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INTELLECTUAL CHALLENGE IN MATHEMATICS TEACHING IN ICELANDIC LOWER SECONDARY SCHOOLS

The aim of this study was to assess the level of intellectual challenge in offered to students in lower secondary mathematics in Iceland as it appears in mathematical tasks and in the enactment of tasks in the classroom. Ten mathematics teachers participated in the study. A total of 34 mathematics lessons in 8th grade were recorded and analysed using an observation system (PLATO). Mathematical tasks in the lessons were analysed using the Task Analysis Guide. The results showed that a majority of mathematical tasks involved applying specific procedures to reach a solution. The teachers faced difficulties in upholding the level of challenge that the tasks offered. When students worked individually according to a list of numbered exercises it was common that teachers reduced the challenge that the tasks offered by asking only closed questions, computing for students or telling them the answer. The results also showed examples of solution of tasks where critical and creative thinking was required by the students and teachers encouraged collaboration and discussion. In those cases where intellectual challenge was at a high level, students worked together on the same tasks.

Keywords**:** Mathematics education, intellectual challenge, instructional quality, video studies, analytical thinking

INTRODUCTION

Student learning is largely dependent on the extent to which teachers offer a learning environment rich with opportunities for students to confront tasks which match their cognitive level and present them with an adequate challenge. By supplying tasks where students need to use analytical thinking and bring their thoughts into words can they be activated to develop new and improved competence (Vygotsky, 1978). In the Icelandic national curriculum, it is assumed that students develop competency in explaining their thinking and using critical and analytical thinking in mathematics (Ministry of Education, Science and Culture, 2013). These guidelines are mainly found in two categories of key competencies of the curriculum: communication and expression, and critical and creative thinking. Teacher views toward tasks where analytical thinking is required are generally positive although it has been pointed out that those types of tasks are generally uncommon in Icelandic textbooks (Sigurjónsson & Kristinsdóttir, 2018). Until now, few studies on teaching practices in mathematics have focused on how teachers implement such tasks in lessons with the aim of students developing competency in critical and creative thinking in mathematics.

Alongside societal changes in developed nations in recent years, from industry economies to knowledge economies, research has increasingly been focused on the quality of educational systems and quality of instruction (Cochran-Smith & Villegas, 2015). Instructional quality is tightly connected to student achievement, which has resulted in researchers focusing on how to measure instructional quality reliably (Hanushek, Piopiunik & Wiederhold, 2014). To answer questions on instructional quality it is necessary to carefully define what constitutes quality (Ding & Sherman, 2006). There is general agreement that instructional quality is both a complicated and multidimensional phenomenon and therefore research on it requires a variety of methods (Croninger, Valli & Chambliss, 2012). It can be argued that “quality” is not a property that a teacher “possesses” or not. Quality is rather something that a teacher shows with her actions in the classroom. A teacher can show high quality teaching in one lesson, lower quality in another, and differently across different dimensions within each lesson. It is unreasonable to expect a teacher to maximize all quality dimensions simultaneously as his actions are dependent on the aim of each lesson. Different instructional methods are chosen with regard to which student competency is aimed at developing in each lesson.

The potential for cognitive activation is one dimension of teaching quality. In the observation framework of this study, PLATO (Protocol for Language Arts Teaching Observation), teaching quality is viewed as consisting of four domains: Instructional scaffolding, disciplinary demand, representation and use of content, and classroom environment (Bell, Dobbelaer, Klette & Visscher, 2019). In this study the focus is specifically on cognitive activation (i.e., disciplinary demand), and the element called intellectual challenge. Intellectual challenge involves both the teacher’s choice of demanding tasks and how they are implemented to stimulate students’ cognitive activity. Where intellectual challenge is at a high level, students need to use analytical thinking, explain their thinking and reason for their results. The study aimed to assess the quality of intellectual challenge by both systematic observation of teaching from video-recorded lessons and by analysing the tasks that students engaged with.

The aim of this study was to develop an understanding of the challenge mathematics teachers offer students at the lower secondary level as it appears during lessons and in the relevant mathematical tasks. The guiding research question was: How can the potential for cognitive activation in lower secondary mathematics in Iceland be described?

QUALITY AND COGNITIVE ACTIVATION IN MATHEMATICS INSTRUCTION

Intellectual challenge

Cognitive activation involves both what kind of tasks a teacher chooses for students to engage with and how they are implemented so that they challenge students to think critically and creatively. To create opportunities for students to participate in tasks that involve intellectual challenge, where students need to apply analytic thinking and draw inferences to develop their thinking and understanding, is an important domain of teaching quality (Praetorius & Charalambous, 2018). The challenge presented to students during mathematics lessons greatly dictates what kind of thinking they need to apply to reach a meaningful result. In PLATO, the intellectual challenge element is at a high level when emphasis is on reasoning, inferential thinking and analytical thinking (Grossman, 2019). It is important to underpin students’ conceptual understanding by supporting them in explaining their thinking and reason for their solutions. The teacher holds responsibility in creating opportunities for students to develop such skills within their zone of proximal development (Vygotsky, 1978).

