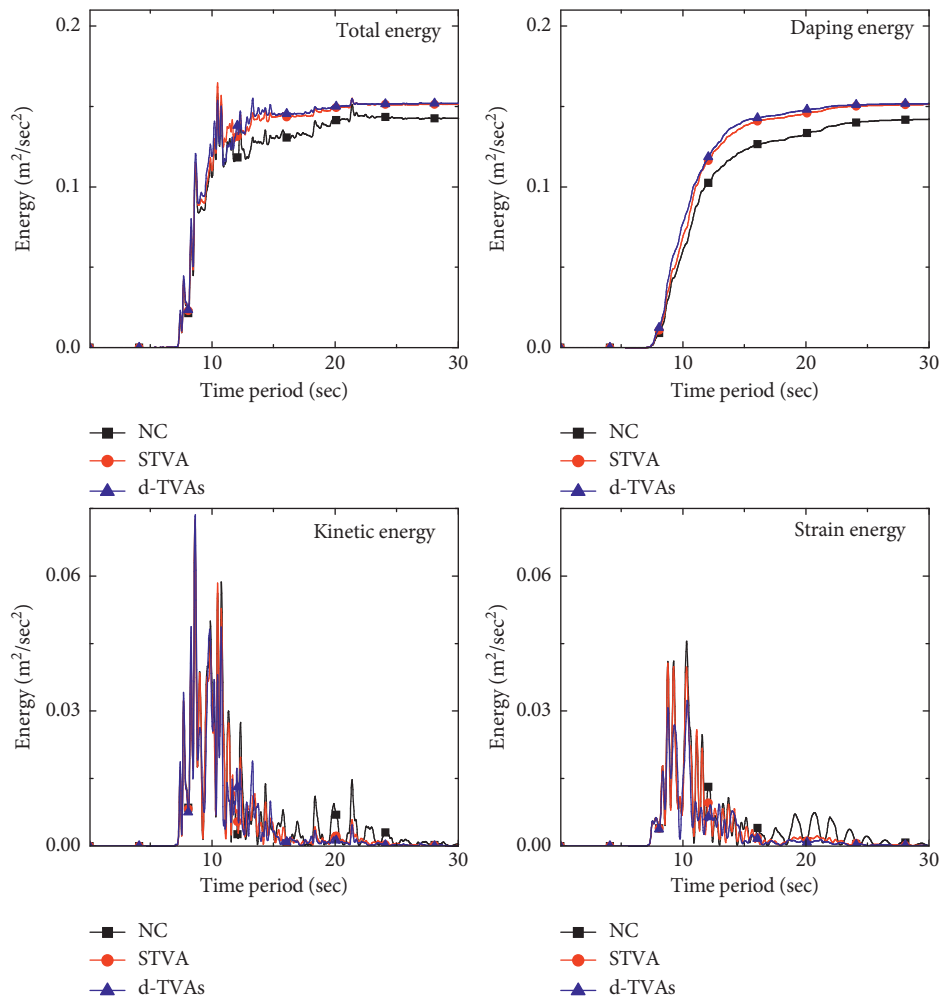
FIGURE 6: Different energies for the NC, STVA, and d-TVAs subject to Kobe, 1995 ground excitation.

parametrics involved in the system, which can be investigated in the future studies.

## 4. Conclusions

Seismic response control of buildings installed with a single tuned vibration absorber (STVA) and distributed tuned vibration absorbers (d-TVAs) is investigated. An assessment is made on the response of the buildings installed with the STVA and TVAs distributed along the height of the buildings. From the trends of the results of the present study, the following conclusions are drawn:

(1) The STVA and d-TVAs are effective in displacement response control of flexible structures as compared to rigid structures. Also, better performance of d-TVAs as compared to STVA is evident based on results presented

(2) The installation of d-TVAs is practically easier as compared to STVA because of its reduced size

(3) The d-TVAs are more effective in controlling the pseudospectral acceleration response of the structures.

(4) The increase in the total energy and damping energy is significantly more in the d-TVAs than the STVA and NC.

(5) The decrease in the kinetic energy and strain energy is significantly less in the d-TVAs than the STVA and NC.

## Conflicts of Interest

The author declares that there are no conflicts of interest.

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