Modernising Mathematics Teaching in the 1960s – Controversies in Nordic Cooperation

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Abstract

Four autonomous countries, Denmark, Finland, Norway, and Sweden, established Nordic cooperation on modernising mathematics teaching in 1960, supported by the Nordic Council and OEEC, later OECD. Working teams wrote directives to teams writing experimental texts to be tested. The work continued until 1967. Soon controversies crept in. There were language problems, there were different opinions about modern mathematics concepts and symbols at compulsory level, and there were questions about authorship. The Swedes and the Danes turned out to be more active than the Norwegians and the Finns. The cooperation created useful discussion on mathematics curriculum, not the least for preparing legislation on nine-year compulsory school, stressing equal access to education for all, which was under development in all four countries, as well as in Iceland, the fifth Nordic country.

Keywords: Agnete Bundgaard, Bent Christiansen, Denmark, Finland, Iceland, Modern Mathematics, New Math, NKMM, Nordic Committee on Modernising Mathematics Teaching, Nordic Council, Norway, OEEC, Royaumont, Sweden.

Introduction

In November 1959, a seminar on new thinking in school mathematics, initiated by the OEEC, Organization for European Economic Co-operation, was held at Royaumont, France. Mathematicians and mathematics teachers from Denmark, Norway and Sweden attended the seminar. They agreed upon organising Nordic cooperation on a reform of mathematics teaching. Finland was invited to join.

The goal of this study is to analyse this cooperation of four independents states with different legislations and school systems, but similarities in many respects, languages of the same North-Germanic origin, and a new platform for cooperation, the Nordic Council, founded in the 1950s. The school systems in all the four countries were undergoing reforms, extending seven years' compulsory school to nine years.

Gradually, each nation took its own direction. The Danes and the Swedes were the most active in the cooperation; others were more hesitating. After ending the cooperation, only little material was published in common to two or more countries. However, some permanent changes in attitudes towards school mathematics were results of this new cooperative thinking.

The study focuses on the cooperation of the Nordic countries and its controverses at compulsory level. The research question is: Could the cooperation of four Nordic independent states create a common school mathematics policy, and which obstacles did it meet? The research method is analysing archived documents, preserved at the Swedish National Archives (SE/RA/2717), official

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reports, and scholars' accounts, focussing on interactions between the actors in the cooperation, and the conditions under which they operated: different languages, traditions, and political and educational systems.

Survey of the literature

Many researchers have written about the school mathematics reform movements of the 1950s and 1960s, also in the Nordic countries. The main source on the Royaumont seminar is the report *New Thinking in School Mathematics* (OEEC, 1961). A report was also written on the Nordic cooperation. Its English version is *New School Mathematics in the Nordic Countries* (Nordisk Råd, 1967).

Gunnar Gjone (1983) wrote a comprehensive work in eight parts on international reform efforts and national curriculum reform attempts in Norway. The latter half of its second part is dedicated to the Nordic regional cooperation (Gjone, 1983, Vol. II, pp. 78–98).

Johan Prytz (2018) wrote a short account of the cooperation, together with an overview of curriculum reform activities and textbook publications in Sweden in continuation of the cooperation. In Denmark, Ole Skovsmose (1979), Jens Høyrup (1979) and others wrote significant papers in a collection of essays on the mathematics during the 1960s in Denmark, edited by Peter Bollerslev. Recently, Kristín Bjarnadóttir (2023) wrote a chapter on the Nordic cooperation in the book *Modern mathematics. An international movement?* edited by Dirk de Bock.

The Nordic cooperation

The Nordic Committee for Modernizing Mathematics Teaching, *Nordiska kommittén för modernisering af matematikundervisningen*, NKMM, was set up in November 1960 under the Nordic Council's Culture Commission. Each of the four countries appointed four persons to the committee. Its members were mathematicians, mathematics teachers, and school administrators. The programme for the common Nordic reform was to analyse the situation within each country, to work out proposals to curriculum plans, and to write experimental texts for testing. Support was gained from the OEEC (U 23)[1], later OECD, and the Nordic Council (I 7). Other costs were divided between the four countries in the proportion 1 : 1 : 1 : 2, where Sweden with the largest population bore the greatest share (U 8; U 9).