The challenge that a teacher offers students in lessons is to a degree dependent on the nature of the tasks that students are to engage in. The effect of differently demanding tasks on learning has long been a topic of research in the field of mathematics education. Despite the general agreement about the importance of including such tasks, statements of a direct connection to student achievement have sometimes been overstated (Otten, Webel & de Araujo, 2017; Stein, Smith, Henningsen & Silver, 2009). Tasks with a high cognitive demand can be difficult to implement, both for students and teachers. To effectively include such tasks in instruction it is considered important for teachers to have strong professional knowledge (Wilhelm, 2014). To maintain demands of reasoning and the challenge that the tasks offer it has been useful for teachers to have support, for instance through professional development programs (Tekkumru-Kisa, Stein & Doyle, 2020). A teacher's implementation of tasks in the classroom is crucial in determining what students learn from them.

A problem that arises with analysis of video-recorded lessons according to quality criteria is when students work independently in textbooks, a common teaching format in Iceland (Jónsdóttir et al., 2014; Þórðardóttir & Hermannsson, 2012). It is possible that students engage in analytical thinking and draw inferences in constructing their solutions while working by themselves in textbooks, but it is difficult to assess from video alone. Thus, it is also important to study the content of the tasks that students engage in, while also considering how teachers implement them. In implementing tasks with limited cognitive demand, a teacher can increase the challenge by asking students to reason and explain their solutions. In the same way, a teacher can reduce the challenge by telling students exactly how to solve them or by revealing the answer.

The use of tasks as a reference point in research on student thinking in mathematics can be traced back to the research by Walter Doyle in the early 1980s. He argued that student work was largely determined by what kinds of tasks they were set to solve and thereby how they thought about and understood mathematics (Doyle, 1983). In the past 40 years, many studies have built on Doyle's ideas. Tekkumru-Kisa, Stein and Doyle argue in a recent article that there are still opportunities for further study in this field. As an example, they mention studies on how discourse between students and teachers in lessons can maintain the challenge of tasks where the solution requires analytical thinking (Russo & Hopkins, 2017; Tekkumru-Kisa, Stein & Doyle, 2020).

In the Task Analysis Guide (TAG), mathematical tasks are divided into four categories. Categories 1 and 2 comprise tasks with low cognitive demand and categories 3 and 4 comprise tasks with high cognitive demand (Stein & Smith, 1998). High demanding tasks are considered to support analytical thinking as students need to justify their answers and reason for their solution path. In a study on Icelandic textbooks in upper secondary schools, over 80% of the tasks belonged to categories with low cognitive demand (Sigurjónsson, 2014). When such tasks are used as a foundation for lessons, the teacher needs to increase the challenge if the aim is to reinforce student conceptual understanding.

Many factors can influence participation and educational achievement and hypotheses on causation between demanding tasks and student achievement are still problematic (Otten et al., 2017). Despite those difficulties, studies have indicated a strong correlation between student cognitive activity and achievement (Baumert et al, 2010). Teachers still have a tendency to reduce challenge, but with increased professional knowledge they are more likely to maintain or even increase challenge (Wilhelm, 2014). In this respect, a point has been made for the usefulness of teachers receiving specific support in developing their skills in maintaining the challenge that supports student's analytical thinking (Norton & Kastberg, 2012).

In the Mathematical Tasks Framework, it is described how mathematical tasks develop in the classroom (Stein & Smith, 1998). Student learning is dependent on what the tasks are, how they are implemented by the teacher, and finally how the tasks are solved by the students (see figure 1). The TAG is based on the Mathematical Tasks Framework. The aim of the TAG framework is to analyse tasks as they appear in curricular or instructional materials, but not how they are implemented during lessons. However, the PLATO framework assesses more specifically how tasks are implemented by teachers and partly how students engage with them, even if it is dependent on what is visible in the video data. By using both TAG and PLATO, a more holistic approach is taken at analysing intellectual challenge, both from the nature of the tasks themselves and their implementation during lessons.

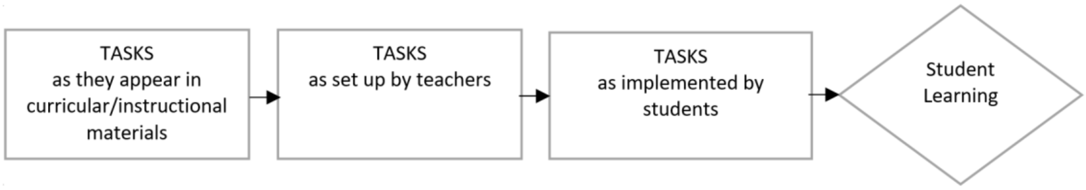

Figure 1. The Mathematical Tasks Framework (Stein & Smith, 1998).

Research on mathematics teaching in Iceland

Research on mathematics teaching in Iceland indicates that an emphasis is put on fluency in carrying out procedures and that students work independently in textbooks. In a report on mathematics teaching at the lower secondary level from 2012 the majority of observed lessons involved student independent seat work in textbooks with assistance from the teacher (Þórðardóttir & Hermannsson, 2012). Around 83% of teachers stated that they used that instructional format rather often or very often. Around a third of observed lessons included some direct instruction from the teacher to the whole class in a discourse with students. Gunnarsdóttir and Pálsdóttir reached similar results, where they concluded that in most mathematics lessons in Iceland students worked independently in books where the teacher walked between and assisted (2015). However, they found some examples of group work and discussions, but it was considered rare. In a large-scale study of teaching methods in Icelandic compulsory schools the results showed that student collaboration or group work is not widely used (Sigurgeirsson et al., 2014). A report on mathematics teaching in upper secondary schools described similar teaching methods. In addition, it was pointed out that tasks where students need to provide their own reasoning to develop conceptual understanding were rare and that tasks involving proof were almost entirely absent (Jónsdóttir et al., 2014). These descriptions of mathematics teaching in Iceland are somewhat in line with the descriptions of Pehkonen, Hemmi, Krzywacki and Laine on mathematics teaching in Sweden, where students were said to work in textbooks at their own pace without any direct instruction (2018).

