Creative Mathematics

Professional Development in an Icelandic Compulsory School

Ósk Dagsdóttir

Dissertation submitted in partial fulfilment of a Ph.D. degree

UNIVERSITY OF ICELAND
SCHOOL OF EDUCATION

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Preface

Research seldom happens in a vacuum. Rather, it tends to be connected to or be based on previous research, with the goal of contributing new knowledge or understanding for the future. However, research is always a social process to some extent and in education the social aspect is crucial. Educational research is intended to shed light on the process of learning and requires the participation of different stakeholders.

Although my name is on the front of this thesis, it is in no way my private work. This educational action research happened within a school system with the cooperation of many people. I am thankful for the part played by the teachers who participated in this research. Without their participation in the professional development and their contribution to the research process, this process would not have been what it is. I am also grateful to the administrators who allowed this research to be possible, and to the students who played a vital part when the teachers were developing their own work. These people all remain anonymous.

For this thesis I had two thesis advisors: Bharath Sriraman, Professor at the University of Montana, and Freyja Hreinsdóttir, Professor at the University of Iceland. They both helped me tremendously in this process, providing invaluable insight, wisdom, and feedback as well as encouragement to keep going and guidance for the necessary steps to complete the research. For that I am very grateful.

Two opponents provided very helpful feedback at my interim evaluation: Kristín Bjarnadótttir, Professor Emeritus at the University of Iceland, and Ólöf Björk Steinþórsdóttir, Senior Lecturer at the University of Northern Iowa. This resulted in improved methodology and a better focus for the research process. Ólöf later joined the thesis committee and has been a very valuable addition to the team. Jónína Vala Kristinsdóttir has also supported me throughout this research process as a critical friend and also joined the committee at a later stage; she offered invaluable support for completing the thesis. Moreover, I am thankful to the staff at the University of Iceland who make the Ph.D. program possible.

The project has received four grants. The first grant was from the developmental fund of The Reykjavik Department of Education and Youth in 2018 and is intended to support their educational policy. The second

grant was from The Icelandic Teachers Union Research Fund in 2019 and is intended to support research on teachers' work, the changeable role of education, and innovative projects and methods. The third grant was from the Steingrímur Arason research fund in 2019, managed by the University of Iceland, and is intended to support innovative educational research that promotes academic and professional contributions in the field. The fourth grant was an assistant teacher grant from the University of Iceland for the school year 2019-2020. These grants were instrumental in my completing the thesis and for that I am thankful.

Furthermore, I am very grateful to the men and women who sparked my interest in mathematics and mathematics education, starting with my father who showed me puzzles, games and tricks, and taught me so much. I also thank many mathematics teachers I have had. Guðbjörg Pálsdóttir, Guðný Helga Gunnarsdóttir and Jónína Vala Kristinsdóttir at the University of Iceland proved instrumental in my ongoing interest in mathematics education and have taught me so much from my first day at the School of Education and continue to do so.

Finally, I am grateful for the support given by my family and friends, both before and during the research process. I would not be who I am without their support and they have certainly showed tremendous support in my work with this research. They listen, read, and never tire of hearing about my endeavours. To my sons, I am grateful for the continual reminder of how important children are and how valuable it is to provide them with excellent education. In Iceland we are truly blessed with education for all and I am grateful for that.

Abstract

Creativity is an important component in education. Modern society with an unknown future calls for creative individuals who develop new ideas and solve problems in a creative manner. Computers have taken over much of the routine work in mathematics and other sciences, but these fields call for creativity, cooperation, and perseverance. However, mathematics taught in schools often lacks creativity and focuses on solving routine problems with given algorithms. Although some teachers see the value of creativity for mathematics learning, they struggle to foster this important component in their classrooms. This action research focused on a professional development program in an Icelandic compulsory school. The program intends to support teachers to foster creativity in their mathematics classrooms. The teachers partake in seminars and work towards bringing their learning into their classrooms. Data was gathered from seminars and work with teachers. The data was analyzed qualitatively and included interviews, surveys, videos, field notes, and reflective journals. Case studies were chosen to see how the teachers explain the effect of the program on their views, pedagogy, teaching practices and student learning. The case studies revealed that professional development on creative learning can influence teachers' pedagogy and practice. The results further brought to light aspects for learning that have been developed into a model for creative mathematics learning. This model brings together the components of play and games, hands-on learning, and conversation in mathematics. These aspects helped the teachers bring the students to deeper learning modalities that connect to creative mathematics learning for the students. By partaking in this process, the teachers found that their views, teaching, and student learning were altered. They described barriers of time, student group size, learning space, standardized tests and limiting teaching materials. These research results and development of the PD program have value for teachers, teacher educators, those planning professional development, school management and policy makers.

Skapandi stærðfræði

Starfsþróun í íslenskum grunnskóla

Sköpun skiptir sköpum fyrir menntun. Nútímasamfélag kallar eftir einstaklingum sem geta þróað nýjar hugmyndir og leyst vandamál óvissrar framtíðar. Tölvur hafa tekið yfir mikið af hefðbundnum aðgerðum í stærðfræði og vísindum en þörf er á sköpun, samvinnu og þrautseigju fyrir framþróun þessara greina. Þrátt fyrir það er stærðfræðin sem kennd er í skólum oft ekki skapandi og byggist á því að leysa hefðbundin verkefni með fyrirfram gefnum reikniritum. Jafnvel þó að sumir kennarar sjái gildi þess að leggja áherslu á sköpun í stærðfræðinámi eiga þeir erfitt með að efla þennan mikilvæga þátt í kennslu sinni. Þessi starfendarannsókn snýr að starfsþróun í íslenskum grunnskóla. Starfsþróunin leitast við að styðja kennara til þess að efla sköpun í stærðfræðinámi. Gögnum var safnað frá námskeiðum og í vinnu með kennurum. Gögnin voru greind með eigindlegum aðferðum og byggðu á viðtölum, könnunum, myndböndum, vettvangsnótum og ígrundunardagbókum. Tilvik voru valin til þess að skoða hvernig kennararnir lýstu áhrifum starfsþróunarinnar á viðhorf sín, uppeldisfræði, kennslu og nám nemenda sinna. Tilvikin leiddu í ljós að starfsþróun í skapandi stærðfræði getur haft áhrif á kennslu og nám. Auk þess komu í ljós þættir fyrir nám sem hafa verið settir fram sem líkan fyrir skapandi stærðfræðinám. Líkanið tengir saman verklega vinnu, leik og samræður í stærðfræði. Þessir þættir hjálpuðu kennurunum að búa nemendum skapandi nám. Með þátttöku í þessu ferli sáu kennararnir að viðhorf þeirra, kennsla og nám nemenda þróaðist. Þeir lýstu takmörkunum eins og tíma, stærð nemendahóps, kennslurými, samræmdum prófum og takmarkandi kennsluefni. Þessar niðurstöður og starfsþróunarprógrammið sem var þróað hafa gildi fyrir kennara, skólastjórnendur og þá sem skipuleggja kennaranám, starfsþróun, kennsluskrár og námsefni.

Table of Contents

List of Figures

List of Tables

1 Introduction

It has become universally accepted that our world needs creativity (Baer, 2013; Daniels, 2013; Grohman & Szmidt, 2013; Kettler, Lamb, & Mullet, 2018; Ranjan & Gabora, 2013). It is associated with better learning outcomes, problem-solving skills, and personal health and growth (Daniels, 2013; Grohman & Szmidt, 2013; Starko, 2018). Many go as far as to say that creativity should be the core focus of school life and education and that it is the best way to help students reach their greatest potential (Baer, 2013; Ranjan & Gabora, 2013). Creativity can make the school experience more interesting and dynamic and better prepare students for life in a society filled with uncertainty and questions (Kettler et al., 2018; Sefton-Green, Thomson, Jones, & Bresler, 2011).

This thesis focuses on action research which takes place within a professional development (PD) program on creative mathematics education. In this introduction, the scene is set for the research by explaining the curriculum in Iceland and the role of PD within the Icelandic school system. The motivation and purpose of the research are detailed, which are followed by an account of the research questions and aim. Towards the end of the chapter, the core concepts for the study are defined in a preliminary way and an overview of the entire thesis provided. The hope is that this chapter will explain to the reader the importance of this research, how it was developed and what to expect from reading the thesis.

1.1 Creativity and the Curriculum

Young children are buzzing with creative energy, fantasy play and making connections unbound by linear systems; these drift away in the formal education system (Bloom, 2014; Glăveanu & Beghetto, 2017). Although it is not fully clear how formal education promotes or represses creativity, some research shows a negative connection between formal education and creativity (Simonton, 2017). This obscure innate trait that everyone seems to have is often not so easy to conjure up on demand.

There has been an increasing interest in creativity, both nationally and world-wide. Although the concept has not been definitively defined, it seems that everyone wants a piece of the creativity cake. Western societies are looking to creativity for business and innovation, and it is a trait widely

sought on the job market. In educational policies there is a widespread focus on creativity (Craft, 2011a). People want creativity for personal life, health, and growth. The world calls for creative solutions for a rapidly changing society, environmental problems, and other dilemmas of the future.

The Department of School and Youth in the capital of Iceland, Reykjavik, recently developed a new educational policy based on letting dreams come true (Reykjavíkurborg-Skóla og frístundasvið, 2019). The policy is intended to be developed until the year 2030 and has five primary components: social efficacy, self-empowerment, literacy, creativity, and health. The goal is that all children can grow together in a community of human rights and respect for diversity. Desired outcomes are self-efficacy and self-esteem, reading for education and leisure, understanding nature and society as well as initiative, creativity and critical thinking. Actions to follow the policy include increasing the teaching of natural sciences, mathematics, and the special subjects, as well as outside learning, simplifying services for students, improving facilities, and creating a developmental fund to support the policy. This is to happen through rich opportunities of professional development, coupled with effective counseling and guidance that supports teachers' professional initiatives.

The national curriculum guides for Icelandic schools set creativity as one of six guiding pillars for learning (Ministry of Education, 2013). The curriculum serves as a framework for all schools in Iceland: play school (ages 1 to 5), compulsory school (6 to 15) and secondary school (16 to 19). It emphasizes in a comprehensive way the educational policy, based on legislation on Icelandic schools. The other five guiding pillars are literacy, health and wellbeing, democracy and human rights, equality, and sustainability. The guiding posts set purpose for Icelandic schools and are to be fostered in all subjects to build good education and school culture.

In the compulsory school curriculum guide, emphases for each subject can be found. For mathematics, the curriculum emphasizes the main objective of students acquiring competence to use mathematics as a living tool for a range of purposes and in varied situations. They should develop the ability to propose and solve problems, reflect on different methods and models, and evaluate findings. Teaching ought to encourage students to adopt positive attitudes towards mathematical learning and help students to become adept at analyzing and solving problems, evaluating solutions, using mathematical concepts, and making connections. Students should learn to choose appropriate tools for every task and become adept at using the language of mathematics. They need to be able to explain calculations, findings, and hypotheses. Supporting teachers to foster creativity in their mathematics classroom can help them meet these goals.

1.2 Professional Development

Teachers are trusted with an enormous responsibility: to create a learning environment and experiences that induce growth, development, and learning. As our society changes, so do schools and policies. These changes make it crucial for teachers to have opportunities to develop as professionals (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). Professional development is important to support school curriculum changes for mathematics teachers (Fiorentini, Miskulin, Grando, Nacarato, & Passos, 2011). All teachers need continuing education after graduation. They gain experience and insight from teaching, but also need to update and increase their skills and knowledge through what is known as professional development (PD) (Loucks-Horsley et al., 2010).

A recent report came out in Iceland on PD for Icelandic teachers and principals (Mennta- og menningarmálaráðuneytið, 2019). The report outlines in detail the need for PD for teachers and encourages legalization of PD. The report focuses on policy changes connected to all aspects of education, including play schools, compulsory schools, secondary schools, and universities. It also notes the importance of the connection of professionality and PD, and advocates for changes that make this important aspect of education a priority on all levels.

In Iceland, all in-service teachers are obligated to partake in PD as part of their job. Professional development means any formal or informal learning to which in-service teachers apply themselves. This can include workshops, courses, formal education, communities of practice, reading and research (Desimone, 2009). Partaking in good PD is important so that teachers can follow up on expectations from the curriculum and society, along with meeting students' needs (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009). Teachers need to continuously further their learning and improve their skills so that students can be best supported to develop their talents. Teachers also need opportunities to take part in successful PD for all subjects (Darling-Hammond et al., 2009). Gunnarsdóttir (2014) found PD for mathematics teachers in Iceland to be limited. Creating and researching a PD program focused on creative mathematics education is therefore valuable for teachers, schools, and students.

1.3 Motivation and Purpose of the Research

Many people think that mathematics is not a very creative subject and to an extent they are right (Ambrose, 2017; Grégoire, 2016). Although the work of mathematicians is highly creative and deals with uncertainty, inventing,

problem finding, problem solving, divergent thinking and other traits connected to creativity, most people never experience mathematics in that way (Sternberg, 2017). They learn mathematics as a set of rules, algorithms, and procedures that they must follow (Leikin & Pitta-Pantazi, 2013; Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). There is little room for cooperation, creative thinking, or the messiness that mathematicians and scientists deal with in their work. Creativity is frequently left on the backburner for content learning or seen as existing outside the curriculum, possibly only for gifted students (Beghetto, 2013). Standardized tests and other content-based tests can be a factor in minimizing this important component of learning (Sternberg, 2017). Fixed ideas of what school should be and how learning should happen can prevent creative learning.

Many teachers do not see mathematics as creative (Panaoura & Panaoura, 2014). Some teachers recognize that mathematics is creative but their conceptions are often narrow and focused on technology or teaching creatively rather than teaching for creativity (Bolden, Harries, & Newton, 2010). Sometimes teachers in mathematics want to teach creatively but lack the means or ability (Shriki, 2010). Since teachers themselves have often not had any experience of creative mathematics learning, they struggle to provide that environment for their students, so they go back to what they know and continue the cycle (Leikin & Pitta-Pantazi, 2013). However, supporting teachers to explore creativity can raise their awareness of this important aspect of mathematics (Panaoura & Panaoura, 2014).

There is an important need to teach and provide pre- and in-service teachers with opportunities to see how creative mathematics can be (Sternberg, 2017). Empirical research on how best to support teachers in this endeavour is vital (Sriraman, 2017). It is important to create programs with intentional experience in mathematical creativity to raise teachers' awareness of creativity in mathematics and support them to incorporate it for their students (Panaoura & Panaoura, 2014).

Mathematics is a core subject and excluding the important creative component can make mathematics uninteresting and fail to provide students with important skills for future learning, work, and life. Creative mathematics learning provides students with valuable tools for understanding mathematics, science, logic and how the world operates. The skills learned in creative mathematics can possibly be transferred to other fields, although a lot remains unclear on how creativity transfers across domains.

I have been interested in creative mathematics education for a while. In my B.Ed. I saw how my lack of interest in mathematics as a child was based on not getting opportunities for creativity in my mathematics learning. For my M.Ed. thesis I developed and researched a course for teachers on creativity in mathematics education. I have since held many workshops and courses for teachers. I have taught mathematics for over five years and continuously develop my own teaching. I have come to see how challenging it can be to change your own teaching and how much support teachers need to do that.

Starting out as a teacher at the research school, which I will call Vinaskóli, I found it interesting how much emphasis they had on PD. All the teachers there partake in weekly PD sessions and cooperate towards better learning. At that school there is also wide basis for collaboration and both social and natural sciences are taught through student workshops with many teachers collaborating, including special subject teachers such as arts and crafts teachers. These workshops struck me as very creative, and I found that intriguing.

During my first year at Vinaskóli there was one teacher who was interested in creative mathematics education who kept coming to me and asking me about her mathematics teaching. During the course of one year, we frequently discussed mathematics education. In that time, I heard her change various things in her classroom and become more conscious of how she taught. Through that collaboration, I learned that I could support other teachers at my school and wanted to do more of that. I approached the principal with the idea of implementing a two-year PD program on creative mathematics education. She received this positively and the program started in the fall of 2018. I soon decided with my supervisors that doing an action research within that program would be my Ph.D. thesis.

The purpose of this study is two-fold. On the one hand the purpose is local at the research site and focuses on supporting the teachers to foster creativity in their mathematics classroom. On the other hand, the purpose is to gain a better understanding of how a PD program on creative mathematics can work to support teachers with creative mathematics learning and what can be learned from developing and carrying out such a program. This purpose extends the local nature of the research.

The PD program in research was based on regular seminars on creative mathematics education and ongoing support for teachers to develop creative mathematics in their own classrooms. The seminars presented teachers with theories of creativity in mathematics and provided them with

methods to promote creative learning opportunities for their students. These seminars included presentations and discussions, and teachers participated in creative learning experiences such as problem solving, working with patterns, and working like real mathematicians. The support between seminars was based on the wishes of individual teachers but included coaching, meetings, and collaboration within their mathematics classrooms.

1.4 Research Questions and Aim

When starting research at a new place, it is important to be clear on what the goal of the research is to be. A researcher must have the goal of learning something new. Educational research generally relates to shedding light on some aspect of learning that needs better explaining, often with the goal of positively influencing educators or other stakeholders (Efron & Ravid, 2020). Newer research in education is generally focused on reform as well as exploring the existing situation (Herr & Anderson, 2015).