The committee appointed teams to work out directives for experimental texts. In continuation, writing teams would be appointed, first for grades 7–9, age 13–16, and grades 10–12, age 16–19. For grades 1–6, age 7–13, the present course content was to be analysed with respect to mathematical as well as psychological

[1] References marked (U #) denote numbered outgoing correspondence, while those marked (I #) denote incoming correspondence of the Nordic committee's central office in Stockholm. The documents are preserved at the Swedish national archives (SE/RA/2717).

and pedagogical perspectives. The committee contacted experts for that grade level (U 8).

Initially, Iceland was considered to join the cooperation as a member of the Nordic Council (U 1). At that time, communication with Iceland from abroad was slow and expensive, and Iceland was not invited. However, experimental texts by the NKMM committee for compulsory level were translated into Icelandic and distributed to large proportions of the cohorts born during 1960–1972.

The archived documents about activities of the committee reveal that many obstacles hindered the realisation of producing common material to be tested in all the four countries. There were language problems, members of the teams had different opinions on the direction of the reform, and there were questions about authorship.

Some recommendations from the Royaumont seminar

Mathematics professor Gustave Choquet of Paris addressed the seminar in Royaumont, introducing arithmetic instruction as basis to all subsequent study of mathematics (OEEC, 1961, pp. 63–67). He remarked that it was important to merge arithmetic and algebra as closely as possible in primary and secondary education. This could be done through the study of structures. The set Z of algebraic integers constituted an excellent basis for study in that it may be regarded as taking concrete form in the child's mind very early. Its "discrete" character made it tangible. It could therefore be profitably used, in preference to lines and planes, for introducing and studying important concepts such as one-to-one correspondence, function, conversion, and the relation of equivalence.

In elementary school, the notion of finite cardinal number and ordinal number could be shown with the help of suitable material of the Cuisenaire type.

> Concomitant with this, *the concepts of the subset* of a set, of complementary set, of union and intersection of two or three sets can be shown. […] The concept of order can be studied from simple examples where order is either total or non-total. […] Exercises with finite collections clarify the concept of one-to-one correspondence which, in turn, leads to the concept of the cardinal number. […] Addition and multiplication are introduced by the union of finite disjoint sets and the product of finite sets, respectively. (OEEC, 1961, pp. 64-65)

Choquet discussed the futility to teach algorithms such as taking the square root which should be dropped completely.

> It will be far more profitable and pleasant for the pupil to do a few calculations in the binary, octad, or duo-decimal systems, than to practice endlessly in the system of decimal numeration. This work should be introduced very early. There can be no doubt that the basic concept of numeration can only be thoroughly understood by the pupil if he has studied several different systems. (p. 67)

Since the turn of the century, a discrepancy in the Euclidean geometric axiom system had been recognized. Among attempts to mend it, Choquet proposed a complete axiomatic system, provided with six basic axioms (Choquet, 1969). Professor Jean Dieudonné held a revolutionary lecture on geometry at Royaumont, proposing to discard the Euclidean approach and replace it with linear algebra for upper secondary level (OEEC, 1961, pp. 31–46). Another approach, proposed at Royaumont by Dr. Otto Botsch, was motion geometry (pp. 76–83). The report from Royaumont does not recount discussion on geometry at lower school levels, except that study of geometry should be initiated on an intuitive experimental basis in the very early primary years (p. 23), while language and notation universally in use should, according to Dieudonné, be introduced as soon as possible (p. 41).

Texts for primary level grades 1–6 – preparations

Language problems

An initial assumption was that the working language in the NKMM committee and the teams, producing directives for writers of experimental texts, would use own mother tongue as working language. Norwegian, Swedish, and Danish all belong to North-Germanic languages. Each of them has several dialects so at least persons working in administration and education were used to listen to and understand a variety of Nordic dialects and related languages. The Finns were later invited to the cooperation. Finnish is, however, a Uralic language, unrelated to the North-Germanic languages. About 5–10% of the population, living on Finland's west-coast had Swedish as a first language, so Swedish is an official language in Finland and taught at compulsory schools. However, the Finn Veikko Heinonen in the working team for grades 1–6 had problems in communicating with others in the group. The Dane Agnete Bundgaard, a member of NKMM, complained about this to the headquarters of NKMM in Stockholm (I 39). Other members were a Norwegian, a Swede, and a Danish mathematics expert. Heinonen soon disappeared from the team.