In the only pre-existing video study where mathematics teaching in Iceland is compared to teaching in other countries, Savola (2012) described Finnish mathematics teachers as "rather traditional", but Icelandic mathematics teachers as "progressive-minded" and mainly using learner-based strategies. A major contrast was found in the independent student learning in Iceland compared to the emphasis on whole-class interaction in Finland. In his conclusion, he found many Icelandic teachers emphasized individualization and learner control at the cost of content-related discourse and reasoning. In the PISA Mathematics Assessment, Finland has in recent years scored highest among the Nordic countries, while Iceland has scored the lowest. It is possible that a part of the explanation can be connected to the quality of intellectual challenge in lessons.

METHOD

The study is a part of a Nordic research centre, Quality in Nordic Teaching (QUINT), which is leading in comparative research on teaching quality in the Nordic countries. One of the aims of QUINT is to assess teaching quality in three subject areas based on video recordings of lessons as well as other data from both students and teachers. One of the benefits of video data is that lesson interactions, communication and environment can be watched repeatedly to analyse more accurately than with direct observations. This article only builds on data from mathematics lessons in Iceland. All protocols regarding ethics in data collection and analysis were strictly followed. All participants, that is both teachers, students and their guardians, gave their informed consent in writing on a consent form. The study was conducted in accordance with the Data Protection Act no. 90/2018 by notification to The Data Protection Authority (Persónuvernd) as well as being reviewed by the Science Ethics Committee of the University of Iceland without comments.

Participants

Ten Icelandic compulsory schools participated in the study. The schools were chosen purposefully with the aim of establishing heterogeneity of the sample in terms of variety in students' social background, as well as school location and type of school. The sample consisted of schools from different parts of the country, both rural and urban, as well as different types of classroom spaces and overall school organization. A total of 11 mathematics teachers participated in the study where in one of the schools two teachers taught together with team teaching. Table 1 shows information about the different schools and teachers participating in the study.

Table 1. School characteristics and location, teacher experience and mathematics specialization.

School	School size*	Location**	Teaching experience (years)	Specialized in mathematics
School A	Large	Large town	16	Yes
School B	Large	Town	16	No
School C	Large	City	11	Yes
School D	Medium	Town	33	No
School E	Large	City	1	No
School F	Medium	Large town	4	Yes
School G	Small	Small town/rural	10	Yes
School H	Small	Small town/rural	14	Yes
School I	Small	Small town/rural	1	No
School J	Large	Large town	28	No

* Large school > 500 students, Medium school = 300– 500 students, Small school < 300 students.
** City > 100,000 inhabitants, Large town = 15,000 – 100,000 inhabitants, Town = 3,000 – 15,000 inhabitants, Small town/rural < 3,000 inhabitants

Data collection

Three to four consecutive 8th grade mathematics lessons were recorded in each participating school, 34 lessons in total. Each lesson was recorded using two video cameras, one located in the back of the classroom and the other in the front, to film the lesson from two different angles. Two microphones were used; one worn by the teacher and the other in the middle of the classroom to record student talk. Additional data was collected in the form of interviews with teachers and a student survey. Contextual data was also collected, such as photographs of tasks and lesson plans. This article only reports on the results of analysis of the video data and mathematical tasks. All tasks that were seen or heard from the video-recorded mathematics lessons were analysed and identified by considering lesson plans. The tasks were analysed as they appeared in instructional materials and the analysis did not consider student implementation or solution paths.

Task analysis

A total of 144 tasks were analysed using the Task Analysis Guide (TAG) as put forth by Stein and Smith (1998). In TAG, tasks are divided into four categories depending on the amount of analytical thinking required to solve them. Tasks in categories 1 and 2 involve little or no analytical thinking but categories 3 and 4 comprise tasks that require students to think analytically (see table 2). Tasks are analysed as they appear in curricular or instructional materials, without considering how they are set up by teachers or implemented by students (Stein & Smith, 1998). In most cases the tasks were analysed by numbered exercises in textbooks but there were also cases where tasks from the teachers were analysed.

Table 2. Four categories of tasks according to the Task Analysis Guide.

Category 1 Memorization	Students reproduce previously learned or memorized facts, rules, or definitions. Time frame may prevent the task from being completed using a procedure. The task is unambiguous and directly states exact reproduction. No connection to mathematical concepts or underlying meaning.
Category 2 Procedures without connection	From prior instruction, experience or placement of the task, the use of a procedure is evidently called for What needs to be done and how is clear, but not why (no analytical thinking). No connection to mathematical concepts or underlying meaning of procedures. Focused on producing correct answers rather than developing understanding. Do not require explanations, or explanations focus on describing procedures.
Category 3 Procedures with connection	Focus students' attention to the use of procedures for developing deeper understanding of mathematical concepts and ideas. Suggest a solution path with broad general procedures with close connections to underlying meaning. Procedures are not followed without thinking. Often represented in multiple ways to develop connections and meaning. Require some cognitive effort. Students need to engage with concepts and ideas that underlie the procedures (some analytical thinking required)
Category 4 Doing mathematics	Require analytical and non-algorithmic thinking. There is no predictable approach or solution pathway explicitly suggested. Students need to explore and understand the nature of mathematical concepts, processes, or relationships. Demand monitoring or regulation of one's own cognitive processes. Relevant knowledge and experiences need to be used appropriately to solve it. Students need to analyse the task and examine constraints that may limit possible solution strategies and solutions. Require considerable cognitive effort. May involve some level of anxiety due to the unpredictable nature of the solution process (Stein et al., 2009, p. 6; Sigurjónsson & Kristinsdóttir, 2018)