Action research in particular focuses on reform (McNiff, 2017; Mertler, 2020; Mills, 2018). Educational action research deals with a cyclical process of improvement and cooperative learning within an educational institute (McAteer, 2013). It is usually conducted by educators or others who work closely with them (Efron & Ravid, 2020). The researchers themselves usually develop their own action plans and research something they feel needs improving (Merriam & Tisdell, 2016).

This research focuses on the process of the PD program to reflect on the learning of the teachers and how they experience changes in their own classrooms. The teachers at Vinaskóli all do an action research project of their own choice each year, as part of other PD. I decided to do an action research dissertation of the PD program I was developing with the teachers. I hold an insider-outsider position that provides me with a unique opportunity to look at a whole school over an extended period of time.

The main research questions are:

1. How do teachers who take part in a PD program on creative mathematics education report how it influences their pedagogy and classroom practice?

2. What can be learned from the active development of a PD program of creative mathematics education?

The main research questions are answered by focusing on four sub-questions:

A. How do the teachers explain the influence of the PD program on their pedagogy and views of mathematics education?

B. How do the teachers who implement creative mathematics in their classroom report its influence on their classroom practice and student learning?

C. What barriers do teachers experience when working with creativity in the mathematics classroom?

D. How can teacher experiences, as seen through the action research, add to the existing literature on how to support teachers with creativity in their mathematics classroom?

The aim of the research questions is to learn how the teachers explained their learning from partaking in the PD program. Special focus is put on how the program influenced their pedagogy and their views on mathematics, as well as how they implemented their learning in their classrooms and the changes they noticed in student learning. The aim was to add to the existing knowledge on how to support teachers in fostering creativity in the mathematics classroom.

The teachers who participated in this research collaborated with me to construct the data that was analyzed to answer these research questions. Case studies were carefully selected to best reflect the learning process of the teachers and how the teachers experienced changes in their classroom practice and student learning. The case studies are the core data in answering the research questions. However, data from all the participants in the PD is used to provide information on how the whole school partook in the PD program and those results further help answer the research questions.

Valuable information can be gained by creating a whole school PD program on creative mathematics education and researching the process. Information on how such a program can actively encourage and motivate teachers to foster creativity in their mathematics classroom has the potential for informing and improving the teaching and learning of mathematics. Analyzing how taking part in such a PD program can influence teachers' views, pedagogy, and classroom practice adds to the exciting literature on creativity in mathematics education. The research provides valuable information that can inform policy makers, teacher educators, and

those responsible for developing curricula and PD opportunities. The research holds implications for other schools, teachers and students who wish to focus on creativity in mathematics teaching and learning.

1.5 Introducing Core Concepts

Throughout this work there are a variety of concepts used and discussed, and although the concepts are explained in better detail in the chapter on the theoretical framework it is helpful to explain the most crucial concepts at this point, albeit in a preliminary fashion. The main concepts are creativity, creative mathematics education and professional development.

As explained in the theoretical framework, creativity can be seen and researched from different perspectives. For the purpose of this thesis, creativity is seen from the viewpoint of the individual and used to explain how students can be creative in their own learning. There is no requirement for a creative product or for the creative endeavour to extend beyond the individual student. The same is true when referring to creative education and creative mathematics education.

In the theoretical framework, mathematical creativity is explained as both the professional contributions of very few and the creative acts or ideas of students, but for the purpose of this research we focus on student creativity. The terms creativity in mathematics education and creative mathematics education are used interchangeably and are a matter of style. There is however a distinction made between creative teaching and teaching for creativity, based on whether the focus is on the teachers' creativity or students' creativity. For this thesis, the term creative learning is generally favoured, with the aim of incorporating both aspects.

Professional development (PD) is another concept of importance for the theoretical framework of this thesis. PD refers to any formal or informal learning in which teachers participate alongside their teaching profession. Professionalism refers to their identity as professionals and the methods they rely on in their profession. Coaching refers to the act of another professional collaboratively supporting a teacher with their classroom practice and professional development.

I believe it is helpful to the reader to give preliminary explanations of the core concepts here at the beginning of the thesis. The reader can then follow the text better and know what is being referred to from the beginning. The same is true for the next section that lays out the overview of the thesis to help the reader understand how it is constructed and to facilitate the reading.

1.6 Overview of Thesis

This thesis is focused on an educational action research that took place in an Icelandic compulsory school over the course of two school years. The purpose of the study is to add to existing literature on how to support teachers to foster creativity in their mathematics classroom by developing a collaborative PD program on creative mathematics education that would be researched through iterative action research cycles. The research focused on investigating the influence of the PD program on creative mathematics on teachers' pedagogy and classroom practice and to explore what can be learned from its collaborative development.

This first chapter has explained the rational for the research and how it connects to the curriculum and PD in Iceland. The second chapter, Theoretical Framework, explains in detail the theoretical framework for the study. Theories of creativity and creative mathematics education are presented, and it is explained how these theories connect to the PD program in research as well as to the data analysis. Theories of PD are also presented with the goal of explaining how the PD program was developed, with the purpose of positively influencing teaching and student learning.

The third chapter, Methodology and Methods, addresses why action research was chosen as methodology. It details information on the research site and PD program and how case studies were used within this action research. I clearly explain my positionality in the research process and how the dialectical action research spiral was used for this study. Methods of data gathering and analysis are explained in detail, as well as the ethics of the study, the role of rigour, limitations, and delimitations.

The fourth chapter, Results and Discussions, dives into how the teachers who partook in the PD program explain the influence of the PD program on their pedagogy and classroom practices. The results focus on how the teachers experience changes in their teaching and student learning as a result of participating in the PD program of creative mathematics education. The chapter explains how the PD program developed and which aspects were important for the teachers to be able to foster creativity in their classroom. The results are discussed in relations to the theoretical framework with a focus on theories of creativity, creative education, and PD. The last chapter focuses on drawing together the main conclusions of the thesis and discusses implications and applications of the research and how the results can be developed further.

2 Theoretical Framework

The connection between theory and practice in action research is important although not always direct or logical (Stringer, 2014). While theory can inform or influence practice, the link can be discursive with ideas, notions and elements seen in development of practice without the claim of being automatically applicable. Theory can inform a process of enlightenment that helps new practices emerge, but theories of the academic world do not always fit comfortably into teachers' everyday reality or practice. Academic theories are embedded in concepts and views of a reality that only makes sense within a particular social context and any theory is just one set of possibilities for explaining or interpreting a social situation (Stringer, 2014).

Stringer (2014) explains that one of the strengths of action research is that it accepts diverse perspectives of different stakeholders and the "theory" each will hold to explain occurrence of events and how a mutual understanding is developed. He notes that imposing outside theories without a deep understanding of the nature of events or the dynamics of the context can lead to misrepresentation or misinterpretation of the situation. Stringer states that although no research is atheoretical it is especially important with action research to rely on stakeholders' descriptions and interpretations of events. He elaborates that in action research:

> More relevant and effective theories emerge from hermeneutic dialectic – meaning-making dialogues – between stakeholders, using the concepts, terminologies, and formulations that make sense to them. This does not mean that academic theories cannot enter the research arena, but it does mean that academic theories may become relevant only when stakeholder theories have been described and elaborated. (Stringer, 2014, p. 39)

The role of theory for this research is two-fold. Theories presented for this research were used in the planning stage of the PD program and research and were developed as the PD program progressed. The theories presented were also used in the analytical process when reflecting on the "theory" that was constructed by the stakeholders or participants in the research. This study is an educational action research which is one type of practical action research. Therefore, the theory is based on practice and connected to the field.

In Table 1 the theoretical framework is laid out in a concise manner to clarify to the reader the use of theory for this research.

Table 1 Theoretical Framework

Theoretical Framework		
Creativity and Creative Education	**Creative Mathematics Education**	**Professional Development**
Creative potential (Runco, 2007) • Person • Press (environment) • Process Creative problem (Cropley & Cropley, 2009) Little-c creativity (Craft, 2001) Flow (Csikszentmihalyi, 1997) Componential model of creativity (Amabile, 2013) Environment for creative learning	Creative mathematics learning Working like real mathematicians Creative problem solving Art, culture and patterns Mathematics and society Evaluating creative mathematics learning • Formative assessment	Design framework (Loucks-Horsley et al., 2010) Socio-cultural model (Jaworski, 2011) Professionalism Collaborative professionalism Adult learning Successful professional development Coaching

The theoretical framework for this study is based on three different theories of knowledge. The first is general theory of creativity and creative education. The second is theories of creative mathematics which are applicable to this thesis. The third comprises theories related to PD and developing an effective PD for teachers of mathematics. These theories have been used to plan and develop a PD program on creative mathematics education and are also used when analyzing the program. The research questions look at how the teachers who took part in the PD program report its influence on their pedagogy and classroom practice and what can be learned from the active development of the PD program. To answer the research questions, the theoretical framework is used for reflection.

Theories of creativity and creative mathematics education were brought into a PD program with the purpose of educating teachers on these concepts and providing them with experiences of creative mathematics. The goal was to show teachers ways to be creative in mathematics that they in turn would be able to bring into their own classrooms, allowing their students to be creative in their mathematics learning. The themes that

were brought into PD seminars include specific ways of mathematizing, such as working like real mathematicians, creative problem solving and engaging with patterns, as well as general ideas for creativity in the classroom such as mathematical conversation, formative assessment, and collaboration.

The PD program was planned and developed in accordance with theories of effective PD, with the teachers partaking in seminars that included joint mathematical work, discussions on mathematics and its teaching, and support for them to bring creativity into their mathematics classrooms. Although I brought theories of creativity and creative mathematics education into the PD program, it was clear from the beginning that the PD program was a collaborative construct based on the teachers' needs and participation. This type of collaboration is important in all PD endeavours and for action research in general (Desimone, 2015; Efron & Ravid, 2020; Loucks-Horsley et al., 2010; Mertler, 2020; Zepeda, 2019).

Although some of the theoretical framework was in place before the research started, it was further developed through the cyclical analysis of Mills' (2018) dialectical action research spiral. This was particularly true for theories of collaborative professionalism and coaching which proved to be fundamental for the PD program. Coaching became a crucial part of supporting the teachers in bringing their learning from PD seminars into their classroom. To foster creativity in their mathematics classroom, the teachers needed and relied on personal support and collaboration.

As explained above, this framework was used in the development of the PD program as well as when reflecting on the results of the research. No research happens in a vacuum and although the data analysis for this action research was done inductively, the results are reflected on in light of previous research. The data was not analyzed through one particular lens but theories of creativity, creative mathematics education and PD were used to reflect on the results. As explained, the participants or stakeholders' words built the "theory" and results for this study, which is why this approach was chosen.

This chapter is divided into three main sections, based on the different theories of knowledge of the theoretical framework. The first section presents theories and definitions of creativity in general and creative education, as these are relevant for this research. The second section details theories of creative mathematics education which are crucial to this study. The third section explains how theories of PD were used in developing the program. These chapters explain theories that were important in developing the PD program and subsequently for analyzing the data.

2.1 Creativity and Creative Education

Creativity is fundamental for this research. This research follows a PD program on creative mathematics education with the goal of improving and informing practice. For this purpose, it was important to explore different theories of creativity and analyze which of these are important for this research. In this section I explain how theories of creativity and a creative learning environment connect to this research. The learning environment is particularly important, as the research is focused on classroom practices.

2.1.1 Creativity Research and Definitions

Long before contemplating the concept of creativity, early humans must have needed creative solutions in order to survive. The pyramids and the seven wonders of the world are testaments to man-made creations that required an immense amount of imagination and organization. This shows that humans have long held the ability of creating something new, whether it be new knowledge, tools, buildings, or artwork. The word creativity, however, is historically new and the research field newer still (Glăveanu & Kaufman, 2019). The focus we now see on creativity in books, magazines, political documents, and educational regulations is recent.

Historically, creativity was seen as coming from a divinity and was connected to mythology (Glăveanu & Kaufman, 2019; Kettler et al., 2018; Pope, 2011; Simonton, 2001). Creativity was often seen as something obscure or even dangerous (Glăveanu & Kaufman, 2019). After Christ, Western societies saw creativity in the story of creation and believed that those who created were working for God (Albert & Runco, 1999; Glăveanu & Kaufman, 2019; Kettler et al., 2018). Eastern views were based on creativity as repetition, imitation, equilibrium and rules (Albert & Runco, 1999). The Western views changed little until the Renaissance. The idea emerged that creativity could come from the person rather than being divinely orchestrated (Kettler et al., 2018). This view took time to gain a firm footing and it was not until the Enlightenment that it had become established.

Creativity research was neglected for a long time and there have been many hurdles for researching the phenomena (Glăveanu & Kaufman, 2019; Pope, 2011; Simonton, 2001; Sternberg & Lubart, 1999). A clear, untangled definition has been lacking and creativity was thought to be too obscure, supernatural, or spontaneous to be researched within the scientific method. Furthermore, creativity has been seen differently from other concepts in psychology and is often viewed from an economic point of view, while psychological research is less pursued (Jones, 2011).

When researching creativity, it is important to know what is being researched. Creativity can be seen as individual or as socially constructed, as the unique contribution of a few or as a trait everyone possesses. Creativity can be researched as qualitative or quantitative and as domain specific or across domains (Kaufman & Sternberg, 2007; Mayer, 1999). Early research of creativity often focused on the genius and the creative individual (Glăveanu & Kaufman, 2019). In the early 20th century the foundation of creativity as a multifaceted phenomenon was laid with a gradual focus on creativity research (Mayer, 1999). In the latter half of the 20th century, many questions were raised with few answers (Mayer, 1999). In the 21st century, research on creativity has increased with more empirical results (Runco, 2007; Simonton, 2017).

Rhodes (1961) wrote about how creativity could be researched from different perspectives. These are often called the 4P's and constitute person or personality; products; processes; and press. Press or pressure is a word he uses for the environment and how that can influence creativity. The 4P model is still being used to describe the different ways creativity can be researched (Kaufman & Sternberg, 2007; Mayer, 1999). Simonton (1995) later added persuasion as an influential factor and Runco (2007) added creative potential as opposed to creative performance. Runco sets out a hierarchical structure with person, process and press under creative potential and product, persuasion, and interactions under creative performance (Figure 1).

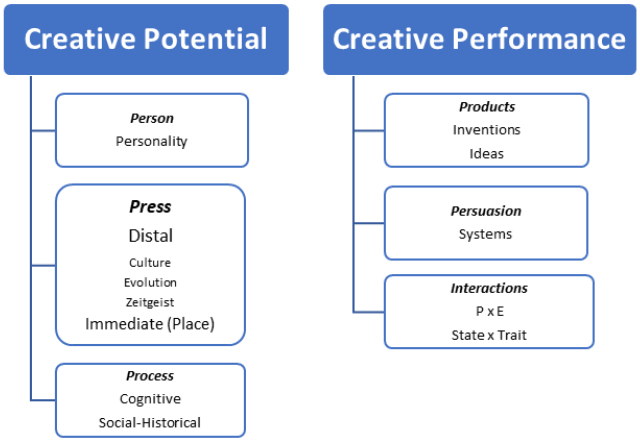

Figure 1 The Hierarchical Framework for the Study of Creativity (Runco, 2007)

Creative potential is an important aspect of creativity (Runco, 2016). By only focusing on the creative product, a creative potential that has not yet been manifested can be missed. Therefore, an important factor in educating for creativity is to realize that all students have creative potential and research how that can be developed. In this connection, Runco's (2007) hierarchical structure for creativity can be beneficial. To develop children's creativity, we need to support them as individuals (persons) to get skills to take part in the creative process and create an environment (press) that does so. These same children may then later show some kind of creative performance, resulting in products, persuasion, or an interaction of the two.

Wallas' (1926) model of creativity is to this day widely used when researching the creative process. The model is based on four stages: preparation, incubation, illumination, and verification. In the first stage, preparation, the person works on the project as she can. In the second stage, incubation, she puts it aside and does other things. Then in the third stage, illumination, the solution comes to her after she has stayed with it for a while. In the verification stage she seeks to confirm and work with what came to her. Although Wallas' model is still widely used when researching the creative process, it is however clear that the creative process is not always linear and can be seen as cyclic, while recursion or recycling through the stages is often added (Wong & Siu, 2011).

Cropley and Cropley (2009) have expanded the 4P model and added problem and phase. Phase is based on Wallas' (1926) model while Cropley and Cropley have added the phases information, communication and validation. Problem is the task to be completed. Person is seen in terms of thinking processes, personality, and motivation. Process is seen as cognition and press is based on social factors such as roles, models, and norms. Product is differentiated into relevance, effectiveness, novelty elegance and generalizability. Phase is seen as an aid to explain better how the different aspects of creativity are seen and reflected in the different stages of the creative process.

Although creativity research has grown considerably in the last few decades, there has been no consensus on a universal definition of creativity, which can be problematic when researching the phenomena (Kaufman & Sternberg, 2007; Mayer, 1999; Simonton, 2017). Most definitions of creativity consist of coming up with something different, new, or innovative; of high quality and appropriate to each task (Glăveanu & Kaufman, 2019; Kaufman & Sternberg, 2007; Lumsden, 1999; Simonton,

2017). Many definitions emphasize usefulness while others focus on value, adaptability, or intense work (Boden, 1999; Gruber & Wallace, 1999; Martindale, 1999; Simonton, 2017; Sternberg & Lubart, 1999).