Differences in opinions

There were also differences in opinion in the working team for grades 1–6. The directive (U 213) was probably a compromise between the opposite opinions of the Dane Agnete Bundgaard and the Norwegian Torgeir Bue. The concepts "equal to", "less than" and "greater than" were to be introduced early together with their symbols, to build up to concept "ordering". The operations of addition as a union of two disjoint sets, and multiplication were to be introduced with reference to the commutative and associative axioms of the number field in grades 1 and 2, figuratively and verbally, as the pupils were not expected to be able to read at the age of seven.

In geometry, pupils were to be introduced to the simplest geometrical objects. By playing geometrical games, computational competence could be trained. The perimeter and area of the circle would be introduced by not too great accuracy, for example by the value of π as 3.

Two appendices were attached to the directive, one written by Torgeir Bue, where he stressed a variety in approach, and another one by Agnete Bundgaard who seems to have had stronger influence on the final version of the directive (see Table 1). They do not seem to have disagreed on geometry as neither of them mentioned that topic.

Table 1. Appendices to directives to experimental texts for grades 1–6.

From Torgeir Bue, headmaster of teacher training school	From Agnete Bundgaard, deputy school master at a primary school.

12 pages where sets were not mentioned.
Mathematics was to be put in context.
Emphasis on differentiating the content, related to children's experiences, e.g.
• using the same operations in different situations,
• differentiating the difficulty level,
• differentiating the forms of exercises, e.g. by
  • self-controlling exercises,
  • fill-in exercises,
  • situation-exercises: pupils must find facts themselves,
  • pupils make up texts themselves with given facts,
  • exercises with texts without numbers,
  • variations of picture-exercises,
  • detective-exercises with errors for children to find,
  • interpreting graphs, etc.
	11 pages + 3 pages on instruction.
	Emphasis was to be laid on building up
	• the set concept and
	• the number concept,
	• using related concepts, such as
	  • pairs,
	  • disjoint sets,
	  • subsets,
	  • sets of sets,
	  • mapping into and onto a set,
	  • one-to-one correspondence mapping.
	This was to be taught by using pins, balls, apples, etc.

The mathematics expert, Erik Kristensen, checking the directive, expressed a pity for the children, receiving such thorough teaching as presented for introducing the number concept. He commented also on new algorithms for subtraction and division, that it reflected a true idealism to have children understand long division in detail (I 285).

Agnete Bundgaard wrote (I 286) that she was not in agreement with Bue about including addition- and multiplication-tables and the metric system; children were to make their own tables. Torgeir Bue disappeared as a member of the working team. He resigned from the NKMM in September 1963 (Nordisk Råd, 1967).

The NKMM decided in its meeting in October 1961 that there were to be two writing teams, for grades 1–3 and grades 4–6 respectively (U 253). Agnete Bundgaard was disappointed (I 286). She wanted to continue in a united group, it was important with respect to continuity. Or maybe, should she just be quiet, and things would be organized in Sweden? she asked.

Agnete Bundgaard wrote experimental texts for grades 1 and 2 in cooperation with the Finn Eeva Kyttä. She wrote alone a text for grade 3, containing place-value systems in base five, four, and twelve, prime numbers, permutation of three digits, and the relation of the transverse sum to the nine times table (Bundgaard, 1969–71). The texts were tested in Denmark and Finland. In Sweden, different experimental texts for grades 1–3 were written and tested there. No experiments were made in grades 1–3 in Norway. For grades 4–6, the series produced by the US School Mathematics Study Group, SMSG, was translated into Swedish for experimental teaching (U 1052), and into Norwegian for a small group (Gjone, 1983).

Agnete Bundgaard completed texts for grades 3–6 alone to be tested in Denmark and at least some of them in Greenland. In grades four and five, she reviewed various place-value systems and introduced geometry with points, lines, and planes in a set-theoretical framework (Bundgaard, 1969–71). Her texts were published by the Gyldendal publishing house from 1966 and were not counted among the experimental projects by the NKMM. Still, Bundgaard continued to report her work to the secretary of NKMM, Matts Håstad (I 1633, 1734, 1780).