Lesson analysis

To ensure an accurate and valid analysis of video data from lessons, a robust and reliable classroom observation system is needed. Such observation systems have two key purposes: to understand and to improve teaching. PLATO, the observation system used in this study, is a well-known system for analysing instructional quality (Bell et al., 2019). The observation system was designed by a team of researchers from Stanford University and is widely used to assess instructional quality. PLATO was originally designed to assess the relation between instructional practices in lower secondary language arts instruction to student achievement. Since its inception it has been adapted to assess instructional quality in other subjects as well as being used as a tool for professional development, supporting teachers in developing effective instruction.

In PLATO, lessons are divided into 15-minute segments for analysis. PLATO consists of twelve elements and this article focuses on the element called intellectual challenge. Each element is scored on a four-point scale for each 15-minute segment of a lesson. Over the 34 video-recorded lessons, a total of 88 segments were analysed. To use PLATO a rater needs to complete a course on the use of the system and receive the appropriate reliability certificate. Reliability between raters in this study was tested by comparing the outcomes of the first 15-minute segments of two lessons from each school. Table 3 shows descriptions of the different levels of the intellectual challenge element.

Table 3. Shortened rubric on intellectual challenge from the PLATO framework.

	Intellectual challenge
1-level	Teacher provides activities that are almost entirely rote or recall.
2-level	Teacher provides activities or assignments that are largely rote or recall, but a portion of the segment promotes analysis, interpretation, inferencing, or idea generation.
3-level	Teacher provides a mix of activity or assignments: most promote analysis, interpretation, inferencing, or idea generation, and a few are focused on recall or rote tasks.
4-level	Teacher provides rigorous activities or assignments that largely promote sophisticated or high-level analytic and inferential thinking, including synthesizing and evaluating information and/or justifying or defending their answers or positions. (Grossman, 2019).

In analysing intellectual challenge, the teachers' questions or comments to students can lead to the level being adjusted by one level depending on whether the challenge is increased or decreased from the original task. Additionally, there is a time factor in the element, such that if less than half of a 15-minute segment involves student work where solution of tasks requires analytical thinking, then intellectual challenge is at the 2-level. If more than half of the segment involves such work, then the segment is at the 3-level.

PLATO has been recommended for a variety of reasons: It resonates with existing research, the four instructional domains replicate the areas outlined in research literature as critical for student learning, it builds on a feasible and applicable observation system, it allows systematic comparison of instruction across subjects, countries and educational settings, and it provides an opportunity to test for possible cultural biases embedded in the system (Klette, Blikstad-Balas & Roe, 2017).

By employing both the TAG and PLATO, a holistic view is taken the opportunities offered to students to engage in analytical thinking in mathematics. In TAG, the challenge is assessed as it appears in instructional materials, without considering how it is set up by teachers or implemented by students. In PLATO the teachers' implementation of the tasks in lessons is assessed. By comparing results between the two analyses, connections can be made between what kinds of tasks teachers choose for their students to solve and how the potential of those tasks for analytical thinking is maintained during lessons.

RESULTS

A total of 144 tasks appeared in the lessons and were analysed using the TAG. The intellectual challenge element was assessed from video recordings of 34 mathematics lessons according to the PLATO in 15-minute segments (88 segments in total).

Task analysis

Out of all tasks from the lessons, 64% belonged to category 2 where a solution required using procedures without connection to underlying concepts or meaning behind the procedures. In six schools out of ten a majority of tasks were in this category. The proportion of tasks in TAG category 3 was 33%. In that category the solution path is not made obvious by a worked example and analytical thinking is required for solving the task as it appears in instructional materials. Such tasks were present in all the participating schools but were only in the majority in school C. Only four tasks belonged to TAG category 4, that is doing mathematics. Their origins were both from the *Átta-tíu* and *Skali* textbook materials, as well as in a mathematical game provided by the teacher. None of the tasks examined belonged to TAG-category 1, that is memorization. In table 1, the distribution of task categories is shown by schools.

Table 4. Distribution of task categories by schools according to TAG.

School	Category 1	Category 2	Category 3	Category 4	Total
School A	0	14	2	0	16
School B	0	9	1	0	10
School C	0	1	9	0	10
School D	0	5	4	0	9
School E	0	8	3	0	11
School F	0	12	9	1	22
School G	0	4	1	1	6
School H	0	5	4	1	10
School I	0	5	2	0	7
School J	0	30	12	1	43
Total	0	93 (64%)	47 (33%)	4 (3%)	144

A noticeable difference is seen in the volume of tasks by schools. School J is an outlier with a total of 43 tasks identified. This is explained by the fact that in all of the observed lessons students worked independently in textbooks according to a plan with numbered exercises, in three different chapters and two different textbooks. In school F, 22 tasks were identified from the same type of instructional format. In other schools there were fewer tasks identified in the lessons. In schools G and H the tasks came from either textbooks or from the teacher, where specific tasks were assigned to all students at the same time. Table 5 shows the origin of tasks by schools.

Table 5. Origin of tasks by schools.