Plucker, Beghetto and Dow (2004) formulated an elaborate definition that many use: "Creativity is the interaction among *aptitude, process and environment* by which an individual or group produces a *perceptible product* that is both *novel and useful* as defined within a *social context*" (p. 90). Yet some argue that a simpler definition is just as good: "Creativity is the process of coming up with a good idea" (Smith & Smith, 2017, p. 31).

However, focusing on the usefulness or value of a creative idea can be a hinderance (Harris, 2014; Weisberg, 2015). Although innovation needs to be of value and is often connected with business and profit, creativity does not need to be (Harris, 2014). Including value or usefulness in the definition of creativity can confound the research process and is always a subjective judgement that can result in creative work being disvalued (Harris, 2014; Weisberg, 2015).

Vygotsky (2004) wrote about creativity and about how the human mind can both reproduce past experiences and create something new, as well as how these two functions work together. He describes the process as both an external and an internal one, based on our own experiences. He emphasizes that creativity is given not only to a few outstanding individuals but also to the ordinary person and takes children's play as an example of that. Children create using disassociation, where parts from different experiences are reworked through imagination and fantasy. Vygotsky explained the creative act:

> Any human act that gives rise to something new is referred to as a creative act, regardless of whether what is created is a physical object or some mental or emotional construct that lives within the person who created it and is known only to him. (Vygotsky, 1978, p. 7)

Vygotsky focuses on creativity as a trait that all humans possess. Closely related to his ideas are concepts coined by Craft (2001) that are referred to as little-c and big-C creativity. The big-C creativity or high creativity refers to when a person is creative on a universal scale, such as when a person comes up with something that changes a certain field permanently. Little-c creativity or everyday creativity is focused on creative everyday solutions that we all come up with. Kaufman and Beghetto (2009) have expanded this

to a 4-C model that includes mini-c creativity and Pro-c creativity. They define mini-c as a novel and personally meaningful interpretation of experiences, actions, and events and believe it to be important in education. They also added the Pro-c category to describe people who are professional creators but have not reached eminent status.

Flow is an aspect often connected to student creativity, which has been researched by Csikszentmihalyi (1997). Csikszentmihalyi has interviewed creative individuals in order to research and describe flow. In flow, an individual is enmeshed in what they are doing, without thinking either about it or about something else. This is often true for children's play but is also seen in adults, such as athletes and artists.

Amabile (2013) describes what she calls the componential theory of creativity, stating that three internal components and one external component are needed for creativity. The internal components are domain relevant skills, creativity-relevant processes, and task motivation. The external factor is a social environment that supports creativity.

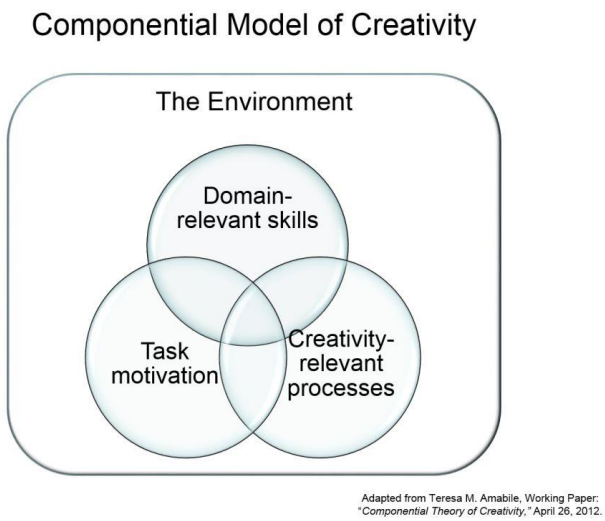

Figure 2 Componential Model of Creativity (Amabile, 2013)

Although being creative within a specific domain requires specific creativity processes or skills, a person's skills within that domain can also influence how creative they can be within that field. A lot remains unclear about how creativity transfers across domains and it is clear that creativity is not always transferable between domains (Baer, 2013; Tinio & Barbot, 2017). A person can be creative with one task but not with a different task,

and many people are creatively talented within one or more domains but lack the ability to be creative in other domains (Tinio & Barbot, 2017). This is sometimes connected to a person's general skills in a certain domain, but it is also pivotal to have creative experiences within a specific field to be able to develop creativity relevant processes.

Creativity relevant processes include personality traits such as independence, risk-taking, adaptability, self-discipline, tolerance for ambiguity and traits of divergent thinking. Divergent thinking refers to four characteristics of creativity described by Torrance (1974). The characteristics are fluency, the multitude of ideas; flexibility, to be able to see situations from different perspectives; originality, to come up with something new; and elaboration, to adapt and expand on an idea. Divergent thinking, which is believed to be an important component in creativity and an indicator of creative potential, can be developed through brainstorming and other methods (Baer & Kaufman, 2012; Runco & Acar, 2012; Runco & Pagnani, 2011). Possibility thinking is another creativity-relevant process and includes self-determination and direction, innovation, action, development, depth, risk;, being imaginative;, posing questions, and play (Craft, 2011b).

Related to self-determination is task motivation. Amabile (1989) writes:

> Although the major hallmark of intrinsic motivation is interest, another is *competence.* Children will seek out activities, persist longer, and enjoy them more if they can get a feeling of mastering something on their own. And when children are told or shown that they have performed well on a challenging task, their intrinsic motivation increases. A second important aspect of intrinsic motivation is *self-determination*, or the feeling that you are working on something for your *own* reasons and not someone else's. Not only do children need to feel that they are succeeding at something, but they also need to feel that it is their choice to do it. (p. 55)

This shows how the three components connect for a creative experience. The fourth component is environment. An accepting environment with reasonable rules and flexibility has been shown to nurture creativity in children (Runco & Pagnani, 2011). Influential environmental factors include physical surroundings, family upbringing, school experience, workplace environment, cultural tradition, and historical milieu. The next section focuses on how teachers can create an environment that fosters creativity.

2.1.2 Environment for Creative Learning

When discussing creative education there is often a distinction made between creative teaching and teaching for creativity (Grohman & Szmidt, 2013; Jeffrey & Craft, 2004; Starko, 2018). Creative teaching means that a teacher is creative and uses creative methods when presenting material to students (Craft, 2011a; Snyder, Gregerson, & Kaufman, 2013). This does not necessarily mean that the students are given the opportunity to be creative themselves (Starko, 2018). However, there is a connection between the two and creative teachers often teach for creativity, so it is best if both can be fostered in the classroom (Jeffrey & Craft, 2004; Kettler et al., 2018). This has sometimes been called creative learning (Craft, 2011a).

Often teachers want to educate their students for creativity but seem to lack the means or ability to do so (Daniels, 2013; Grohman & Szmidt, 2013; Kettler et al., 2018; Novotna & Sarrazy, 2014; Ranjan & Gabora, 2013). Teachers are insecure about how to promote creativity, feel there is no time or feel they are themselves not creative enough (Daniels, 2013). Standardized tests, emphasis on correct answers, and teaching for traditional tests can hinder creativity in schools (Bloom, 2014; Ranjan & Gabora, 2013; Runco, 2007; Snyder et al., 2013; Sternberg, 2017). Creativity enhancement is often viewed as extra-curricular and not part of the everyday subject matter (Beghetto, 2013). This supports the idea that creativity is something that can be turned on and off and is not a part of all aspects of school and life.

Sometimes the responsibility of developing creativity is placed mainly within gifted education, which gives the impression that creativity only belongs to a few students instead of all of them (Beghetto, 2013). However, creativity learning has been shown to be important to the diverse learning group and is therefore of vital importance in terms of supporting teachers to teach for creativity (Starko, 2018).

It is important for teachers and schools to create a positive environment for creative learning (Lucas, 2001; NACCCE, 1999; Starko, 2018; Sternberg & Williams, 1996). A creative teacher can be a good example for students on how to be creative (Cropley, 2001; Fautley & Savage, 2007; NACCCE, 1999; Simonton, 2013; Snyder et al., 2013; Sternberg & Williams, 1996). Creative teachers need not be artists or inventors but it helps if they are curious, bold and open (Grohman & Szmidt, 2013). They need to respect their students, encourage them in their studies, follow their interests and make room for diverse learning (Lucas, 2001). Creativity can be encouraged by paying attention to feelings, asking open questions and allowing room for

the new (Lucas, 2001). Teachers who are kind, listen and pay attention to their students' efforts are usually better at creating a classroom that allows creativity (Grohman & Szmidt, 2013).

Creativity can be supported in various ways and it is possible to be creative in more than one way (Snyder et al., 2013; Sternberg & Williams, 1996). Teachers can support creative learning by helping students accept ambiguity and to break out of black-and-white thinking (Levenson, 2014). It is important that students have the option of taking creative risks, exploring creative ideas and developing their talents as part of everyday classroom activities (Ranjan & Gabora, 2013; Sternberg & Williams, 1996). Teachers do well to encourage playfulness, flexibility, and collaboration, while an encouraging environment can help students grow and learn to be more creative (Baer, 2013; Daniels, 2013; Ranjan & Gabora, 2013). Many teachers emphasize the importance of asking questions and encourage students to doubt, define problems, search for ideas and work across the curriculum (Fautley & Savage, 2007; NACCCE, 1999; Sternberg & Williams, 1996).

An important factor in creativity education, but one that is often forgotten, is allowing enough time for the creative process (Fautley & Savage, 2007; NACCCE, 1999; Ranjan & Gabora, 2013; Sternberg & Williams, 1996). Another aspect is to know that mistakes are part of the process. When teachers and schools focus on the right way to do things and correct mistakes in a harsh way, it can make students afraid of taking chances in their learning (Ranjan & Gabora, 2013; Sternberg & Williams, 1996). With creativity there are inevitable obstacles and it is important to encourage students to be resilient in the face of adversity (NACCCE, 1999; Starko, 2018). Self-efficacy can be important for students developing creativity (Starko, 2018; Sternberg & Williams, 1996). Teachers can support their students with encouragement and help them believe in their own creativity (NACCCE, 1999; Starko, 2018; Sternberg & Williams, 1996).

Although creativity cannot be forced, setting up a classroom environment that encourages diversity, interactions and instability can allow for creativity to emerge even when not planned for. Beghetto (2013, p. 5) writes: "Creative micromoments are brief, surprising moments of creative potential that emerge in everyday routines, habits and planned experiences." The micromoments happen when the lesson deviates from the original plan and when a student shares some unexpected idea. When teachers are faced with these moments, they have two options: Trying to understand the student or having the student understand their original plan. In those moments lies an opportunity for creativity. Choosing the

option of trying to understand what the student brought up can result in a dead end but also in development of creative potential. In choosing to try to have the student understand what the teachers originally planned, there is also a risk: the risk of missing out on following up on unexpected ideas and the risk of discouraging the students.

As explained in Amabile's (2013) componential model, intrinsic motivation can be conducive to creative success (Amabile, 1996). Intrinsic motivation can, however, be challenging to promote (Baer & Kaufman, 2012). Although some extrinsic motivators can work in a positive way on intrinsic motivation, extrinsic motivators commonly affect intrinsic motivation in a negative way. Intrinsic motivation can be supported by offering choice, showing interesting aspects of tasks, and planning towards interests without the constant pressure of tests or evaluation of students' work (Baer, 2013). Students are often afraid to produce something they have invested in on a personal level if they expect to be judged harshly for it (Ranjan & Gabora, 2013).

Collaboration and communication are important for creativity education. Learning is a social process and it is important that students can take part in a learning environment that encourages that (Moran & John-Steiner, 2003). This is reflected in Vygotsky's (1978) socio-cultural theory. Students use internalization to create their own personal meaning out of the learning content, based on the external stimuli. This is closely related to the concept zone of proximal development and how children can learn more in a socially encouraging setting than by themselves (Vygotsky, 1978). When teaching for creativity, communication also opens the possibility of externalization when students bring something to the collective learning process which is a crucial aspect. Students need to be active learners and participants in a creative classroom while teachers need to be mindful to hear what students bring to the table (Beghetto, 2013; Moran & John-Steiner, 2003).

Attitude is important in developing creative potential (Grohman & Szmidt, 2013). Whether a teacher holds a negative or a positive attitude towards creativity can influence students (Grohman & Szmidt, 2013). If a teacher has the attitude that creativity is within everyone's grasp and is an important quality to nurture, this can have a very different effect from believing that it is not for everyone and is possibly a waste of time. This is closely related to the concept of the growth mindset that has in recent years been connected with educational success (Clarke, 2014; Dweck, 2016). One way to work with teachers' attitudes is to have them do

assignments related to finding their own and their students' creative attributes, in order to notice and nurture creative behaviour in themselves and study creative potential in their students (Daniels, 2013). Working on teachers' attitudes can influence them to show students' creativity in a positive light (Grohman & Szmidt, 2013).

Many general teachers have many different subjects that they teach and although they teach for creativity in one or more of them, it does not follow that they teach for creativity in all of them. Teachers are perhaps very used to teaching language and history in a very creative manner but teach mathematics in a very restricted, uncreative way. Teachers need to have training and experiences of creative learning within the different subjects they teach. They need to learn ways to promote creativity across the curriculum and create an environment that fosters creativity in all school subjects and projects.

2.1.3 Summary

The goal of this thesis is to explain how teachers experience the influence of a PD program on creative mathematics on their classroom practice and student learning. The theories that were presented in this section on creativity and environment for creative learning are important for the PD program. These theories are presented to the teachers in the PD seminars and connect to theories of creative mathematics education.

Although creativity is a popular concept today, creativity research is recent and no consensus exists on its definition. In this chapter I have presented the different aspects from which creativity can be researched, along with other theories that relate to the creative process and tie in with education. Runco's hierarchical structure for creativity research (Figure 1) shows the different ways in which creativity can be researched. In terms of the 4P model, the focus for this research is on the creative potential of students, the creative classroom environment that the teachers create (press) and how that is inducive to each student's (person) creative learning process. The focus is not on creative performance per se, but some teachers in the study notice a change in student ideas and inventions (products).

Cropley and Cropley's (2009) notion of problem is important for this research as mathematical creativity is often focused on problem solving. The teachers in the study explain what they notice on a personal level, such as thinking processes and motivation, as well as social factors in the classroom. The creative process is not researched in this study, which is

based on one specific model. Instead, the data is used to share the story of how the teachers developed through the PD program and which effects they noticed for their students and in their mathematics classroom.

Some use the 4-C model to help teachers understand how creativity can be experienced in the classroom (Daniels, 2013; Kettler et al., 2018). This was done for the teachers in the PD program, but for the purpose of this research the focus is on little-c or mini-c creativity. This is in accordance with the idea that all students can be creative in their own mathematics learning. The next section focuses on theories of creativity in education which were central for the PD program and this research.

2.2 Creative Mathematics Education

For this educational action research, the theoretical framework is closely connected to the practical nature of the study. The theories presented in this section focus on mathematical creativity and fostering creative mathematics learning. They lie at the heart of the PD program and are the focus of the themes that are presented and explored in each of the seminars.

The first subsection focuses on mathematical creativity and is based on presenting how creativity manifests within the field of mathematics. This is done to explain how creative the subject of mathematics is and ties in with how the teachers in the PD program are presented to the difference between school mathematics and the work of mathematicians and other scientists.

The second subsection focuses on fostering creative mathematics learning and is based on theories of practical ways of developing creativity in the mathematics classroom. The focus is on different ways of supporting creativity in the mathematics classroom and is directly related to how the teachers worked in PD seminars and which methods they were introduced to for their own classroom practice.

2.2.1 Mathematical Creativity

> The genesis of mathematical creation is a problem which should intensely interest the psychologist. It is the activity in which the human mind seems to take least from the outside world, in which it acts or seems to act only of itself and on itself, so that in studying the procedure of geometric thought we may hope to reach what is most essential in man's mind. (Poincaré, 1910)

Creative mathematics is a concept that raises a lot of eyebrows in society at large. Many do not see mathematics as creative at all. Or if people see mathematics as creative, they generally see it in connection to the visual arts. Many are aware of how mathematics plays a part in many forms of art and see that as the only creative side of mathematics. The connection of art and mathematics can be seen in painting, sculpting, music, writing, patterns and fractals (Sharp, 2012). Besides the connection to art, it is a common belief that mathematics is not a creative subject (Ambrose, 2017). Most mathematicians disagree with this notion and find that they combine creative and logical thinking in their working processes (Pehkonen, 1997).

To better understand the concept of creative mathematics, it is helpful to first think about what mathematics is. Schoenfeld (1992) characterizes the mathematical enterprise as follows:

> Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns—systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics"). The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. However, being trained in the use of these tools no more means that one thinks mathematically than knowing how to use shop tools makes one a craftsman. Learning to think mathematically means (a) developing a mathematical point of view—valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure—mathematical sense-making. (p. 1)

He argues that the mathematical enterprise is a social activity, based on experimentation and the search for unknown patterns and principles, and that in order to be successful it is imperative to develop a mathematical point of view and use mathematical tools to make sense within theoretical or real-world mathematics.