Bundgaard's texts for grades 1–6 were translated into Icelandic, first for experimental teaching, and later as an official textbook series for large proportions of cohorts born in 1960–1966. The enthusiasm quickly waned, however. The criticism mainly concerned the implementation of new, detailed algorithms, e.g. for division as mentioned by Kristensen (Kristjánsdóttir, 1996).

Texts for grades 7–9 – Preparations

New school system – New symbolic language

In all the four countries, there was an ongoing preparation for a uniform nine-year compulsory basic school for age 7–15 in grades 1–9, aiming at equal access to education for all. Previously, schools in the Nordic countries were differentiated in the middle grades, grades 5/7–9. The main criteria for differentiation were the pupils' plans and their conceived capability to enter gymnasium studies. The experiments with modern mathematics in grades 7–9 had impact on curriculum development in the new uniform basic school in Norway and Sweden (Gjone, 1983, Vol III; Prytz, 2018).

For this level, the directive emphasized that the topics were to be presented in terms of the set concept, its derived concepts and their symbols: e.g. the union, $\cup$, the intersection of two sets, $\cap$, the subset, $\subseteq$, the empty set, $\varnothing$, the symbol for element, $\in$, a complementary set, and some symbols from logic. The experimental texts were to be on two topics: algebra and geometry (U 81).

Differences in opinions

Algebra. The algebra text, ready in 1962, was written by Bent Christiansen (D), Matts Håstad (S) and Ragnar Solvang (N), while Bent Christiansen had the prime responsibility for volume one. Experiences were different in Denmark and Sweden. A shorter version, written in a simpler language would be used in continued experiments in Sweden. A new Danish algebra version, written by Bent Christiansen and his collaborator Allan Christiansen, became the basis for experiments in Denmark. Experience had shown that the material was suitable for grade 6 where experiments were performed in 65 classes (I 1607).

At one point, Bent Christiansen expressed dissatisfaction with the central office of the NKMM, located in Stockholm, Sweden (I 1607). He regretted that a request from UNESCO to publish in its *New Trends* an abstract of a report of the experimental activities in grade 7 in Denmark was turned down. He reminded that he had mandate over his own texts, created under the Mathematics Institute at The Royal Danish School of Educational Studies where he himself was a leader. His name had also been omitted from a Swedish version of the text created in common. The friction was resolved. The Swedish algebra version was completed in 1966 (U 1536). The authors listed were all the members of the initial writing team, while the Danish algebra version was attributed to Allan Christiansen and Bent Christiansen only.

Geometry. Two geometry series of different difficulty levels were made, both written in Swedish. The more extensive one was written by two Swedes, Bertil Nyman and John Amundsson, and the Finn Inkeri Simola. It was translated into Finnish and Norwegian and tested in all three countries. Geometrical objects were introduced by set-theoretical concepts. A complete axiomatic system was provided with six basic axioms, based on Gustave Choquet's (1964) axiom system.

The less extensive geometry, written by the Swedes G. Bergendal, O. Hemer, and N. Sander, was successfully used for experiments in Sweden and Finland. The grade 7 volume was translated into Icelandic. No documents have been found regarding controversies concerning experiments in writing and testing geometry in grades 7–9. Neither of the texts were written by a Dane nor were they tested in Denmark. Looking at the experiments for grades 7–9, the Danes obviously went their own way, both in algebra and geometry, which points to some disagreement.

Texts for gymnasium level – Grades 10–12

The gymnasium level was originally the main concern of the NKMM. In October 1966, texts were available, on (1) algebra, (2) geometry, (3) functions and calculus, (4) statistics and probability, and (5) differential equations. These were written by six Swedes, including Matts Håstad, two Danes, and one Norwegian. The texts became the basis for some commercially published textbooks, such as a well-known Swedish series by Gunnar Bergendal, Matts Håstad and Lennart Råde (1966–1968), and a Danish series by Erik Kristensen and Ole Rindung (1962–

1964). The Finns did not participate in writing experimental texts for the gymnasium level, which reveals no common policy at that level.

Did this cooperation have any permanent impact?