School	Origin of task	Content
School A	*Almenn stærðfræði II*, tasks from teacher	Ratios, geometry
School B	*Skali 1B*	Algebra and expressions
School C	Tasks from teacher and digital quiz	Algebra
School D	*Átta-tíu 2*	Algebra
School E	*Skali 1B*	Statistics
School F	*Skali 1B*	Statistics, algebra
School G	*Átta-tíu 1 & 3*, *Skali 1B*, tasks from teacher	Expressions
School H	*Átta-tíu 1 & 2*, tasks from teacher	Ratios
School I	Tasks from teacher, *Skali 1B*	Expressions
School J	*Skali 1B & 2B*	Statistics, geometry, algebra

Lesson analysis (intellectual challenge)

Intellectual challenge in mathematics lessons in the participating schools was at the 2-level in a majority of segments according to PLATO, or 63%. Intellectual challenge was at the 3-level in 14% of segments and at the 4-level in 7% of segments. This indicated that only a portion of the lessons included an emphasis on student's analytical thinking, interpretation or idea generation. In most segments students worked in textbooks while the teacher circulated and assisted. It was common to see teachers reduce the challenge of tasks by doing the intellectual work for students in their assistance. For this reason, most segments at the 2-level were placed there because the score was adjusted down from the 3-level. The proportion of segments at the 1-level of intellectual challenge was 16%. In those cases, the most common reason was students silently listening to the teacher lecturing, watching the teacher solve tasks or writing notes on the whiteboard, either for revision or introducing new material, and therefore were not active participants in the lesson segment. Also, at the 1-level were segments where students silently watched instructional videos each on their own tablet without any critical or open questions accompanying the activity.

Figure 2 shows the distribution of levels of intellectual challenge by schools. In schools D, E, F, I and J, there were no cases of intellectual challenge above the 2-level in each 15-minute segment. In schools A, B and C, the highest level of intellectual challenge was at the 3-level. Only schools G and H had segments with intellectual challenge at the 4-level, where students had to both use analytical thinking and evaluate information to reason for their solutions. In those two schools the teacher comments and questions caused the score for intellectual challenge to be adjusted up by one level while it was generally more common to see the challenge reduced by teachers.

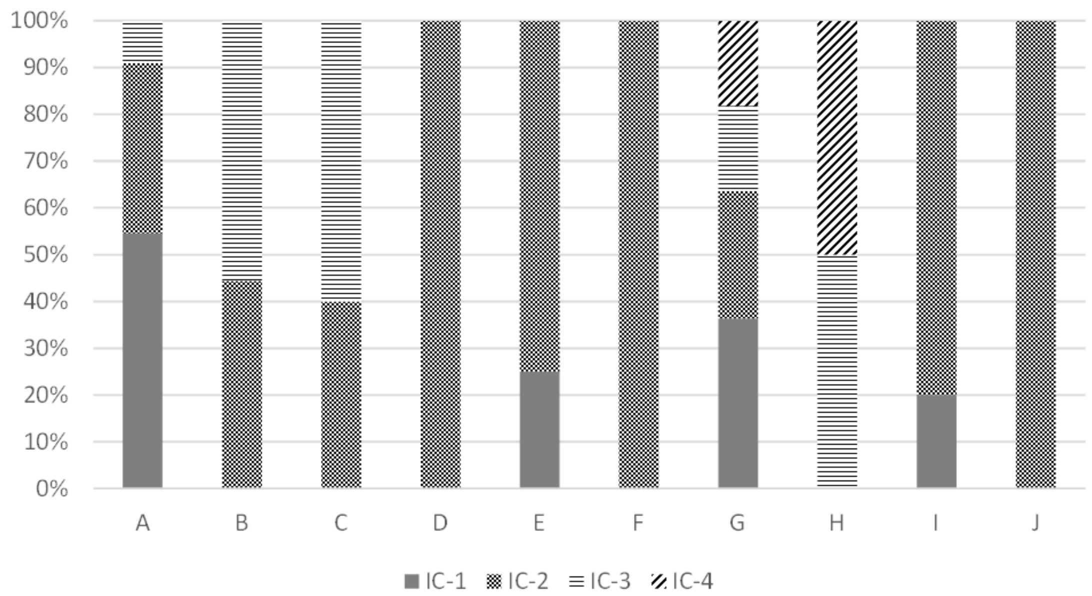

Figure 2. Distribution of levels of intellectual challenge by schools A-J according to PLATO.

Intellectual challenge reduced

In schools D, F and J the instructional format in all lessons involved students working according to a plan from the teacher with numbered exercises from textbooks that they were set to solve. The teacher walked between and assisted students who were often at different places in their plans. In all cases in these three schools, the teachers reduced the challenge, e.g., by telling the students exactly what to do to solve the tasks or by telling them the answers, resulting in intellectual challenge at the 2-level. The teacher's assistance primarily aimed at producing a correct answer but less at student's conceptual understanding. The following is an example of how a teacher reduced intellectual challenge as a student requested assistance in solving the equation 12 – 3x = 6 (TAG: Category 2):

Teacher:	"Wait a second, how did you get 9x?"
Student:	"Because I minused."
Teacher:	"Is this x?"
Student:	"No."
Teacher:	"Are these like terms?"
Student:	"No."
Teacher:	"Here you have twelve apples and three oranges, eat three oranges, you then have nine what? The apples are untouched."
Student:	"Yes."
Teacher:	"You can't... there is no x here. These are not like terms. So start by taking... to try to get x."
Student:	"I never know where to start. "
Teacher:	"We always begin by combining like terms if we can, but I do not want to know what minus x is. To change minus to a plus what do we do?"
Student:	"We bring it over."
Teacher:	"We bring it over, yes. So just erase this. Then you have this here 12 = 6 + 3x. Then you continue fixing this. Okay? "

In the example the teacher executed the procedure of adding 3x to both sides of the equation for the student without eliciting the student's thinking or understanding on what it means to "bring it over".