Ervynck (1991) notes that in general mathematicians are not particularly interested in examining how they think and research on creativity in

mathematics is uncommon. For instance, little research was carried out on mathematical creativity in the 20th century within the fields of psychology, mathematics and education (Leikin & Pitta-Pantazi, 2013; Sriraman, Haavold, & Lee, 2013). However, Ervynck (1991) describes creativity as a crucial part of the mathematical process:

> Creativity plays a vital role in the full cycle of advanced mathematical thinking. It contributes in the first stages of development of a mathematical theory when possible conjectures are framed as a result of the individual's experience of the mathematical context; it is [*sic*] also plays a part in the formulation of the final edifice of mathematics as a deductive system with clearly defined axioms and formally constructed proofs. It is an essential factor in research mathematics when new ideas are formulated in a manner previously unknown to the mathematical community (p. 42).

The few mathematicians who have studied their own thinking generally see creativity in their work and it appears that Wallas' (1926) model of creativity can be used to reflect on their process. In 1910, Poincaré wrote the article Mathematical Creation and described the experience of illumination, which is similar to Wallas' (1926) model for creativity. Hadamard (1945) describes this same experience being seen in mathematicians and scientists.

Sriraman (2009) researched how Wallas' (1926) model for creativity can be used when researching mathematical creativity. He researched five mathematicians and looked at how the model fitted their work. He found that the mathematicians went through a process of preparation, incubation, illumination, and verification. They would often work on a problem intensely, set it aside for a time and come back to it after some sort of illumination, at which point they would verify and write up the work. They mostly thought that the illumination was a result of their work, although one perceived it as almost God given, and another saw it as a coincidence. Liljedahl (2013) looked at how mathematicians described illumination as part of their work and as an affective experience. This is similar to the description given by Poincaré (1910) and Hadamard (1945).

There has been a stronger research focus on mathematical creativity in the 21st century with workgroups, books and journals devoted to the subject. There is however no consensus on a definition of creative mathematics and a multitude of different definitions exists (Sriraman, 2017;

Sriraman et al., 2013). Sriraman (2008) points out how the trait of usefulness that some of the general definitions include does not apply to mathematical creativity. As an example, he points out Andrew Wiles' proof of Fermat's last theorem that is an important contribution in the mathematical community while not useful per se. Some of these mathematical creativity definitions focus on little-c while other focus on Big-C creativity (Sriraman et al., 2013).

Sriraman and Liljedahl (2006) discuss a possible definition focused on creativity in mathematics for students at the school level. They define mathematical creativity at the school levels as:

> 1. the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or
> 2. the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle (p. 19)

This definition focuses on the creative problem-solving process which is important for school mathematics. It is however unclear whether students' "insightful solutions" or "new questions" are new or insightful only for them as students or whether they can be interpreted in a broader sense.

Nadjafikhah et al. (2012) came up with the definition:

> According to some of [sic] definitions, a creative act in mathematics could consist of: creating a new fruitful mathematical concept; discovering an unknown relation; and reorganizing the structure of a mathematical theory. Mathematical creativity is not only related to the novel work of mathematicians but also discovering something not already known by one even if the result is hitherto known to others.

This definition attempts to include both little-c and big-C creativity and explains which mathematical processes can be seen in the creative process.

For this research, the focus is on little-c or mini-c creativity as mathematics students are not expected to come up with a professional or a world-altering aspect of mathematics. They create new knowledge and connections that hold meaning for their own learning. Usefulness or value

are not considered crucial components in this aspect; rather, the focus is on fostering creative potential and the creative process without an expectation of a creative performance or product. The goal is to support the participating teachers with creative mathematics in their classroom. The following subsection takes a closer look at what that entails.

2.2.2 Fostering Creative Mathematics Learning

Although many teachers believe that creativity is important for mathematics education, they often lack the means or ability to foster creativity in the mathematics classroom as they may not have had prior experience or adequate education to follow up on this important goal (Haavold & Birkeland, 2017; Shriki, 2010; Sternberg, 2017). There seems to be a connection between teachers' ability to teach for creativity, their mathematical background (domain-relevant skills) and their views and experience with creativity in mathematics (creativity-relevant processes) (Haavold & Birkeland, 2017; Lev-Zamir & Leikin, 2013). In that way, Amabile's (2013) componential model can be applied to mathematics education.

There is an important need to teach and provide pre- and in-service teachers with opportunities to see how creative mathematics can be practised (Sternberg, 2017). In order to support students with creativity in mathematics, teachers need to have had creative experiences in their own mathematics learning (Nadjafikhah et al., 2012; Sheffield, 2013; Shriki, 2010). In this section, some concrete examples of how to foster creativity in mathematics are presented; by bringing these into a PD program for teachers, the teachers get an opportunity to learn about creative mathematics by partaking in creative endeavours themselves. The following sub-sections focus on themes that were presented and explored in the seminars of the PD program, which are better outlined in the methodology chapter.

The first subsection focuses on creative mathematics in general, which entails the theme of the first PD seminar along with the theories presented previously on creativity and a creative learning environment. The next subsection explores how working like a real mathematician can foster creative mathematics learning, which is the theme for the third PD seminar. The following subsection focuses on how problem solving supports creative mathematics learning, which is the theme for the second PD seminar. After that comes a subsection on how geometrical patterns, art and culture can help students be creative in their mathematics learning – this is the theme

for the fourth PD seminar. Next there is a subsection that focuses on mathematics and society, which is the theme for PD seminar seven. This is followed by a subsection that focuses on evaluating creative learning through formative assessment, which was the focus of PD seminars five and six. The specific PD seminars are detailed in Table 2 in the methodology chapter.

2.2.2.1 Creative Mathematics Learning

Teachers are vital in facilitating a creative approach in the mathematics classroom (Panaoura & Panaoura, 2014). However, some teachers tend to underestimate the value of creativity in mathematics education (Panaoura & Panaoura, 2014). Despite some teachers acknowledging the importance of creativity in mathematics, many do not include it in their teaching (Kattou, Kontoyianni, & Christou, 2009). Teachers tend to view creativity in mathematics as more based on teaching practices, whereas mathematicians focus on the nature of mathematics and its creative aspects (Yazgan-Sag & Emre-Akdogan, 2016).

Mathematical abilities of students include fluency, flexibility, originality, elaboration, visualization, imagination, and intuition (Kettler et al., 2018). For students, extending boundaries, transformation, evaluation, sensitivity, and the ability to take another perspective are important. Convergence, affect, and ambiguity can also be important (Beghetto & Schreiber, 2017; Mann, Chamberlin, & Graefe, 2017; Tan & Sriraman, 2017). Ways to support these traits include allowing students to generate multiple ideas and approaching problems in various ways (Kettler et al., 2018). Allowing students to develop their solutions, the use of illustrations, problem posing and viewing problems from different angles fosters creativity (Kettler et al., 2018). It is important for teachers to encourage students to doubt and try out different ideas, and then to evaluate those throughout the creative process (Sheffield, 2013).

To support students' mathematical abilities, teachers should ideally create an intellectually safe learning environment that encourages discovery, ambiguity, and allows students to take risks and make mistakes (Luria, Sriraman, & Kaufman, 2017). Teachers can help students by offering an environment that provides time and space for developing understanding and experiencing a flow of thought (Nadjafikhah et al., 2012; Sriraman, 2009). The creative process can be helped by providing students with projects that pose a mathematical struggle, together with learning environments where they are encouraged to view problems from different

aspects, reflect on their own ideas and try different methods (Nadjafikhah et al., 2012; Robinson & Koshy, 2004).

Student-centred instruction focuses on addressing a student's concerns with the goal of incorporating concept-based learning (Luria et al., 2017). Moving from procedural-based learning to concept-based learning is crucial in developing student creativity and helping students become active learners who can analyze and generalize (Luria et al., 2017). Creativity provides better knowledge of concepts and is fundamental to conceptual learning in mathematics (Yazgan-Sag & Emre-Akdogan, 2016).

Students benefit from encouragement from teachers and schools to be creative in their mathematics learning (Sternberg, 2017). They need to be provided with opportunities to solve problems, come up with their own ideas and develop divergent solutions (Nadjafikhah et al., 2012). Students need to understand and represent problems, discern patterns, make generalizations, draw analogies and connections, see old problems from different perspectives and find personal solutions to existing problems while evaluating and reflecting on these solutions (Yazgan-Sag & Emre-Akdogan, 2016).

Many believe that focusing on creative mathematics means abandoning all arithmetic, but that is far from the case. Rather, it means abandoning the idea that students should be taught algorithms that they use to mindlessly solve problems that make no sense to them and to instill to students that mathematics is more than just arithmetic. Ways to helps students become more creative with arithmetic include showing them how to use number systems with other numbers than 10, doing modular arithmetic, using number series to spark interest and conversation, and having students create imaginary number operations other than the standard addition, subtraction, multiplication, and division (Sternberg, 2017). As a result, students can use creative learning to better understand numbers, our base system, operations, and other mathematical concepts, while honing both their mathematical and creativity skills.

Motivation is a crucial aspect for promoting creativity, as previously discussed (Amabile, 1989, 1996, 2013). Many have a history of feeling discouraged or even hating math, so working with children on making mathematics more fun, interactive and enjoyable is essential (Boaler, 2016; Pound & Lee, 2015). Ways to do this include making mathematics real by connecting it both to children's experience and to everyday life, using structured materials, hands-on learning, puzzles and games (Pound & Lee, 2015).

Creativity can be fostered through art, stories, music, dance, architecture, play, outdoor work and working cross-curriculum (Pound & Lee, 2015; Sharp, 2012). Rhymes and rhythm can be used to promote creativity in mathematics, particularly for younger students (Karwowski, 2017). Showing students role models in science or mathematics can open them to the creative nature of mathematics (Nadjafikhah et al., 2012). The following section explores how mathematical creativity can be fostered by allowing students to work like real mathematicians.

2.2.2.2 Working like Real Mathematicians

School mathematics generally differs from the work of mathematicians (Sheffield, 2013; Sternberg, 2017). Students work on well-defined problems, searching for one correct answer using known algorithms and rules (Khalid et al., 2020; Parker, 1993). The qualities of being quiet and fast are awarded with little room for cooperation or communication (Nadjafikhah et al., 2012). Mathematicians, however, are rewarded for working intensively on challenging problems and creating new knowledge (Sternberg, 2017). They work for an extended period of time on projects with no tangible solutions, they communicate and cooperate as needed, and are rewarded for resilience and finding connections in a messy world (Parker, 1993).

Mathematics is an ever-changing science and it is important for students to learn how creative it can be (Sharp, 2012). This can be done by presenting students with examples of how mathematicians work and then moving their learning closer to that process (Sriraman, Yaftian, & Lee, 2011). This can give students an idea of how the mathematical process can be one of different attempts and mistakes with no magic solutions or simple answers. Seeing how mathematicians work on the same project for an extended period of time can also help students break away from the idea that mathematics should consist of quick, short problems with one correct answer. A classroom that allows for mistakes, experiments and trying to create your own connections can be created. This ties in with what has been written about the creative process, internalization, and divergent thinking.

Presenting students to the work of mathematicians and other scientists and having them explore related tasks can be reflected in the 4-C model (Kaufman & Beghetto, 2009). Students can be shown Big-C work that has permanently changed the field of mathematics and science and successful Pro-C creations done by professional mathematicians or scientists. Students can work in ways similar to professional mathematicians, scientists or

engineers and foster their little-c creativity potential (Kettler et al., 2018). They come to their own learning through a mini-c creativity process. Students can learn how creative mathematics really is.

Shriki (2010) researched how prospective mathematics teachers developed their understanding of the creative mathematics process by working on activities that resembled the work of real mathematicians. They invented their own geometrical shape, worked with its properties for some time, developed theories and rules that applied to the concept and presented it to others. With this, they also reflected on what is entailed in a "good teacher". Shriki found that the student teachers became more conscious of creativity in mathematics, developed mathematical and pedagogical skills, and saw the nature of mathematics in a new light. After an initial struggle, many enjoyed the work and felt that their views on how mathematicians work and what constitutes a good teacher changed. They generally felt better equipped to promote creativity with future students. This type of task can be used with in-service teachers as well.

2.2.2.3 Creative Problem Solving

> Setting students ill-posed and open-ended problems and allowing them a prolonged period of engagement and independence in their way of working are necessary to stimulate their mathematical creativity. (Grégoire, 2016, p. 25)

Creative problem-solving has been widely used to promote creativity across the curriculum (Cropley, 2001). This process is sometimes divided into six different steps that can happen in any order (Baer & Kaufman, 2012). These steps are mess-finding, data-finding, problem-finding, idea-finding, solution-finding, and action planning. Mess-finding entails exploring a situation and looking at what needs to be done; data-finding means gathering all information and data that you may need; problem-finding means exploring the different problems presenting themselves within the situation and choosing a focus; idea-finding means brainstorming on how to solve the chosen problem; and solution-finding means looking for a solution for the specific problem. Action planning means that you have an overview over the process from beginning to end and can analyze how what you are doing can fit.

Problem-solving or open-ended problems have been crucial in mathematics education for the last decades (Kettler et al., 2018). The mathematician Polya (1981) writes:

> Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. Solving problems is the specific achievement of intelligence, and intelligence is the specific gift of mankind: solving problems can be regarded as the most characteristically human activity. (p. ix)

Polya describes problem-solving as swimming, skiing, or playing the piano. He says that even if you can give students tools and ideas, ultimately if you want to swim you have to go into the water and if you want to solve a problem you have to do it.

Problem-solving is a way to promote creativity in mathematics learning (Khalid et al., 2020; Nadjafikhah et al., 2012). Problem-solving can help students approach tasks in a new way and connect knowledge in a creative way (English & Sriraman, 2010). Problem-solving develops cognitive skills, fosters creativity, is part of the mathematical process and promotes interest (Pehkonen, 1997). Many teachers are more focused on how problem-solving develops cognitive skills than it being a way to foster creativity (Pehkonen, 1997). However, creative problem solving has the role of enhancing students' problem solving skills as well as their creativity (Khalid et al., 2020).

Sriraman, Yaftian and Lee (2011) researched how Wallas' (1926) model can be reflected in the problem-solving process but that model can be quite linear. Another model developed by Sheffield (2013) describes a more flexible, creative problem-solving process (Figure 3) and assumes that the process is fluid with five aspects: creating, relating, investigating, communicating, and evaluating.

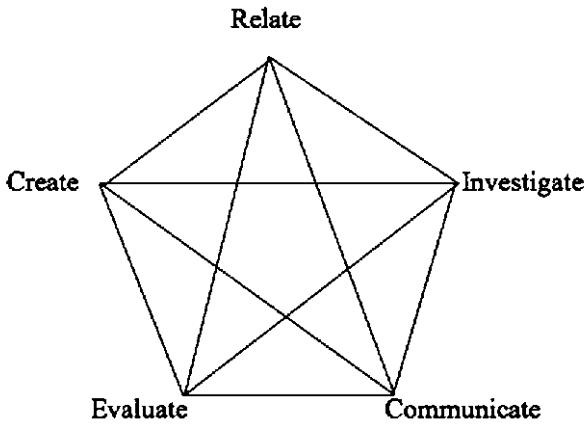

Figure 3 Model for Problem Solving (Sheffield, 2013, p. 326)

Working on challenging problems that require mathematical struggle can develop creative potential, while moving away from procedures, rapidity and precision helps students experience creative mathematics learning (Nadjafikhah et al., 2012). Rather than following specific algorithms, rules and procedures for a single correct answer, a student can search, explore, make assumptions, adapt strategies, devise plans, reason, reflect, justify and come to their own conclusion (Khalid et al., 2020; Nadjafikhah et al., 2012). This can be reflected in Sheffield's (2013) model and the six stages of creative problem-solving (Baer & Kaufman, 2012).

It is important to create a learning environment where viewing problems from different aspects is appreciated, and where students are encouraged to ask suitable questions so they can develop their own ideas and concepts and can reflect on their emerging ideas (Nadjafikhah et al., 2012). By acknowledging multiple solutions and supporting students to take risks, make mistakes, interact, change their minds, and share their thoughts freely, students can become more creative and active learners in their own studies.

Teachers can provide helpful hints as students come to new insights for themselves that they can then apply to new problems (Nadjafikhah et al., 2012). This is closely connected to Vygotsky's (1978) idea of the zone of proximal development, where the teacher's purpose is to support learning to bring the student a little further than they could have gone on their own. In that way, students can use internalization to make sense of their own learning and externalization when they communicate while also interacting with other learners and their teacher.

The problem with problems is that there are two types of problems: routine problems and non-routine problems. "The nonroutine problem demands some degree of creativity and originality from the student, the routine problem does not." (Polya, 1966, pp. 126-127). When speaking of problem solving and creativity, we are looking at non-routine problems. Non-routine problems are problems that hold uncertainty (Beghetto, 2017). Students need to be presented with problems that require them to perform some thinking or action that is not immediately known to them. They need to try out new ideas, test them and find their way within a situation that is not completely familiar.

One type of problem-solving is insight problems that involve selective encoding, comparison, and combination (Sternberg, 2017). This means that the student needs to identify useful information from the problem and from their previous learning, and then combine these to reach a solution.

Such problems are sometimes called brain teasers because they require thinking outside of what you would normally suspect. Martin Gardner (1986) is a mathematics enthusiast who has gathered and published many insight problems that can be used with students to encourage them to think in a different fashion. Such problems can also spike very lively conversation that can be conductive to collective mathematizing.