The plan of the pan-Nordic cooperation was to evaluate the results of the use of experimental texts in a limited number of classes. According to the final report, 90 classes took part in experimental instruction at grades 1–6, 450 classes at grades 7–9, and 770 classes at grades 10–12. In most classes more than one experimental text was used (Nordisk Råd, 1967, pp. 45–46). In the 1960s there were many children in the families. Assuming 30% of the 20 million inhabitants, 6 million, to be at the age level 7–16, and 25 pupils in each class makes $540 \cdot 25 = 13{,}500$ pupils affected, which is less than 1%. The direct effect of the experiments in grades 10–12 in 770 classes may have been greater as only relatively small proportion of the cohort attended gymnasium at that time.

Another plan was to write proposals on content of mathematics syllabuses (Nordisk Råd, 1967, pp. 50–88). Those proposals probably had impact on the writing of national curricula, as mentioned earlier (Gjone, 1983; Prytz, 2018).

The greatest impact and controversies appeared when the experiments were materialised in commercially published textbooks according to the revised national curricula. We shall look briefly at the cases of Sweden and Demark, the most active participants in the cooperation. The information and impact filtered to Iceland from there, and we shall have a more detailed look at the situation it met.

Sweden

According to Johan Prytz (2018), modern mathematics was introduced on a broad scale in grades 1–9 in Sweden when the national curriculum of 1969, written under the influence of the cooperation, took effect. The overall difference from previous curricula was that topics were moved to earlier grades, such as equations, statistics, functions, the coordinate system, etc. New topics included place-value number notation with bases other than ten in grade 3, vectors in grade 7, and trigonometry in grade 9, in addition to set theory. Several textbook series for the compulsory school were published from 1969, including the modern mathematics ideas, while traditional series were in a clear minority. Among the authors were Matts Håstad, the executive of the NKMM committee. The impact dwindled before the mid-seventies.

Denmark

Ole Skovsmose (2016) listed six textbook series for compulsory level, written under the impact of modern mathematics. His assessments of the mathematics in the 1960s in Denmark has a wider reference in the Nordic countries:

> The 1960s mathematics is a clear and radical breakthrough of the mathematics teaching that over a long period had stagnated around a limited set of methods and problems […]. The mathematics reform has slit the teaching out of a dead and archaic tradition, radically changed its content and enlarged its sphere. (P. 152, author's translation)

Iceland

In 1966, Iceland was still in considerable contact with Denmark, its former ruler, 2100 km away, but news about educational currents were ad hoc and slow. News about the activities of the NKMM cooperation came through Guðmundur Arnlaugsson (1913–96), a former colleague of Svend Bundgaard (1912–84), mathematics professor at the University of Aarhus, who was a guest speaker at the Royaumont seminar.

In the 1960s, the Icelandic authorities were still struggling to build schools around the country, a huge area in proportion to population, in continuation of school legislation of 1946, prescribing 8-year compulsory school, for age 7–15. The content had been devoted minimal attention. A national curriculum document was published in 1960, fourteen years later. The mathematics curriculum added the rule of three, equations, interests, and area- and volume-computations to the earlier 7-year programme of the four arithmetic operations on whole numbers, decimal and common fractions. The small size of the population, about 4000 children in each cohort, caused that there was only choice of one textbook series, printed and distributed by a state publishing house for grades 1–6, while textbooks from grade 7 were financed by the families.

When the Bundgaard-series was translated for an experiment in seven classes in grade 1, and the results were presented to school leaders and teachers in spring 1967, it struck as a bolt of lightning. Next year, 86 new classes were added, and more the following year. It was no longer an experiment, but a regular textbook series for more than half the cohort for several years. The short version of geometry and the Swedish version of algebra were translated for grades 7–9. The original plan was to translate and implement Bent Christiansen and Allan Christiansen's *Algebra*. Bent Christiansen came up to Iceland for discussion in February 1969, but finally in 1970, the authorities backed out with lost confidence in the Danish version of modernising mathematics for compulsory school level (Borgarskjalasafn Reykjavíkur, askja 328).

Arithmetic. The content of the Bundgaard series was highly theoretical. In addition to the content of for the first three grades, previously described, a collection of set theoretical concepts and symbols was introduced in the fourth grade.