There were also cases where the challenge reduced by tasks that otherwise offered opportunities for analytical thinking and belonged to category 3 according to TAG. The following quote from the data shows an example of that. The student has a number pattern with numbered pictures of a growing number of blocks, marked with m, where m_1=5, m_2=9 and m_3=13. The student's task was to create a formula for m_n and compute m_{80}. The student asks for assistance:

Teacher:	"Yes, create a formula. What changes in between here?"
Student:	"One plus ..."
Teacher:	"It adds 4. Then you write 4x. If you take 4 of this, how much is left?"
Student:	"One."
Teacher:	"Plus one. And then the x is the m-number."
Student:	"So if I were to find 80, would I then just do four… "
Teacher:	"Four times 80 plus one"
Student:	"Okay."
Teacher:	"You do like this [teacher enters into the student's calculator] four times 80 plus one."

In this example the teacher essentially solved the entire task without the student having to show any understanding or own thinking.

In many cases teachers also told students exactly what procedures to carry out. As an example of that, a student asked for assistance in finding the area of a semicircle. The teacher answered: "If I were to do this task then I would do three squared times pi divided by two." The teacher has fully given a solution path to the student. The only thing left for the student to do is to enter what the teacher said into a calculator and write down the resulting value. In some cases, students were not exactly given the solution, but little space was given for thinking and use of concepts was questionable, as the following example shows. A student asks for assistance in solving the equation 75 / G = 15 (TAG: Category 2):

Teacher:	"I think it's much more comfortable instead of writing it like this that you write this like equal to 15. You do not want to know what division G is, how can you change it into a regular G?"
Student:	"Move it over."
Teacher:	"No no, then you have 75 and 15 and then the G comes and what does it do?"
Student:	"Times."
Teacher:	"Right, and then you need to move 15, here it is multiplying. On the other side it does what?"
Student:	"Divide."
Teacher:	"Right, away with 15, then the answer appears."

Unlike the example before, the student was not directly given the answer. Nevertheless, the challenge was reduced by leading the student through the procedure with closed questions where only few options were possible. The emphasis seems to be producing a correct answer at the cost of eliciting the student's thinking and understanding.

Intellectual challenge maintained or advanced

In schools A, B and C there were segments with intellectual challenge at the 3-level. In those schools this was the case when students worked on tasks and teachers did not reduce the challenge in their assistance via questions or comments. However, in these schools the instruction never advanced the intellectual challenge.

In schools G and H there were segments with intellectual challenge at the 4-level. In those segments, students had to reason or explain their answers to others, and therefore intellectual challenge was advanced. In school G, a segment of the lesson was scored at the 4-level where the math game 24 was played. In each round the teacher wrote four numbers on the whiteboard and students were to construct an expression with the value 24 by using arithmetic operations, parentheses, or powers (TAG: Category 4). The following discussion shows an example of intellectual challenge being advanced:

	Teacher writes: 8 7 9 1 [10 seconds pass]
Student 1:	"Okay, I figured it out."
Teacher:	"Okay."
Student 1:	"One minus... or, then seven minus one."
Teacher:	"Seven minus one is six."
Student 1:	"I know. Nine minus six."
Teacher:	"Yes."
Student 1:	"And then three times eight."
	Teacher writes: (7–1)
Teacher:	"You said seven minus one. Then what would you do?"
Student 1:	"Then I would do, that, nine minus six, which is the answer, seven minus one, such that ..."
Teacher:	"Yes, but how would you write this?"
Student 1:	"I would put the nine in front."
Teacher:	"Nine minus, parentheses, seven minus one."
Student 1:	"And that's three, and that times eight. "
	Teacher writes: 9–(7–1)
Teacher:	"Okay. And then I say times eight here?"
	Teacher writes: 9–(7–1)*8
Student 1:	"Uuh, you put another parenthesis in front of nine, and then parenthesis after."
Teacher:	"Why do you do that?"
Student 1:	"Because otherwise that would be six times eight."
Teacher:	"Because if I would start by computing these brackets, then I would get this here."
	Teacher writes: 9–6*8
Teacher:	"And what would we start by doing there?"
Student 2:	"Six times eight."
	Teacher erases 9–6*8 and adds a new line stating: (9–(7–1))*8

The teacher asked students to explain their solution and wrote on the whiteboard how the expression should look according to their explanations. At the end of the next lesson, a few rounds of the same game were played. However, that segment was only at the 3-level because the teacher did not advance the challenge in the same way. The teacher accepted student solutions without offering them to explain how the expression should be written or seeking further reasoning for their solution.