2.2.2.4 Art, Culture, and Patterns

An aspect of our culture that most people connect with creativity is art. Artists use mathematics such as the golden ratio in their work to create symmetry and equilibrium. They also rely on many techniques based on mathematics, geometry, and scaling. Patterns are one connection of mathematics and arts. Nature follows the laws of mathematics in flowers, rock formations, salt flats and other natural patterns. Whether visiting the amazing Alhambra in Granada, Spain, or sitting in a knitting club in Iceland, patterns seem to be part of every culture. All cultures have their patterns, and all have mathematics as the base of construction. Patterns show up in buildings, clothes, kitchenware, curtains, purses, books, and napkins. From Babylon to Timbuktu, patterns are everywhere.

Bringing patterns into mathematics education can be a way to help students realize the creativity in mathematics. Massarwe, Verner and Bshouty (2011) researched how working with student teachers with cultural ornaments can help them to see how mathematics can show up in arts and culture, and can even be connected to heritage and religion. Teacher students who worked with cultural ornaments were able to connect this important part of their culture to mathematics and see both the beauty and mathematics in the ornaments. This is closely related to the ideas of Goldin (2017) on emotions in connection to mathematical creativity and the connection to attitudes, beliefs and values.

Other ways to incorporate patterns in mathematics include using fractals or ideas from a mathematically inspired graphic artist, M.C. Escher, and having students create their own patterns based on these ideas. Students can benefit from working with patterns as this connects to algebraic construction and more evolved mathematizing. There are numerous ways to paper folding, such as origami and kirigami that connect geometry to art, hands-on projects, and mathematics. This can also be seen in using geometrical computer programs and in letting students work with simple coding to realize the rules, patterns, and cause/effect relationship in computer programs.

The Bridges Organization (https://www.bridgesmathart.org/) has held yearly conferences since 1998 that are intended to foster research, practice and interest in mathematical connections to art, music, architecture, education and culture. The conferences are interdisciplinary, with contributions from mathematicians, scientists, artists, and educators, and gives them a unique opportunity to connect, share and inspire each other. This conference hosts talks, papers, hands-on workshops, a visual art exhibition, a family day and arts-focused performance events in music, film, poetry, and theatre. Visiting the family day in Stockholm 2018 was a treat, seeing everything from complex paper folding and computer systems to a mathematically expressed dance. Using connections like these can bring students to an awareness of mathematics in the arts, sciences, and our current culture. This can help them see mathematics in a broader light and spike their interest in mathematical connections.

2.2.2.5 Mathematics and Society

Many students get to know mathematics as a discipline wholly unconnected to the world they live in (Hersh & John-Steiner, 2017; Kettler et al., 2018). Geometry, social and natural sciences, and other subjects are often taught in connection to the world, society, and nature. As mathematics is not connected to any one aspect of our world, it is easy to teach it as loosely related abstract concepts.

However, many students connect more with mathematics if it is connected to their world, life and interests (Pound & Lee, 2015). This is how mathematics learning connects to real life. Our culture and society are filled with mathematics and using that connection can be a way to promote creativity (Hersh & John-Steiner, 2017; Pound & Lee, 2015). Having students work on real-life projects, connecting two or more disciplines, and using peer groups to find ways to solve any problem that might arise can be a creative experience (Boaler, 2016; Pound & Lee, 2015).

Mathematical modelling can bring mathematics closer to students' actual lives and encourage creativity for mathematics students (Palsdottir & Sriraman, 2017). Mathematical modelling is a cognitive activity that aims to describe aspects of the real world and their relationships and interactions in mathematical terms (Kettler et al., 2018). Mathematical modelling tends to offer problems scenarios more complex than traditional textbook-driven classroom exercises. Mathematical modelling allows for more complexity and mathematical structure of ideas, concepts or prototypes. Mathematical modelling supports students in problem-posing and problem-solving (Singer & Voica, 2017).

Although it is important to bring aspects of society into the mathematics classroom, it is also vital to create a supportive society in the mathematics classroom. Grégoire (2016) wrote:

> One common stereotype about scientific creativity is that of the lone scientist. Many people imagine the creative researcher as someone working alone in his or her laboratory and suddenly making an amazing discovery. We have seen in the preceding text that such a representation is never correct. Scientific creativity is the result of an interaction with other researchers who are working or have worked on the same problem and similar ideas. Creative insights rely on discussion confrontation, collaboration and social support. (p. 32)

Collaborative learning is crucial for the development of mathematical ideas and creativity for students (Boaler, 2016; Pound & Lee, 2015). Mathematical conversation is a particularly important component in fostering students' creativity in the mathematics classroom (Boaler, 2016; Kettler et al., 2018). Using talk to promote creative thinking involves asking open-ended questions; valuing and praising specific examples of what children do and say; and establishing an atmosphere in which children feel safe to take risks, make mistakes, have fun and relax as they explore new ideas, make new connections and try original ideas (Pound & Lee, 2015).

2.2.2.6 Evaluating Creative Mathematics Learning

> Creativity is undefinable, possibly unteachable, largely unassessable, and becoming the most valuable commodity in the 21st-century market. (Harris, 2014, pp. 2-3).

One of the barriers to creative education is that many find it obscure and difficult to evaluate. Mathematics teachers commonly rely on testing for evaluating. Sternberg (2017) notes that standardized testing and other tests that emphasize one correct answer discourage mathematical creativity. Torrance's (1974) test of creative thinking is based on fluency, flexibility, originality and elaboration. However, the test focuses solely on identifying creative individuals and outcomes, much as standard tests do, rather than focusing on the creative learning process.

Formative assessment is a term that has been in use for several decades. Some prefer the term assessment for learning, but there is no consensus on one definition for either term. William and Leahy (2015) have seen that there are four main points that various definitions from different authors share. These are:

1. The amount of time that elapses between the collection of the evidence and the impact on instruction.
2. Whether it is essential that the students from whom evidence was elicited are beneficiaries of the process.
3. Whether students have to be actively engaged in the process.
4. Whether the assessment has to change the intended instructional activities (William & Leahy, 2015, pp. 6-7).

Formative assessment can work well to evaluate creative learning, as it is process-based and, along with teaching, provides continuous feedback with the goal of improving learning (William, 2011). It can be a powerful tool for raising achievement and preparing children to be lifelong learners (Clarke, 2014). It can also help students become active in their own learning (Keeley & Tobey, 2011). At Vinaskóli, teachers are encouraged and taught to use formative assessment. They work with talk partners, feedback, success criteria, and other tools to help students become active learners (Clarke, 2014). Supporting teachers to use these tools and other formative assessment tools in mathematics can further support creative learning environments.

Keeley and Tobey (2011) argue that in a formative assessment-centred classroom, students learn to play an active role in their learning and support the learning of other students: "They come to realize that learning has to be done *by* them; it cannot be done *for* them" (Keeley & Tobey, 2011, p. 12). Closely related to formative assessment is the idea of growth mindset, a term coined by Dweck (2016) and connected to the idea of self-efficacy. In her book *Mathematical Mindset*, Boaler (2016) writes about how this can be crucial for mathematics learning and emphasises the importance of allowing and valuing mistakes, showing students the beauty in mathematics with rich mathematical tasks and working towards a growth mindset in mathematical learning.

For mathematics education, the goal of formative assessment is to strengthen active students, encouraging them to think and be curious (Keeley & Tobey, 2011). Formative assessment can help the teacher and students to reflect on learning, encourages discussion and helps students with inquiry and well-thought answers. In that way it can support mathematical struggle, concept learning and transferred learning. Moreover, it helps to determine how students can use mathematical ideas in new situations, as well as encouraging mathematical dialogue and social

construction of concept understanding in mathematics. Formative assessment in mathematics can be done with interviews, reflection journals, writing, tests that require analysis, and videos.

2.2.3 Summary

This section has explained the practical theories of the PD program on creative mathematics education which are at the centre of the theoretical framework. Exploring theories of mathematical creativity and ways to foster creative mathematics learning ties into the action research process which is better explained in the methodology chapter. In that chapter, it is laid out how each PD seminar was planned with these theories in mind. The results are reflected on with the existing theories of creativity, creative learning environment, and creative mathematics education in mind.

In this section we have seen that there is not just one way to foster creativity in mathematics. We have seen how mathematics is in its essence creative, how mathematicians work creatively and how students' creative potential can be fostered by moving it closer to the work of mathematicians and scientists. Creative problem-solving is a good way to foster creativity in the mathematics classroom and allowing students to develop their own solution is vital in that context (Grégoire, 2016; Nadjafikhah et al., 2012). Art, culture, and patterns can also be used to foster creativity in mathematics and interdisciplinary learning, while mathematical modelling and collaboration are yet other ways (Boaler, 2016; Kettler et al., 2018; Palsdottir & Sriraman, 2017; Pound & Lee, 2015). Formative assessment works well to evaluate creative mathematics learning as it is process based (Keeley & Tobey, 2011). In the next section, I present theories of PD and explain how those were used to develop the PD program on creative mathematics education.

2.3 Professional Development in Mathematics Education

The ability of teachers to foster creativity in their mathematics classroom is based on their own experiences and self-efficacy, as well as how they are able to reflect on their actions with self-reflection (Panaoura & Panaoura, 2014). Bringing theories of creative mathematics into PD seminars can support teachers to foster creativity in mathematics. Teachers need to have experienced creative mathematics in order to support their students with it (Nadjafikhah et al., 2012). Therefore, it is important to give teachers opportunities to explore ways of working in creative ways in mathematics. For mathematics teachers, it is particularly important to give them

opportunity to analyze, research, and problem-solve so they can improve their knowledge, understanding, and skills (Loucks-Horsley et al., 2010). Hands-on projects can be beneficial for teachers to connect better to their classroom and to understand student learning better (Darling-Hammond et al., 2009).

This section focuses on the theories of PD which are important for the development of the PD program in research, with sub-sections on the framework used for designing the program, professionalism for teachers, collaborative professionalism, adult learning, successful PD, and coaching in PD. The theories presented here make up the third part of the theoretical framework for this study and were crucial for the development of the PD program as well as the analytical process of this action research.

2.3.1 Designing Professional Development for Mathematics Teachers

Loucks-Horsley et al. (2010) write about professional development for teachers of mathematics and science. They emphasize the importance of a connection between teachers, projects, and students to improve teaching and learning. When designing PD, it is important to be open to the different needs of teachers. Diversity and professionality are important. It is important for teachers to develop their knowledge and leadership skills through collaborative learning. Including teachers in the learning process is crucial, as is the use of learning groups or communities to gain a joint language, insight, and vision.

Loucks-Horsley et al. (2010) have developed a design framework for PD of mathematics and science that shows the process and influential inputs (Figure 7). The design process draws on the research, practice, and beliefs of educational reform and assumes that PD is complex and reflective. Loucks-Horsley et al describe the PD process as cyclical and under constant reflection and revision. The framework assumes that the foundation is the knowledge and beliefs that result in the vision on which the PD is based. By committing to that vision and those standards for learning, the PD can be designed within the learning context. Goals are made, based on which critical issues are important. The plans and action are based on strategies that are deemed appropriate. This process is then evaluated and repeated. The researchers emphasize the importance of teachers being active participants in the process of design and how they should have the opportunity to communicate their learning and application in teaching.

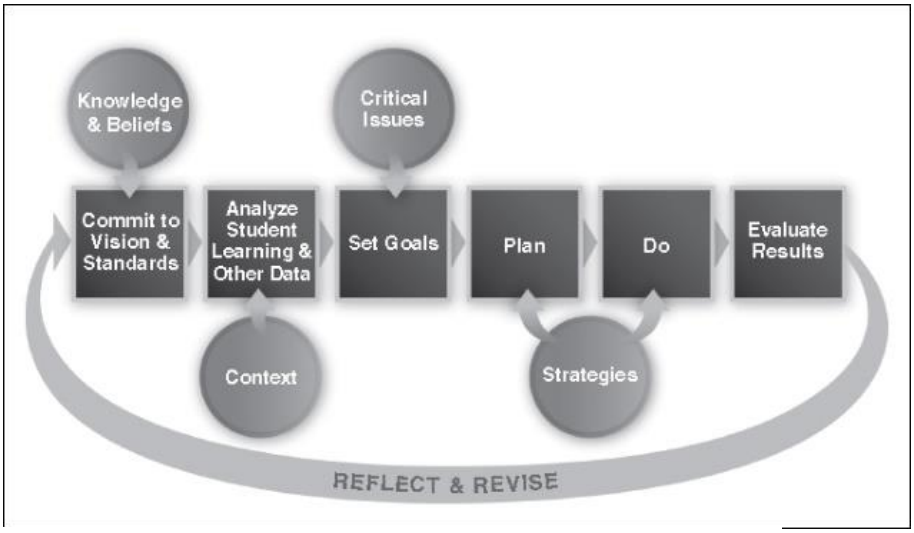

Figure 4 Professional Development Design Framework (Loucks-Horsley et al., 2010, p. 18)

Loucks-Horsley et al. (2010) design framework was used for designing and developing the PD program on creative mathematics education. The PD program was based on the idea that creativity is important for mathematics learning. The teachers and I collaboratively committed to the vision and standard to embark on a joint journey of discovery and learning where the emphasis was on fostering creativity in the mathematics classroom.

The context for developing the PD program was based on collaborative professionalism of the teachers within their existing school culture. Through the action research, data was gathered from the teachers, such as interviews, surveys, and videos from seminars. The teachers also gathered data from their students and developed lesson plans with the goal of increasing creative learning in mathematics.

The PD program took place within the context of a school that was already focused on creative learning in a few other subjects and where teachers participated in regular PD, including creating and carrying out their own action research projects each year. Within the school context and by analyzing the relevant data, goals were made for the PD program on how the PD could benefit the teachers in their own classrooms, considering any critical issues within the school.

The PD program was planned as part of the existing structure for PD at Vinaskóli, which already had weekly PD sessions in which all teachers participated. The PD seminars on creative mathematics were set up as part

of those weekly sessions that were planned for the teachers. Having the PD seminars at a time when the teachers were used to having PD was beneficial for the joint commitment and follow through of the program. Each teacher then proceeded as they saw fit with the PD between seminars while receiving support from me and collaborating with other teachers.

Strategies were used in the PD program that were based on theories of adult learning and successful professional development. In collaboration, the teachers and I planned and carried out lessons for their students which were based on strategies for creative mathematics learning that had been brought into the PD seminars. The lessons were evaluated, reflected on, and revised in collaborative meetings, with the focus of learning from the process and supporting student learning.

Through the collaboration with the teachers and the use of Mills' (2018) dialectical action research spiral, the PD program was continuously reflected on and revised. Through that revision, the importance of coaching became evident, and coaching was added as an important part of the work with the teachers on fostering creative mathematics in their classrooms. The design framework from Loucks-Horsley et al. (2010) was chosen, as it assumes the active participation of the teachers in the development of the PD and the framework fits well with the cyclical nature of the action research.

Jaworski (2011) identifies four different models of PD programs for mathematics teachers. The first is a direct instruction model of teachers showing, telling, demonstrating as metaphors of transmission, or conveying knowledge. The second is a constructivist instructional model where the teacher as an individual makes personal sense of what is offered in the program. The third is a reflective practice model where teachers reflect on their teaching, aiming at development or change. The fourth is a socio-cultural model, which is based on learning as participation and teachers' learning by interactions in social settings that are based within their school culture and context.

The PD program on creative mathematics education is based on the socio-cultural model with the teachers as collaborators in the entire PD process, using their collective experiences to learn within their school setting. This fits well with Loucks-Horsley's (2010) design framework as it assumes a flexible development based on joint vision and standards. Use of a socio-cultural model also fits well with the school culture and context, as it is based in collaboration and in the active participation of teachers in PD.

Furthermore, the socio-cultural model fit well to address the main concern of creative mathematics education, as creativity in the mathematics classroom is based on allowing students to actively engage in their learning, develop their own understanding and collaborate with fellow students. Developing a PD program that allowed the teachers to work in such a manner was a crucial factor to support them in fostering creativity in their own classrooms. The next subsection focuses on professionalism for teachers, with the following subsection focused on the collaboration needed for PD.

2.3.2 Professionalism for Teachers

As with most professions, teachers need vast training and experience to develop as professionals. Hargreaves and Fullan (2012) argue that to meet the demands of the 21st century and teaching for creativity through a variety of learning styles, it is important to train highly qualified teachers with a deep, broad repertoire of knowledge and skills. They argue that teaching is an important profession focused on constant improvement.

Hargreaves and Fullan (2012) explain how a profession constitutes specialized knowledge, expertise and language, shared standards of practice, long and rigorous processes of training, a protected or licenced service, ethical conduct, self-regulation, autonomy, collaboration, and a commitment to continuous learning and professional upgrading. They argue that for quality in teachers' professionalism, PD must connect theory and practice. They further reiterate that peers become the strongest source of innovation for teachers' professionalism.

High-quality peer interaction includes conditions for professionals to meet; expectations and frameworks of learning; and curricula that are challenging and open for teachers to inquire into their practice, coupled with ongoing and timely data that enables teachers to diagnose and learn from student learning (Hargreaves & Fullan, 2012). Leadership is important for these quality interactions and supports teachers participating in teamwork, where they can learn from their colleagues.

Hargreaves and Fullan (2012) argue that professional capital is a cornerstone concept that brings together and defines quality in professional practice and teaching. It deals with what a person knows and can do as an individual and what can be accomplished collectively. In this context, human capital, social capital, and decisional capital are the discerning factors. This must be based on collaboration, feedback, and

continued practice, mirrored by quality coaching, mentoring, and allotted time for reflection.