Geometry. The national curriculum, published in 1960 prescribed for grade 6: Area and perimeter of a square and rectangle, area of a right-angled triangle. Bundgaard's (1969–1971) textbooks took a qualitative rather than a quantitative approach to geometry with new concepts and topics, introduced in grades 4 and 5 in a set-theoretical framework: lines as sets of points; number of lines through three or more non-colinear points; partition of the plane into half-planes; angle; vertex; quadrangle; rectangle: polygon; pentagon and its diagonals; hint at Euler Circuit; volume of liquids but not area. This different approach may have troubled teachers who were more used to direct pupils into calculations than into discussions, while no sources support this statement.

Who was Agnete Bundgaard?

Agnete Bundgaard worked at a primary school in Frederiksberg, Copenhagen. Only little biographical information about her is available. She was born in 1909, and completed her teacher training in 1930. From 1934 until at least 1967 she served as teacher at primary level, head teacher from 1952, and deputy school leader from 1957 (Boisen Schmidt, 1957). No source has been found that she attended higher education. The highest education of girls of her generation was often teaching or nursing (Kjartansson, 2022). Both could mean a choice between a family and a career. Agnete was single in her life.

Agnete Bundgaard became engaged in the development of mathematics teaching in Denmark, as a member of Denmark's Mathematics Teaching Commission, a platform for representatives from all school levels, from primary schools to universities. Denmark had new school legislative acts by which compulsory education became stepwise a nine-year homogeneous school: The 1958 Act, followed by curriculum guidelines, popularly called *Blå betænkning*, the *Blue Memorandum*; and later the 1975 Act. Agnete Bundgaard represented the commission in preparing the *Blue Memorandum* (Høyrup, 1976), and she became a Danish representative in the NKMM.

Agnete's brother was Svend Bundgaard, the guest speaker at the Royaumont Seminar where his topic was teacher education. His talk is not included in the seminar report (OEEC, 1961). There have been assumptions that Agnete and her brother cooperated. Their relatives do not think it likely (Jørgen Bundgaard, personal communication, June 17, 2022). Agnete had, however, ample opportunities to learn about discussions at the seminar through her work at the mathematics teaching commission, such as studying the seminar report (OEEC, 1961). Some expressions in the directive for the primary level resemble the recommendations of the seminar, such as the sentence "Addition and multiplication are introduced by the union of finite disjoint sets" in Gustave Choquet's presentation at Royaumont. It is echoed in Agnete's appendix, as is Choquet's recommendation to do a few calculations in the binary, octad, or duodecimal systems, so she may have studied the seminar report thoroughly.

Agnete Bundgaard's influence in Iceland

Agnete Bundgaard had an enormous influence in Iceland. She came repeatedly to Iceland, held courses, and gave interviews to newspapers. She stated in an interview that the main emphasis was on promoting pupils' understanding of the nature of the tasks and on training them to use their own judgement in solving tasks and problems. Modern mathematics had been introduced in many countries and influenced the way mathematics was taught. Other nations' experiences suggested that its concepts and symbols would be of great use in training pupils in clarity of thinking and communicating. It could have very bad consequences for the child if its parents were trying to help, being more willing than able to guide the child. That would only lead to confusion. Therefore, it had been decided not to assign homework to the children and not even allow them to bring their books home. Later, the children would reach enough understanding of the project to be able to explain it to their parents (*Morgunblaðið*, September 13, 1970, pp. 23–24). Meanwhile, the authorities were busy engaging authors to write a new home-made textbook series to meet waves of anger by upset parents and confused teachers (Ragnhildur Bjarnadóttir, personal communication, September 16, 2003).

Earlier arithmetic textbooks consisted mainly of a page after page with long columns of exercises. To relieve the layout, small drawings were inserted, with birds, boys playing with balls or skiing etc., often without relation to the content. This habit was continued in the translations of Bundgaard's textbooks. In a letter attached to the handbook for the third-grade textbook, Agnete Bundgaard expressed her discontent that the Icelandic edition of the textbook was illustrated with drawings, irrelevant to the text, saying:

> Dear Icelandic colleagues. It is you who shall try to show the children that the subject is fun in itself, and for that aim one can surely only use items that are relevant for the subject. (Bundgaard, 1969, a letter attached to a handbook. Author's translation)