In school H the intellectual challenge was never scored below the 3-level, and in some segments, it was at the 4-level. The teacher had students work in pairs and commonly told them to explain to each other. In the teacher's assistance to students, student questions were rarely answered by what they "should do". The teacher rather answered their questions with her own questions to encourage their own thinking. The following shows a discussion between the teacher and a student which contributed to the reasoning behind advancing intellectual challenge to the 4-level. The student's task was to find the scale ratio of a picture which showed a 9 cm long insect whose actual length is 1.5 cm (TAG: Category 2):

Student:	"So, I did 9 cm divided by one and a half. That's six, so is it then one and a half over six?"
Teacher:	"Yes, see, its actual length is one and a half, but it's drawn as [Student: 9 cm] nine, so then it is one and a half over …"
Student:	"Each nine."
Teacher:	"Yes."
Student:	"Is that the scale then?"
Teacher:	"Yes, then you can reduce if you can to one. One over what?"
Student:	"Huh?"
Teacher:	"To show, if it's one and a half over nine, what is it one over? So one cm is how many cm on the picture?"
Student:	"Uuh..."
Teacher:	"What do you need to divide one and a half by to get one?"
Student:	"Uuh zero point seventy ..."
Teacher:	"No, what is four divided by four?"
Student:	"Four divided by four? That's one."
Teacher:	"Yes, then by what do you need ..."
Student:	"Aaah yes, then it's one point five."
Teacher:	"Yes, precisely."

The student then reached a correct result. The teacher gave the student an opportunity to figure out the answer on his own by supporting him on by questions and having him connect it to another result that he knew. In these circumstances it was common to see teachers directly tell students what to compute or even telling students the answer. By encouraging student's analytical thinking in this way, the intellectual challenge was advanced and their conceptual understanding supported.

DISCUSSION

The results of the study show that there is room for improvement in the quality of intellectual challenge in mathematics teaching in Iceland. The most common teaching format in the lessons was student's individual seatwork where they commonly worked alone and not at the same tasks at the same time as other students. This is in alignment with results of previous research which showed that students' individual seatwork in textbooks most strongly characterises mathematics teaching in Iceland (Jónsdóttir et al., 2014; Pálsdóttir & Gunnarsdóttir, 2015; Þórðardóttir & Hermannsson, 2012). This teaching format may hinder tasks that involve analytical thinking to be implemented successfully with little space for collaboration or discourse. Teaching was commonly driven by study plans with numbered exercises from textbooks. The exercise numbers seemed to control student work rather than particular learning goals. In the school with the strongest evidence for intellectual challenge the teacher referred to such a plan but did not let it control the classroom activity. The teacher clearly organized lessons with more specific goals in mind than merely covering particular exercises from a plan. More opportunities for discussions were created in these lessons than others and students worked in pairs on the same tasks. This led to students explaining their thinking to each other which advanced intellectual challenge. When students work on the same tasks at the same time, richer opportunities seem to be created for intellectual challenge at a high level. In discourse with the teacher and fellow students on the tasks, students get an opportunity to develop their competency in reasoning and conceptual understanding of mathematical concepts. The results showed that where students worked according to a plan at different places in the textbook with many tasks appearing in each lesson, intellectual challenge tended to be at a low level.

The way challenge appeared in the tasks and in the lessons corresponded to an extent but did not always match. No tasks were in TAG category 1 which includes memorization tasks. This indicates that at the lower secondary level, the emphasis is not that students memorize procedures or other information, but rather that they apply procedures from solved exercises or follow teacher directions on how to solve the tasks. In TAG, a task is in category 2 if there is a solved exercise which clearly shows how the task is to be solved, as is common in many textbooks. In PLATO, it is considered evidence of analytical thinking to work on such tasks despite the solved exercises, and a segment is at a 3-level if both a majority of the segment involves such activity, and the teacher maintains the demand for analytical thinking.

The results also showed that teachers tend to reduce the intellectual challenge of tasks by only asking closed questions, giving students limited opportunities to try for themselves or frankly telling them the solution. In many cases there was not much left for students to do but to enter into a calculator what the teacher said or to write down the answer. The results thus indicate that it is problematic for teachers to maintain demands for reasoning and the challenge that tasks offer. This is in accordance with the results of previous studies which show that teachers lack professional support in maintaining challenge (Wilhelm, 2014; Norton & Kastberg, 2012; Tekkumru-Kisa, Stein & Doyle, 2020).

School culture and the way mathematics teaching is arranged plays a key role in this respect. Students at the lower secondary level that are used to the idea from the beginning of their schooling that mathematics is a subject in which work is done in collaboration with others no less than individually undoubtedly find it easier to engage in group work in mathematics. Continuity in mathematics learning and collaboration between mathematics teachers between school levels can support mathematics teaching toward more student collaboration on the same tasks. Conversely, students who like individual work and find it useful are worth discussing. High-achieving students are commonly in the position to work individually in more advanced material, and it is worth questioning how well such an instructional format nurtures their development and opportunity for positive learning experiences. With a shrinking proportion of Icelandic students in the highest achievement group, an assessment of the status of this student group in Iceland seems necessary (OECD, 2019). It is worth considering what the results of this study mean for this group of students and how their needs for intellectual challenge can be met.

Although video data from lessons may give a vivid picture of instruction on general, such data alone only provides limited information on student learning and activity, specifically in the case of silent individual seat work. Samples in video-based studies are usually small because of the amount of time video analysis takes. These issues limit to an extent the inferences that can be drawn from the results. Contextual data plays a key role in understanding how the lessons relate to the instructional arrangement in a wider sense.