For mathematics teachers, knowledge development is a continuous process which includes pre-service education and should be based on reflective daily activities to improve knowledge, understanding and teacher skill (Passos & Vézina, 2011). Teacher reflection is particularly important for mathematics teachers and reflection needs to be connected to classroom practice (Bednarz, Fiorentini, & Huang, 2011). Mathematics teachers need opportunities to develop as professionals. Collaborative and investigative practices are important in facing challenges that are encountered by mathematics teachers in school practices (Fiorentini et al., 2011). The next section focuses on collaborative professionalism.

2.3.3 Collaborative Professionalism

> Collaborative professionalism is about working well together in a professional way. It is hard work for a good cause, but it is not self-abnegating or joyless. Collaborative professionalism makes teaching more interesting and engaging for everyone who is involved in it. (Hargreaves & O'Connor, 2018, p. 15)

Professional collaboration benefits students and teachers and its merits are irrefutable (Hargreaves & O'Connor, 2018). Professional collaboration can boost student achievement, increase teacher retention, and enhance innovation and changes. Hargreaves and O'Connor (2018) use the concept collaborative professionalism to describe when teachers create a stronger and better professional practice together. The professional aspect of collaboration centres upon good judgement, commitment, sharing, and deepening of expertise. The collaborative aspect refers to how members of a profession work together, as well as how they talk, share, and reflect together.

Methods for collaborative professionalism include open class or lesson study; collaborative curriculum planning; cooperative learning and working; collaborative pedagogical transformation; and professional learning communities (Hargreaves & O'Connor, 2018). Collaborative professionalism is based on strong cooperation and supports positive individuality. Hargreaves and O'Connor (2018) have formulated 10 tenets of collaborative professionalism that are shown in Figure 5.

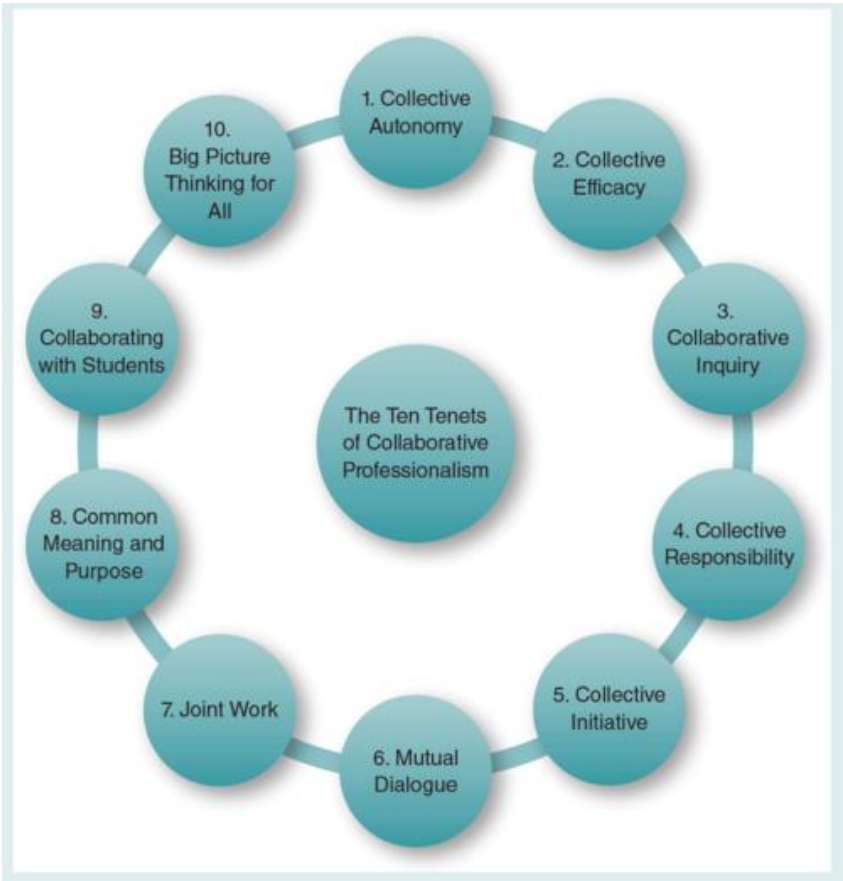

Figure 5 Ten Tenets of Collaborative Professionalism (Hargreaves & O'Connor, 2018, p. 109)

Collective autonomy values teachers' judgement in an open dialogue with others, including feedback, inspiration, and assistance (Hargreaves & O'Connor, 2018). Collective efficacy is the belief that teachers can together make a difference to the students they teach. Collaborative inquiry refers to teachers exploring problems, issues, or differences together to improve or transform their work. Collective responsibility is centred on a mutual obligation to collectively serve the students. Collective initiative reflects how educators are inspired and empowered to try out innovations that engage students and create richness in teaching. Mutual dialogue is based on valuing and fostering an honest and genuine dialogue that embraces differences. Joint work is how teachers share the responsibility of teaching and work. Common meaning and purpose reflect the goal of education,

which enables and encourages young people to grow and flourish. This becomes possible through collaborating with students and actively engaging them in constructing change. The big picture is focused on all being connected in the educational process.

Hargreaves and O'Connor (2012) note that good leadership, time and resources are needed for successful collaborative professionalism. However, they take this a step further and focus on what is beneficial on a practical level. They note that it is always important to work within the existing culture and that all work, relationships, and innovation is shaped within a school system. They note that for success, teams need to collaborate through beneficial and purposeful feedback, striving towards positive change and including their students in the collaboration.

Ferreira and Miorim (2011) found that PD programs with mathematics teachers are generally more successful when based on partnerships where decisions are made collectively. Participants then feel they can share growth through trust and respect in a collaborative group. They feel free to express ideas, listen to criticism and change their points of view. Participants can have different levels of involvement and different interests and viewpoints which contribute to a greater variety of ideas. The next section focuses on adult learning as it applies to the development of PD.

2.3.4 Adult Learning

Professionalism and collaborative professionalism are related and dependent on professional development or professional learning. This learning is often collaborative and dynamic, but as professionals are adult learners it is helpful to look at andragogy or adult learning theory.

Zepeda (2019) explains how andragogy is important when planning professional development (PD). Adult learners are often self-directed, motivated by internal variables, and performing a variety of roles based on their experience. They become ready to learn as they are based on a need to perform better. Strategies for adult learning include active and interactive learning, providing concrete hands-on experiences, and employing novelty while connecting to the learners' prior experience and knowledge. Figure 6 shows the interactive factors that are important for PD based on adult learning.

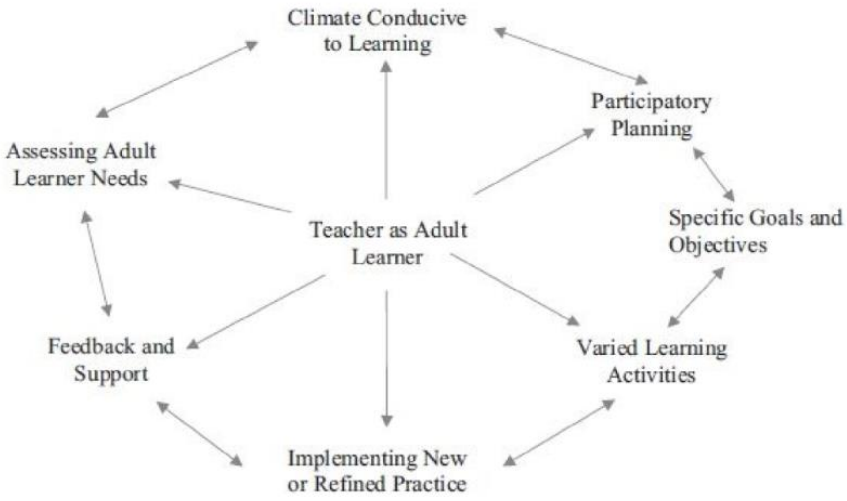

Figure 6 Adult Learners and Professional Development (Zepeda, 2019, p. 39)

For teachers as adult learners it is important to create a climate that is conductive to learning by assessing the teachers needs through continuous feedback and support (Zepeda, 2019). Participatory planning is important where teachers can collaboratively set goals and objectives. Offering them varied learning activities can support them to implement new or refined practice. Suurtamm (2011) found that engaging mathematics teachers in mathematical content and pedagogy through problem solving activities including manipulatives, questioning and sharing of solutions is important. PD is a non-linear process based on interplay between persons and situations, where agency of learning is important for building a practice based on inquiry and collaborative learning (Ferreira & Miorim, 2011).

2.3.5 Successful Professional Development

Successful PD is instructive, reflective, active, collaborative, and substantive (Martin, Kragler, Quantroche, & Bauserman, 2014). With the goal of positively impacting teaching and learning, PD must be connected to teachers' practice and happen over an extended period of time (Desimone, 2015). PD opportunities include learning communities, coaching, study groups, critical friends, lesson study, and action research (Zepeda, 2019). Effective professional development is usually content-specific and related

to subject matter, while offering multiple modalities of active learning and including planned follow-up, coherence, and collective participation (Desimone, 2015; Zepeda, 2019). PD needs to be job-embedded, relevant and connected to the teachers' work, and it is imperative that teachers be given opportunities to use what they learn in any PD endeavour in their own classroom (Zepeda, 2019).

Student learning should always be a primary focus of professional development (Zepeda, 2019). Professional development needs to be connected to teachers' work and they should have opportunities to learn from their own work, their students' work, and the work they do with other teachers. It is particularly important to give teachers opportunities to apply the new knowledge, skills or experience they acquire to their classroom.

Desimone (2009) describes the process of successful PD. The first step should be the PD program or opportunity, which should result in new knowledge, ability, and views for the teachers. This change should then result in improved pedagogy and teaching, which in turn should promote better learning for students. I have drawn up a chart that shows this process (Figure 7).

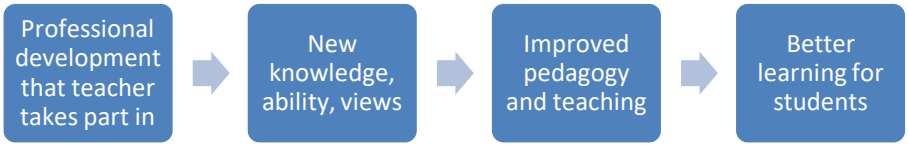

Figure 7 Successful Professional Development (Desimone, 2009)

PD can support teacher voice and agency if teachers are included in decision-making (Zepeda, 2019). Effective PD values teachers' prior experiences while offering them coaching and expert support. The goal is not to fix perceived weaknesses within teachers but to provide them with opportunities to further grow as professionals within a social context of the school and their classroom. Allowing mathematics teachers to move back and forth from PD sessions and their classroom over a year or more can support them in testing new ideas and connecting theory and practice (Suurtamm & Vézina, 2011).

2.3.6 Coaching in Professional Development

> Effective coaching makes it easier for teachers to learn and implement new ideas. Indeed, without follow-up such as coaching, most professional learning will have little effect.

> When professional learning is central to a school's culture and when coaching is woven into all professional learning, most staff members won't need to be told to work with a coach. Most will choose to work with someone who makes it easier for them to learn new strategies, improve their skills, and reach more students. (Knight, 2011, p. 20)

Knight (2011) argues that coaches who work with teachers take a collaborative approach. He has developed seven principles of collaborative coaching: equality, choice, voice, reflection, dialogue, praxis, and reciprocity.

Knight (2011) explains how the seven principles are seen in a collaborative coaching relationship. He explains that equality focuses on the importance of all parties being equal with no hierarchy in place. Choice refers to the teachers' autonomy and ability to decide how they conduct themselves. Voice refers to how coaches can support teachers to find their voice through open and candid conversation and the need to respect that voice. Reflection is an integral part of any PD and coaches do well to encourage it. Dialogue serves as a mutual way of communication, learning, and reflection. Praxis is how teachers choose to use what they learn in their classroom. Reciprocity refers to the joint learning and that everyone learns something new for themselves when in a collaborative coaching relationship.

Knight (2011) further reiterates how important it is that coaches act in accordance with the participation relationships they wish to foster. He clarifies helpful actions that include enrolling teachers, identifying their goals, listening, asking questions, explaining practice, and providing feedback. Feedback should be based on discussing teachers' impressions rather than stating what teachers should do. Practice can be explained in a clear way but should always consider the teacher's views and experience. Active listening and open questions support teachers to both find and work towards those goals. It is helpful if the teachers themselves choose to work with a coach and see the benefit from it.

Partnerships are important in PD programs for mathematics teachers. They should be based on mutual respect and avoid hierarchy (Jaworski, 2011). There is a shared power base where decisions are taken jointly on what is needed and how to fulfill needs. When researchers are directly involved in a PD program, it is important that the teachers are seen as possessing their own practical pedagogy and are allowed to give input into

the process of improving education and learning (Ferreira & Miorim, 2011). With that view, collaborative work can be interesting and fruitful in creating an atmosphere of collaboration and collegiality, where teachers feel safe in trying out new ideas and sharing them (Suurtamm & Vézina, 2011).

With any PD, it is helpful to consider ways that can support teachers put learning into practice. Desimone (2009) is very clear that the goal for all PD is to positively impact teaching. This is followed by a change in a teacher's views, knowledge, or skills that can happen through collaborative learning with other teachers. Moving this new knowledge, skill, or view into a classroom requires effort, and coaching is one way to support teachers with it.

2.3.7 Summary

This action research takes place within a PD program on creative mathematics education that has the goal of positively influencing the teaching and learning of mathematics and investigating what can be learned from the developing PD program and the teachers' experiences. Teachers are professionals who grow, evolve and emerge through the long-term day-to-day work that they do in schools (Zepeda, 2019). All teachers need support from school leaders and peers within job-embedded learning opportunities, whereby efficacy needs to be the focal point of all such efforts. Supportive cultures that increase efficacy include feasible, important, and clearly defined goals; supportive and insistent supervisors; and good peer support.

In this section we have looked at professionalism and collaborative professionalism for teachers, theories on adult learning, what entails a successful PD, and the role of coaching in PD. The theories that have been presented are an important part of the theoretical framework for the study and were used in the design and development of the PD program as well as for reflection when analyzing the data and answering the research questions.

2.4 Summary of Theoretical Framework

In choosing a theoretical framework for this thesis, it was important to stay true to the type of research done. Since this is an educational action research of a practical nature focused on a PD program of creative mathematics education, the theoretical framework needed to be focused on creativity, creative mathematics education and PD.

Theories of creativity and creative learning environments are important for the PD program and the research thereof. The PD program was based on little-c creativity and fostering a creative potential for all students (Craft, 2001; Runco, 2007). The focus was on the learning rather than products or performance. For student creativity, internal motivation and flow are important, as well as having a supportive teacher who fosters an environment that allows students to grow and learn (Amabile, 1989, 1996, 2013).

In mathematics education, creativity can be fostered through problem-solving, through working in similar ways to mathematicians or other scientists, and through working with patterns and art (English & Sriraman, 2010; Khalid et al., 2020; Massarwe et al., 2011; Nadjafikhah et al., 2012; Sheffield, 2013; Shriki, 2010; Sriraman et al., 2011). Connecting mathematics learning to students' interests and the society we live in can foster creativity, and mathematical modelling is a way to incorporate that (Boaler, 2016; Palsdottir & Sriraman, 2017; Pound & Lee, 2015). Students need opportunities to develop ideas, collaborate, make mistakes, and find their own solutions to a variety of mathematical problems (Kettler et al., 2018; Sternberg, 2017). Evaluation of creativity in mathematics can be based in formative assessment as it is process based (Clarke, 2014; Keeley & Tobey, 2011; William & Leahy, 2015).

Presenting teachers with ways to foster creativity and allowing them to work on creative projects can support them to develop creativity in their mathematics classroom (Panaoura & Panaoura, 2014; Sriraman, 2017). The PD program of creative mathematics was based on seminars focused on educating teachers and collaborating with them to provide their students with opportunities to develop their creative potential in mathematics. This was based on Loucks-Horsley et al.'s (2010) design framework for PD. Theories on collaborative professionalism, adult learning, successful PD, and coaching were used in that development.

As explained previously, the theoretical framework connects directly to the development of the PD program and the action research process. In the following chapter, I further explain the action research paradigm and the rational for choosing that methodology for the research. Moreover, I explain the methods used for meeting the research goals.

3 Methodology and Methods

Action research deals with researching change and collaborating with stakeholders (Herr & Anderson, 2015). The methodology focus chosen for this research is that of an educational action research, as the study focuses on researching and implementing improvement at an educational institute (Efron & Ravid, 2020). Action research is often decided on by teachers who collect, analyze, and interpret data, which is followed by developing action plans based on their findings (Mills, 2018). Many argue that action research is best done in collaboration with others who have a stake in the problem under investigation (Herr & Anderson, 2015).

This research is based at one compulsory school and focuses on a PD program on creative mathematics education for all the teachers at that school. These teachers are considered co-researchers and take part in a collaborative learning and research process. The cyclical nature of action research allowed for development of the two-year program through the iterative research process of the dialectical action research spiral (Mills, 2018).

As stated in the introduction, the research questions for this study are:

1. How do teachers who take part in a PD program on creative mathematics education report how it influences their pedagogy and classroom practice?