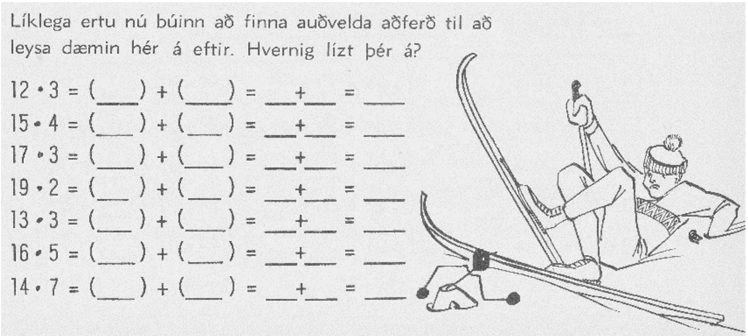

Fig. 1. Problems on dissolving multiplication into two terms according to the distributive law are decorated by drawing a boy falling while skiing. (Bundgaard & Kyttä, 1968, p. 82)

In the Icelandic arithmetic textbooks that followed, all illustrations were related to the texts and supporting them.

As the account of her interview and the letter above to Icelandic teachers show, Bundgaard's work was systematic and purposeful, even prophetic. Some would say that she had a dominating character. One could say that she was hardly problem-solving orientated or cooperative, when she denied parents to see her textbooks and assist their children with homework, nor did she compromise in her teamwork with Heinonen and Bue. Agnete Bundgaard died in 1995.

Concluding remarks

The Nordic experiments on modernising mathematics teaching created a long-needed discussion about curriculum, which had ossified into a collection of particular routines and topics. At some points of the processes in the four countries, they influenced the process of creating new curricula in a radical way, but sooner or later the most radical items were withdrawn, and sets and set-theoretical notation disappeared gradually.

One should not forget that what was called "modern mathematics" in Europe and "New Math" in the United States was a large international experiment that made impact in many countries. Jeremy Kilpatrick (2012) wrote:

> Before the new math era, no one thought of school mathematics as something to be reformed or updated; it simply was what it was. The new math reformers knew almost nothing about the school mathematics curriculum in other countries or, in some cases, in their own country. By the time the new math era ended, in contrast, everyone concerned with school mathematics had a much better sense of what was going on around the world. (p. 569)

Kilpatrick spoke here in general terms. Reform activities in school mathematics at different levels had taken place earlier on smaller scales, but the general international awareness of a need for reform and ensuing movement of the modern mathematics/New Math was unique. Ole Skovsmose (1979) spoke in similar vein about this period in Denmark when he wrote that the reform had slit the teaching out of a dead and archaic tradition.

The concluding paragraph in Kilpatrick's article contributes to answering the questions if the four independent nations could create a common school mathematics policy, and which were the obstacles:

> From a distance, school mathematics looks much the same everywhere. Countries include many of the same topics in their syllabuses and expect pupils to solve many of the same sorts of problems. ... Up close, however, each country has a unique school mathematics—a product of its history, culture, and traditions, and conforming to its social, political, and educational systems. Instructional materials and practices in school mathematics cannot be transported across borders as if they were a common currency. The new math era taught us the paradox of curriculum change: The more school mathematics is internationalized, the more clearly its national character is revealed. (Kilpatrick, 2012, pp. 569–570)

The Nordic cooperation on modernising mathematics teaching met obstacles. Despite their related languages and close neighbourhood, Nordic nations had developed each their own language, culture, tradition, and history, and they adhered each to its own established system. It was only natural that controversies between personalities crept in. The Nordic Committee for Modernising Mathematics Teaching, NKMM, which only had an advisory status, was not unanimous on any of its projects at three school levels, so it could not provide advice on a common school mathematics policy. But, according to Skovsmose, the 1960s mathematics served to clarify former situation and renew traditions and content, and thus permanently change attitudes and thinking on school mathematics.

Acknowledgment. A more detailed account of the Nordic cooperation is available in Bjarnadóttir (2023).

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U 81. Directives to writers of experimental texts for grades 7–9 from the NKMM.

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U 253. Minutes from a meeting of the NKMM committee in Helsingfors on October 12–13, 1961, dated October 14.

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A memorandum by Jónas B. Jónsson on Bent Christiansen's visit to Iceland on Feb. 10–15, 1969.

A letter from Jónas B. Jónsson to Bent Christiansen dated Feb. 25, 1970.