An interesting result is that two of the tasks in TAG category 4 appeared in schools where intellectual challenge in the lessons did not exceed the 2-level in PLATO. This shows that even though the tasks had rich potential for intellectual challenge the teacher did not maintain the challenge in the implementation in the lessons. In both cases, the students encountered these tasks individually in a textbook while their classmates were working on other tasks in the textbooks according to the plan. Only two teachers out of ten advanced the intellectual challenge with their pedagogical approach to the tasks and communication with students. A majority of teachers reduced the intellectual challenged at some points and most did so repeatedly.

Conclusion

Although it seems clear that there are opportunities to improve mathematics teaching in Iceland, and especially in creating opportunities for students to engage in analytical and creative thinking, one must be careful in drawing inferences about teachers and schools that participated in the study. Even though intellectual challenge was at a different level in the schools, it is not necessarily possible to state that the teachers who maintained or created the greatest opportunities for thinking are “better teachers” than those teachers who reduced it. The teaching was certainly rated higher in this particular dimension, but other dimensions of teaching quality, such as classroom management, time management, or feedback, were not a topic of study in this paper.

To support intellectual challenge at a high level in mathematics teaching in Iceland, it is necessary to increasingly emphasise the value of meaningful tasks that students all work on at the same time where discourse, explanations of thinking processes and reasoning are central. Correspondingly, lesson time dedicated to strictly following a plan with numbered exercises from mathematics textbooks could be greatly reduced. Further, when students work together toward well-structured tasks the goal of the lesson becomes clearer than when students work in different places in textbooks. Such teaching practices can be supported to a greater extent with professional development programs that increase professional knowledge in this area, a key for maintaining intellectual challenge (Tekkumru-Kisa, Stein & Doyle, 2020; Wilhelm, 2014). It is not questioned that students can benefit from regular exercise lessons where they work individually to develop their procedural fluency, for instance in solving equations and simplifying expressions. However, it is vital to work toward conceptual understanding alongside. The results of this study and previous studies in mathematics teaching in Iceland indicate that the volume of such exercise lessons is at the expense of analytical thinking, intellectual challenge and classroom discourse.

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INTELLECTUAL CHALLENGE IN MATHEMATICS TEACHING IN LOWER SECONDARY SCHOOLS

EXTENDED ABSTRACT

The aim of this study was to assess the level of intellectual challenge offered to students in lower secondary mathematics in Iceland as it appears in mathematical tasks and in the enactment of tasks in the classroom. The national curriculum explicitly states that students should learn to explain their thoughts to others and engage with mathematical tasks in which both critical and analytical thinking is required (Ministry of Education, Science and Culture, 2012). Previous research has shown that teachers generally have a positive view towards cognitively demanding tasks, but such tasks are scarce in Icelandic textbooks (Sigurjónsson & Kristinsdóttir, 2018). How teachers enact tasks in the classroom with the aim of developing students' competency in critical and creative thinking has not been the object of much study in Iceland.

Ten Icelandic lower secondary schools participated in the study. Schools were purposefully chosen with the aim of establishing heterogeneity of the sample and included different school types in terms of location, size, and students´ background. Three to four consecutive mathematics lessons were video recorded in 8th grade in each school, in total 34 lessons. Two frameworks were used to analyse the data. The Protocol for Language Arts Teaching Observations (PLATO) was used to analyse the level of intellectual challenge offered to students in the classroom on a 4-point scale in 15 minute segments (Bell et al., 2019; Cohen, 2015; Grossman, 2020). The Task Analysis Guide (TAG), was used to analyse whether the solution of mathematical tasks required analytical thinking (Stein, Smith, Henningsen & Silver, 2009).

For the study 144 mathematical tasks were analysed. The tasks were identified from both the videos and lesson plans. Findings showed that the majority of tasks involved procedures without connections to underlying mathematical concepts, or 64%. The tasks that involved procedures with connections counted 33% and only four tasks were in the highest category and involved doing mathematics or 3%. No tasks were in category 1 that involves only memorization.

In a majority of segments, or 63%, intellectual challenge was at the 2-level which

indicates limited evidence of intellectually challenging activities. A common reason for segments scoring at the 2-level was that teachers reduced the challenge of activities in their assistance to students. Teachers commonly told students exactly how to solve tasks and, in some cases, told them the answers. About 20% of segments scored at the 3 or 4-level, which require teachers to provide activities that to some extent prompt high level analytic and inferential thinking. Only in two schools did the intellectual challenge of activities reach the 4-level in some segments. In those cases, teachers advanced the intellectual challenge by asking students questions where explanation and reasoning was emphasized to develop conceptual understanding.

The results show that mathematics teaching in Iceland focuses mostly on tasks where procedures for how to solve the tasks are either fully explained by the teacher or by examples in the learning materials. In lessons where intellectual challenge was found to be at a high-level, students worked in pairs and the whole class was working on the same tasks, giving room for mathematical discussions among students. However, in a large portion of the observed lessons, students worked individually on a variety of different tasks at their own pace with limited interactions with other students.

The findings of this study indicate that there are opportunities to improve intellectual challenge in mathematics teaching in Icelandic classrooms by organizing lessons with specific tasks or learning goals in mind. Such a lesson structure seems to create richer opportunities for student reasoning and explanations. Lessons where the goal is to develop procedural fluency, for example in solving equations and simplifying expressions, may be of some benefit to students but it is imperative to place emphasis on conceptual understanding. The results of this study and previous studies on mathematics teaching in Iceland show that too much time is devoted to procedural lessons at the cost of critical thinking, intellectual challenge and classroom discourse.

Keywords: mathematics education, intellectual challenge, teaching quality, video-based studies, analytical thinking

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