2. What can be learned from the active development of a PD program of creative mathematics education?

With the sub-questions:

A. How do the teachers explain the influence of the PD program on their pedagogy and views of mathematics education?

B. How do the teachers who implement creative mathematics in their classroom report its influence on their classroom practice and student learning?

C. What barriers do teachers experience when working with creativity in the mathematics classroom?

D. How can teacher experiences, as seen through the action research, add to the existing literature on how to support teachers with creativity in their mathematics classroom?

Since the research questions are process-based and examine the teachers' experience and explanations, action research is appropriate. Action research can be quantitative, qualitative, or mixed. With the research questions focused on experience and change, qualitative methods are appropriate for this study. Qualitative methods provide an extensive description and can lead to deep answers (Bloomberg & Volpe, 2019; Creswell & Poth, 2018).

The goal of this study is twofold. The first goal is to inform the action within the PD program at the research site. The second goal is to generate a theory or knowledge of creative mathematics education which can be useful for other teachers, those planning PD opportunities, teacher educators and policy makers. The study represents the documentation of a successful collaboration and serves as an account of both the process and the product of that collaboration. It has implications for explaining similar problems and holds potential to be applied to similar situations. This further results in products or instruments for other settings.

In this chapter I begin by clarifying the rationale behind choosing action research as methodology. I then describe the research site, the choice thereof, and how it connects to the PD program. I explain how the PD program was designed and developed and how the case studies were chosen. I clarify my own positionality in the research. To explain the process of the research and the use of Mills' (2018) dialectical action research spiral, a special section focuses on research design and development. This is followed by detailed accounts of the methods used for data gathering and data analysis. After that I explain the ethics of the research, rigour, limitations, and delimitations of this research.

3.1 Rational for Action Research as Methodology

> Action research is inquiry that is done *by* or *with* insiders to an organization or community, but never *to* or *on* them. It is a reflective process, but is different from isolated, spontaneous reflection in that it is deliberately and systematically undertaken, and generally requires that some form of evidence be presented to support assertions. (Herr & Anderson, 2015, pp. 3-4)

The methodology of action research is open-ended and developmental (McNiff, 2017). Unlike traditionalist enquiries, action enquiries do not aim for closure or certain answers. The process is multidimensional, experimental, and innovative. Answers are provisional and learning is never complete: learning involves letting the story evolve. This is not to say that action research is any less rigorous or less methodologically clear than other research, it just means that this process needs to be clear in its design and findings. The main distinctive features of action research are commitment to reflection, knowledge generation, participative and collaborative work, and practice transformation (McNiff, 2016).

Within the traditional positivist research paradigm, knowledge is seen as certain, out there and waiting to be discovered by using scientific tools (Bloomberg & Volpe, 2019). Action research, however, assumes that there is no one answer; rather, knowledge is uncertain and ambiguous with many questions and answers (McNiff, 2017). That way, knowledge is created as well as discovered through dialogue and it is always tentative and open for critique or modification. Researchers are not searching for one answer or theory for all situations but aim to generate personal theory and to invite others to learn with them and join the dialogue. This knowledge creation is a collaborative process.

The action research process is based on doing research to gain new understandings, to create new knowledge and to take action to improve practices that are rooted in this new knowledge (McNiff, 2017). The knowledge pertains to information on how and why you should act in the situation and helps evaluate the effects of those actions. However, there is some disagreement about the balance between action and research (McNiff, 2017). Sometimes when doing action research, the researcher focuses too much on the action with not enough focus on the research. Although action researchers are commonly interested in generating knowledge for a local setting, such research can make knowledge claims that are generalizable or transferable beyond the immediate setting (Herr & Anderson, 2015). In that way, the results can have external validity or transferability.

Action research can be seen through two theoretical lenses (Mills, 2018). One is critical action research and the other practical action research. Critical action research is also known as emancipatory or participatory action research and is based on theory, with the goal of liberation through knowledge gathering (Mertler, 2020). This process is participatory and democratic and focuses on reform (Mills, 2018). It focuses on improving the quality of life of individuals in organizations, communities and families, and thus bringing about social change (Mertler, 2020).

Practical action research focuses more on “how to” approach a situation and is often focused on professional development and school improvement (Mertler, 2020; Mills, 2018). Educational action research is a common type of practical action research (Mills, 2018). Educational action research is reflective and focused on practice at an educational institute with the goal of generating knowledge and theory to inform and improve practice (McAteer, 2013; McNiff, 2016). The process intends to improve education by incorporating change and does so by involving educators working collaboratively to improve practice through empowering relationships (Mertler, 2020). This type of action research is participative, practical and uses a planned systematic approach to understand the learning process in a critical and open way (Mills, 2018). It is connected to school improvement, teacher empowerment, intellectual engagement, and professional growth (Mertler, 2020).

Action research in education can be reflected in Dewey’s (1938) theory on human experience and active learning for the generation of knowledge (Herr & Anderson, 2015). Dewey (1938) wrote: “We are brought to the conclusion that it is modes of active response which are the ground of generality of logical form, not the existential immediate qualities of that which is responded to” (p. 252). In this light, professional experience of teachers can be seen as a source of knowledge about teaching.

The idea of teachers gathering data in their own school goes back more than a century, when teachers were frontline data gatherers for scientific research on education. However, originally this data was analyzed by others using statistical methods. It was not until the mid-20th century that the action research method was first promoted, with the view that teachers would find their own results more useful than those of outsiders. This was not received well by those working with the traditional positivist research standards of that time and was not pursued much until later that century.

In educational action research the researcher assumes that there are multiple school realities, shaped by individual, historical, and cultural factors (Efron & Ravid, 2020). The purpose is to understand school experience from the perspective of those involved and to improve schools with a researcher who interacts extensively with the setting and acknowledges subjectivity and biases. The research process is immersed in the setting and describes the subjective meanings that individuals place upon their actions and experiences.

For educational action research, it is helpful to look at how it differentiates from traditional educational research (Efron & Ravid, 2020). It

is constructivist as the action researchers are perceived as generators of knowledge rather than receivers and make informed decisions based on the enquiries. It is situational as researchers aim to understand the unique context and the participants and conclusions are understood within the complexities of the setting. It is practical as research questions are chosen from concerns and professional areas of interest and the results are immediately relevant to improvement of practice. It is systematic as the research process is intentional, thoughtfully planned, and methodical, and intended to produce trustworthy and meaningful results. The cyclical nature is reflected in how the application of knowledge leads to new questions and a cycle of research.

The methodology chosen for this study is educational action research. The research focuses on reform at a compulsory school, so this research form fits well. Action research is particularly beneficial in collaborative situations such as that of the PD program on creative mathematics education. The research presumes that much can be learned by exploring the teachers' experiences of the PD process and analyzing how a PD program can be beneficial for teachers who wish to foster creativity in mathematics education.

The teachers in this research can be considered co-researchers as they are themselves studying their students' learning and their own process throughout the PD program. The research is practical in the sense that it aims to improve and develop teaching and learning at the site where it takes place. Yet there is also a transferable aspect, as the results are of importance for other teachers, students, schools, and those preparing PD for mathematics teachers. In the next sections we explore the site and PD program and its connection to the research.

3.2 The Site

For educational action research, the site is generally an educational institute and the research is often done by teachers or other stakeholders (Efron & Ravid, 2020). For this research the site is a school where I teach mathematics. I taught at Vinaskóli for one school year before beginning the research. During that school year I was regularly approached by other teachers who wanted to improve their mathematics teaching. One teacher in particular wished to discuss her mathematics teaching and student learning with me. She was aware that I had researched and focused on creativity in my mathematics teaching and how I had provided courses to teachers on that subject. During that school year, that teacher and I had frequent discussions about our teaching. We both wished to move away

from rigid lesson plans and sought to encourage our students with flexible problem solving and creativity in their mathematics learning.

Through that collaboration I realized how I was able to support and encourage others at Vinaskóli with creativity in their mathematics teaching. I approached the principal with the idea of a PD program on creative mathematics education and we decided to apply for developmental funding that was available in our area. We received a grant to host a PD program on creative mathematics with the teachers at Vinaskóli.

Vinaskóli is a compulsory school in the capital area of Iceland for students aged 6 to 16. Vinaskóli hosts 286 students and 28 teachers. Each class has between 17 and 40 students, and many are taught by two collaborating teachers. Vinaskóli is partially divided into three levels: primary level (grades 1-3), middle level (grades 4-7) and lower secondary level (grades 8-10). For the two younger levels, the general teachers generally teach languages, mathematics, natural sciences, social sciences, and information technology, with other teachers taking care of education in art, design, and physical education. For the lower secondary level, each class has one or two class teachers, but subject teachers generally teach one or two subjects to various groups.

Vinaskóli is a progressive school that provides its teachers with weekly PD sessions. These are often focused on formative assessment. Furthermore, the teachers do other PD provided by the school and outside resources. Vinaskóli also uses action research as PD. The school administration assists teachers in choosing a research topic and the teachers gather data and analyze it over the course of each school year. As the classroom teachers generally co-teach a class at Vinaskóli, they collaborate with their co-teacher in the action research process.

Vinaskóli focuses on creativity and interdisciplinary learning for chosen subjects. For example, social sciences and natural sciences are generally taught through creative student workshops. At the school, all students take music, design and carpentry, textiles, and visual art. However, for mathematics there had not been a special focus on creative learning, and I became aware of how I had an opportunity to bring creative mathematics education to the school and to support the interested teachers by fostering creative learning in the mathematics classroom. The next section focuses on how the PD program on creative mathematics education began and unfolded.

3.3 Professional Development Program

The PD program on creative mathematics education started in the fall of 2018 at Vinaskóli. The program was planned as a two-year collaborative

program and all teachers from grade one to grade ten at Vinaskóli participated in the program. Regular seminars were attended by all teachers at Vinaskóli who teach various classes and subjects, including teachers of languages, sciences, arts, crafts, home economics and physical education. Data was gathered from those seminars. All participating teachers had completed teacher education, but their age and teaching experience varied greatly, with the majority of participants women.

The participating teachers who taught mathematics worked on implementing creativity and creative mathematics in their classrooms. The teachers who sought support with these implementations received coaching by me between seminars. The coaching included me meeting with teachers or teams of teachers, going into their classrooms with them as they wished, and coaching them in the process of incorporating more creative learning opportunities for their students. The teachers who I worked with the most were later chosen as case studies. The subject teachers who taught no mathematics focused on general creativity in their classroom and the special subject teachers on creative mathematics within their subject, as applicable.

The PD program was originally designed in accordance with the literature on creativity and creative mathematics education and theories of PD. However, the PD program continued developing through the iterative action research cycle based on data from the teachers. The teachers explained how they experienced the PD program through interviews, surveys and seminars, and this information was used to evaluate and develop the program as it unfolded. This development is further explained in the section Research Design and Development and is crucial for the results of this research.

The PD program started with an initial seminar at Vinaskóli. During this initial seminar I explained to the teachers how I saw the structure of the PD program. I told the teachers that the seminars were based on presentations and discussion, but also by the teachers working in groups on mathematical projects. I made it clear to the teachers from the beginning that their input during seminars was important, and that I wanted the sessions to be free-flowing discussions and raising hands was not needed. I also acknowledged that many of the teachers had much longer and often different teaching experiences than I did. Finally, I emphasized that this process was a joint effort, and that we were together on this learning journey.

As part of the initial seminar, I explained the methodology of the research to the teachers, including forms of data collection and the roles

within the research. Teachers are an integral part of the action research process, which is something I explained to the teachers during that initial seminar. All of the teachers gave their consent to participate in the research and data gathering. It was very important to be clear from the beginning what the goal of the PD program was and how it related to the research. The following quote is something I shared with the teachers during that initial seminar:

> I find it inspiring to bring the PD program to this school, where I see many avenues of creative teaching. I see this in the student workshops in natural and social sciences and with the use of formative assessment and positive discipline. There are so many good things happening and I want to use the opportunity to offer creative mathematics to the teachers. I can come in and introduce new ways to teach and learn mathematics, because I am so pleased with what you are all already doing. (Ósk, September 2018)

Although Vinaskóli had not specifically focused on creative mathematics education in the past, the ground was fertile, as described in my comment above. The teachers were willing and open to participation in this project, as well as being ready for discussions and possible changes in views and instructional practice. The project was always seen as a collaboration and as focused on student learning.

After the initial seminars, I met with the teachers from the whole school as a group regularly over the course of two years. The total number of seminars was seven and all the teachers from Vinaskóli participated in these. The seminars were usually structured in a similar way and were based on presentations, discussions, and mathematical projects. Apart from the initial seminar, the seminars generally started with a discussion on how the teachers had fared with the creative mathematics PD in their own classrooms since the previous seminar. After that, I presented a specific theme related to creative mathematics learning (Table 2). The theme was explored through a presentation, discussion, and group work where the teachers collaborated on mathematical tasks. After that, we discussed how this might be applicable to their classroom. Towards the end of each seminar there was a summary, and goals were set for continued use of the PD in classroom practice. Teachers were always encouraged to use reflective journals as part of their learning process. A more detailed account of which themes were discussed during each seminar is found in Table 2 on professional development and corresponding goals.

Table 2 Professional Development Seminars and Corresponding Goals

Seminar	Content	Goals
YEAR ONE Seminar 1 Creative Mathematics Learning	1. Presentation of the research and planned data gathering to teachers: a. Videos b. Projects from seminars c. Reflective journals d. Interviews e. Action research 2. PD project initiated. Presentation and discussion on: a. Research of creativity and creative learning b. The national curriculum c. Creative mathematics learning	• Presenting the project to teachers • Initiating a PD program on creativity in mathematics learning • Beginning to gather data for an action research within the PD program
Seminar 2 Creative Mathematics Learning through Problem Solving	1. Group discussion where teachers share and reflect on how they have worked with creative mathematics in the classroom between seminars 2. Presentation on problem solving a. Reasons for using problem solving b. How problem solving can promote creative learning 3. Group work where teachers engage with problem solving in groups 4. Goals set for classroom work with creative mathematics	• Presenting problem solving for creative learning • Supporting teachers to experience creative mathematics activities • Gathering data from teachers work and discussions during the seminars
Seminar 3 Creative Mathematics Learning through Working Like Mathematicians	1. Group discussion where teachers share and reflect on how they have worked with creative mathematics in the classroom between seminars 2. Presentation of how creativity appears in the work of professionals and comparing that to school mathematics 3. Group work where teachers design their own geometrical shape or concept 4. Goals set for classroom work with creative mathematics	• Presenting the work of mathematicians and comparing it to traditional school mathematics • Supporting teachers to bring creativity to their classrooms • Gathering data from teachers work and discussions during the seminars
Seminar 4 Geometrical Patterns for Creative Mathematics Learning	1. Group discussion where teachers share and reflect on how they have worked with creative mathematics in the classroom between seminars 2. Presentation of patterns and art as creative mathematics endeavors a. Cultural patterns b. M.C. Escher c. Fractals 3. Group work where teachers work on projects related to patterns, geometry, reflection, translation, fractals, etc. 4. Goals set for classroom work with creative mathematics 5. Completion of a survey for teachers on their work with creative mathematics (Appendix A)	• Presenting art and patterns as creative components of mathematics • Supporting teachers through discussion • Having teachers partake in creative projects • Gathering data from the teachers work and discussions

Table 2 (continued)

YEAR TWO Seminar 5 Assessment for Creative Mathematics Learning	1. Introduction for new teachers and review of the PD program and research a. Creativity and creative learning b. Creative mathematics education and connection to the National Curriculum 2. Presentation on assessment for creative mathematics a. Evaluating creative learning processes b. Formative assessment and recording of student learning c. National curriculum and competence criteria	• Continuing and presenting the PD project for new teachers starting the second year • Presenting assessment in creative mathematics • Connecting creative mathematics to formative assessment • Gathering data from the teachers
Seminar 6 Tools for Creative Mathematics Learning and Assessment	1. Group discussion where teachers share and reflect on how they have worked with creative mathematics in the classroom between seminars 2. Continued discussion on formative and summative assessment of creative mathematics 3. Discussions and reflection on the connections between competence criteria from The National Curriculum and the creative mathematics learning 4. Demonstration for the teachers on how they can use the assessment systems of the school to evaluate creative mathematics learning	• Supporting teachers with the PD program • Allowing teachers time to discuss creative mathematics learning • Providing teachers with an atmosphere to work with other teachers • Showing teachers ways to evaluate the creative mathematics
Seminar 7 Mathematics in Society and the Society in Mathematics for Creative Learning	1. Presentation of mathematics in society and the society in mathematics a. Mathematics in our society and how to connect learning to students lives b. Learning communities to encourage creative mathematics in the classroom 2. Discussions and reflections on the value of mathematics education and the connection to how the teachers have worked with creative mathematics 3. Goals set for classroom work with creative mathematics	• Introducing teachers to ways of integrating mathematics in different subjects and connecting it to student interests • Emphasizing and encouraging teachers to use discussion and learning communities for creative learning • Gathering data from teachers on their experience of the PD program • Encouraging teachers to set goals going forward

Table 2 details the content of each PD seminar and the theme of creative mathematics learning for each seminar. The themes are based on theories that have been presented in the theoretical framework on how to foster creativity in the mathematics classroom. The first seminar focused on learning about the teachers' initial ideas of creativity and creative mathematics and presenting to them general theories of creativity, creative education, and creative mathematics education.

The second seminar focused on the theme of creative problem solving, as explained in the theoretical framework. Teachers were presented to problem solving and partook in collaborative problem solving themselves. The third seminar focused on fostering creativity through working like real mathematicians and the teachers collaborated to create a geometrical concept. The fourth seminar focused on how to use patterns and art to encourage creative mathematics learning. In groups, the teachers created different patterns based on ideas from M.C. Escher, fractals, and traditional Icelandic patterns.

The second year of the PD seminars focused on practical ways of bringing creative mathematics into the classroom. The fifth seminar was based on describing how to use formative assessment to evaluate creative mathematics learning. The sixth seminar focused on demonstrating how the tools the school uses for evaluation can be used for the formative assessment process. The seventh seminar focused on bringing all the learnings together to create a mathematical community in each classroom, focused on creative learning and collaboration of students.

Although many of the seminars were based on the theoretical framework, neither was the plan for each seminar fixed nor the PD program fully developed before starting the program. Development of the program was based on the needs of the teachers as seen through the active PD development and use of the dialectical action research spiral (Mills, 2018). The teachers themselves were active participants who helped shape each seminar with their ideas, discussions, and collaboration.

Also, the content of seminar six was based on teachers' needs. That seminar focused on using practical tools on which the school relies for evaluation and for the formative assessment of creative mathematics learning, based on criteria from the National Curriculum (Ministry of Education, 2013). The teachers and the principal specifically asked if we could have a seminar related to this theme, so the program was adapted accordingly. Another way that the teachers helped develop the program happened between the seminars, where the support for their classroom work was based on their needs and how they expressed those.

It is of great importance that teachers become active participants in their own PD and their involvement in action research is vital (Desimone, 2015; Efron & Ravid, 2020; Herr & Anderson, 2015; Loucks-Horsley et al., 2010). At Vinaskóli the teachers actively participated in constructing knowledge in the PD seminars and in practising the ideas from the seminars

in their classrooms. Between seminars, the teachers worked on how to better support their students with creative mathematics.

I worked with teachers as they wished and provided them with support and coaching to bring their learning into the classroom. As the teachers generally co-teach, I usually met with a team of two teachers. This was an organic process based on the needs of each team. These meetings mostly focused on creating lesson plans for creativity in mathematics and discussing the changes that the teachers notice in student learning. I went into the classroom with the teachers who requested that, supporting them in their teaching. We either co-taught a larger group or else each teacher worked with a smaller group of students. The teachers who I worked with the most are the teachers who were chosen as cases for the research. In the next section I explain these case studies.

3.4 Case Studies within the PD Program

Case studies are often used in qualitative research and focus on either developing an in-depth understanding of a single case or exploring an issue or problem using the case as specific illustration (Creswell & Poth, 2018). This study does the latter. Case study research intends to understand a particular phenomenon such as program, process, event or concept, by selecting a particular example of the phenomenon as the study focus (Efron & Ravid, 2020).

Case study research involves the study of a case or cases within a real-life contemporary context or setting (Creswell & Poth, 2018). A case can be an individual, a small group, an organization, project, or any other system that is bounded by time and place. The researcher usually uses this case to shed light on the larger phenomenon by exploring in depth the selected entity, actions, and reasons for these actions (Efron & Ravid, 2020). A case study searches for meaning using inductive and investigative strategies (Merriam & Tisdell, 2016). The researcher is usually the primary instrument of data collection, and the case study is an in-depth description of a bounded system.

Case study is sometimes seen as a comprehensive research strategy or methodology. However, it can also be used as choice of what is to be studied and can exist within a different overarching methodology. For this research, case studies are used within the methodology of educational action research with the goal of generating in-depth understandings of how the participating teachers experience the influence of the PD program on their pedagogy, views, and classroom practice.

The cases were chosen purposively, based on the data that best answers the research questions. The case studies were chosen based on which teams of teachers had most often approached me between seminars and to whom I had provided coaching for an extended period of time. These were teachers who actively worked on bringing creativity into their mathematics classrooms. The case studies tell the stories of how these teachers implemented their learning of creative mathematics from the seminars into their teaching and what changes they saw as a result in their own views of mathematics learning and in their students' learning.

The three cases I chose were all teams of teachers who worked with grades two to four. As this choice was based on which teachers approached me for support, I have no way of knowing if the fact that they were teaching at the same grade level is a coincidence or whether the PD particularly appealed to teachers at this grade level. It does, however, make for an interesting analysis as there are some commonalities to the cases. Two cases are based on teams of two teachers and one case on one teacher who had a co-teacher who was not able to participate in the research. Teachers from two of the case studies did their own action research related to creative mathematics and all the teachers worked with me inside and outside their classrooms to promote creativity in their students' learning. I have conducted interviews with the five teachers in the case studies, gathered detailed data from their process, and relied on their stories to paint a picture for the findings.

Anna is presented as a one-person case as her co-teacher from the first research year was not available to partake in the research. She taught grade three in the first research year and grade two in the second year. In the first year I did creative student workshops with her and her co-teacher who had approached me for support after the students scored low on a common evaluation test. She and her co-teacher did an action research project on the mathematics workshops in the first research year.

Jenny and Silvia are co-teachers and presented together as one case. Jenny is a visual arts teacher and both Jenny and Silvia commonly teach at the younger grade level. For both years of the research they taught grade level four, with two different student groups. Jenny and Silvia worked with me both years and did their own action research on their mathematics teaching in the first research year.

Inga and Sunna are also co-teachers and are presented as the third case. They actively participated in the seminars through both research years, although our collaboration outside seminars happened in the second

research year. Inga is educated in leisure studies and Sunna previously taught physical education. They were both new to general teaching at the beginning of the research. During the two research years they taught the same group of students in grades two and three.

The results that are presented in this thesis are focused on the chosen cases, how the teachers describe their learning process, and how they see changes in their views, teaching and student learning. This is then brought together in themes and experiences that are analyzed across these cases, as well as results from all participating teachers.

3.5 Positionality of Researcher

Action research is pluralistic and relational (McNiff, 2017). Action research always happens in context and the researcher undertakes enquiries with others. The research process is dialogical, inclusional, and relational (McNiff, 2017). Action research is often conducted by or with insiders involved in the research site and context (Efron & Ravid, 2020). Researchers are subjective, involved and engaged, and are a natural part of the inquiry setting. Research questions arise from local events, problems and needs, and findings are directly applied to the practice. Practitioners collect, analyze, and interpret data to develop action plans (Herr & Anderson, 2015).

The position of a researcher in action research is important for focus. It can range from an insider studying their own practice to some insider-outsider collaboration, to outsiders studying insiders, to multiple positionalities (Herr & Anderson, 2015). In this research I am an insider working collaboratively with other insiders. Such research often contributes to the knowledge base, improved practice, professional or organizational transformation (Guskey, 2014). This can be called second-person research as I can interact with the participants face-to-face on issues of mutual concern (McNiff, 2017).

I hold in fact three roles. My first role is that of a mathematics teacher at the participating school, Vinaskóli. After working with a teacher at Vinaskóli, I saw that I had something to offer in the school and approached the principal with the idea of a PD program on creative mathematics, to which she agreed. The second role is that of the facilitator of the PD program. I plan seminars and support the teachers inside and outside their classrooms to promote creativity in their classrooms. The third role is that of a Ph.D. student at the University of Iceland. I realized that this PD program would make a very interesting research topic and it became the

focus of this Ph.D. study. For this research it is fundamental to see the teachers as agents, not recipients or onlookers. The idea of agency is that people are able and should take an active part in decisions about how they act and operate in the community (McNiff, 2017).

Participation in action research is most effective when it deals personally with people and enables significant active involvement, enabling people to perform significant tasks and providing support for people as they learn to plan, act and accomplish things for themselves (Stringer, 2014). Effective communication is crucial (Stringer, 2014). It needs to be truthful, sincere, appropriate, informative, and understood by everyone. Key factors involve attentive listening, acceptance and acting on what people say. Relationships should promote feelings of equality, together with maintaining harmony, avoiding conflict, resolving disputes openly, accepting people as they are, and encouraging cooperation with sensitivity to everyone's feelings (Stringer, 2014).

In any learning episode the influence of learning can happen on multiple interrelated levels. A person develops their learning to inform their actions. Their learning and their actions then contribute to other people's learning and actions. However, other people's learning and actions can in turn influence their own learning as well (McNiff, 2017). These mutually reciprocal processes amount to the transformation of existing knowledge into new knowledge, with potential influence in a broader context. Figure 8 shows the transformational relationship of mutually reciprocal influence.

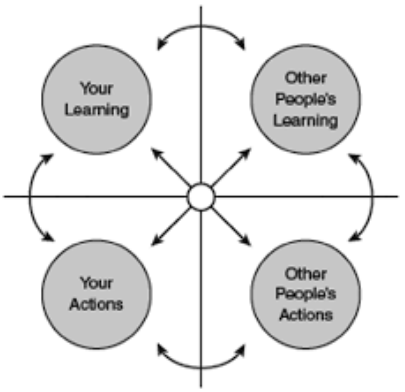

Figure 8 Transformational Relationships of Mutually Reciprocal Influence

As seen in Figure 8, the transformational relationship of mutually reciprocal influence in action research exists in community and collaboration. The focus of this research is the teachers' experiences and how they report on the influence of the PD program on their views, pedagogy, and classroom practice. However, the teachers' learning is directly connected to my actions and vise versa. I am a part of the research process, and it is important to be mindful of that.

The insider position can be valuable for access, knowledge of the site, and the relationship with other participants (Herr & Anderson, 2015). An insider working with other researchers can have a greater impact on the setting in a democratic fashion. Nevertheless, power relations operate even when insiders think they are collaborative, and researchers need to be mindful of this (Herr & Anderson, 2015).

Although bias and subjectivity are natural and acceptable in action research, they need to be critically examined (Herr & Anderson, 2015). The insider position can be helpful when reflecting on data but the researcher needs to be clear on how their position can influence the setting (McNiff, 2016). Insider bias is certainly possible in this research as I want the PD program to be successful. In such cases it is possible for the subjectivity to affect the results (McNiff, 2016). I have therefore put mechanisms in place so that the bias I bring to the situation does not distort the outcomes. In writing this dissertation I am careful to hold conclusions lightly and to recognize dilemmas of context and personal bias.

Action research involves interrogation, deconstruction, and decentring (McNiff, 2016). Interrogating and deconstructing your own thinking can reveal issues in a different pluralistic light. Decentring means to understand that I am not the centre and am actively challenging that idea. In that context, critical reflexivity of how one thinks and dialectical reflexivity in the wider context need to operate jointly (McNiff, 2016). Reflective and dialectical critique begins with questioning assumption and is present throughout the action research and within the cyclical process.

For the purpose of transparency, I wish to make it clear where I stand in regard to mathematics education. As a child I disliked routine tasks in mathematics while finding them relatively easy and straight forward. However, I flourished within creative subjects as well as in mathematical games, origami, and other visual fields of mathematics. When I reached secondary school, I started to like the challenge of mathematics and the logic behind proofs and problems. This led me to wanting to become a mathematics teacher.

In order to become a teacher, I had to learn more mathematics as well as theories of mathematics education. I became fascinated by progressive ideas on how to support students to reach their own understandings and develop their ideas through problem solving. In my B.Ed. I did an autobiographical study that looked at my mathematical journey, and I not only became aware of the fact that creativity was a vital part of my own appreciation of mathematics but I also decided that I wanted my students to learn mathematics through creative avenues. For my M.Ed. I developed and held a course for teachers on creative mathematics education and learned the importance of collaborative professionalism for teachers, besides realizing the importance of extended ongoing support for teachers who wish to change their teaching.

When starting the PD program with the teachers at Vinaskóli, my stance was that of believing in fostering creative mathematics learning and wanting to support teachers with that. I knew I wanted the project to be collaborative and saw I had certain aspects to bring to it and other aspects I needed to learn from the teachers. When I decided to do the research, I chose the focus to be on the teachers' learning, but of course I learned a great deal from the collaboration myself. My learning particularly related to how to better support teachers and how to approach the youngest learners in compulsory school. However, the main research focus is on the teachers' learning and what can be learned from their experiences of the PD program on creative mathematics education.

It is clear that I wanted the PD program to be successful, but I use the data and not my wishful thinking to answer the research questions. I use the method of producing an evidence base and interrogating how I have deconstructed my thinking to make sure the results are trustworthy. I have openly discussed my personal experience, biases, and subjective judgment. I have gone through successive cycles of analysis and included the participating teachers and the thesis advisors in that process. I rely heavily on the participants' descriptive words, explanations, and interpretations. Thesis advisors, coworkers and students at the University served as critical friends during the entire process of study design, data gathering and data analysis.

3.6 Research Design and Development

Action research is informed, committed and intentional, with the research questions generally reflecting the transformational and cyclical process. Educational changes occur bottom-up in a democratic way, as opposed to a

top-down hierarchical process (Efron & Ravid, 2020). Sagor and Williams (2017) argue that strong action planning is seen as a direct and logical extension of the findings and is based on available data. The theory behind the plan is clearly outlined and addressed and the assessment plan provides valuable evidence of the effectiveness of the plan, with the goal that the researcher educators are likely to benefit from data of the eventual implementation of the plan.

Sagor and Williams (2017, p. 8) have identified the following stages of action research:

Stage 1: Clarifying Vision and Targets

Stage 2: Articulating Theory

Stage 3: Implementing Action and Collecting Data

Stage 4: Reflecting on the Data and Planning Informed Action

These stages can be seen in different chapters of this thesis. Stage 1 is explained in the Introduction chapter and Stage 2 in the Literature Review. Stage 3 is explained in this chapter and Stage 4 in the Findings. Although not all action research theorists rely on these same stages, it is a common understanding that the action research process is circular and based on repeated action that is based on findings. For this dissertation, Mills' (2017) dialectic action research spiral (Figure 9) is used with the focus on creativity in mathematics education. The action plan develops alongside the gathering and analysis of data.

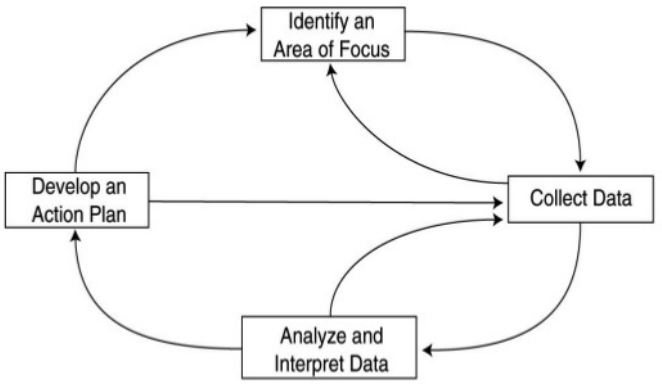

Figure 9 The Dialectical Action Research Spiral (Mills, 2017, p. 26)

As previously explained, the research questions for this action research deal with researching the active development of the PD program and the teachers' experiences as they participate in the PD program on creative mathematics. The goal is to gain a better understanding of how a PD program on creative mathematics can work to support teachers with creative mathematics learning in their classroom and what can be learned from developing and carrying out such a PD program. To explain better how the research unfolded, I have created Table 3 that shows how the dialectical action research spiral was used for this research.

Table 3 The Research Cycles

	First Cycle Before the PD program	**Second Cycle Beginning the PD program**	**Third Cycle First PD Year**	**Fourth Cycle Second PD Year**	**Fifth Cycle After the PD program**
Identify an Area of Focus	Creative mathematics education and professional development	Starting out with the teachers at Vinaskóli	Introducing teachers to ideas of creative mathematics	Bringing creative mathematics into the classrooms	What can be learned? Answering the research questions
Collect Data	Literature gathered on the chosen topics	Data on teachers' initial ideas from the first seminar	Videos from seminars, open surveys, and researcher's journal	Videos from seminars, interviews, open surveys, and the researcher's journal	Data from all previous cycles used
Analyze and Interpret Data	Reading, synthesizing and criticizing the literature	Using the data to develop the program with the teachers	Iterative analyzing of data to better support the teachers	Iterative analysis of data on how to support teaching	Summative analysis of all the data. Bringing together learning from all cycles
Develop an Action Plan	Designing an original structure for the PD plan, based on literature and liaison with teachers and principal.	Beginning the program with teacher collaboration	Going into the classrooms with teachers who requested that, and holding more frequent seminars	Coaching teachers to foster creative learning in their mathematics classrooms	Providing information on how to better support teachers with creative education

As seen in Table 3, Mills' (2018) dialectical action research spiral was important for this research. The first research cycle began before the PD program and study thereof began. That first cycle was based on reviewing the literature and evaluating the needs of the school, keeping in mind past experiences of communicating with the teachers and principal as well as referring back to other PD I had developed and researched in the past. This cycle was continued throughout the research process, thus allowing the theoretical framework to develop along with the PD program,

The second research cycle was based on analyzing the initial ideas of the teachers as they were expressed in the first PD seminar. In the first seminar, I met the teachers with the goal of hearing their ideas of creativity in mathematics and committing to a joint vision and standards for the program, as explained in the Professional Development Design Framework in the theoretical framework (Loucks-Horsley et al., 2010). The second research cycle was also based on evaluating and deciding how the program could fit in with the school culture.

The third research cycle took place within the first year of the PD program. That year, I introduced the teachers to many ways of fostering creativity in mathematics education, which have been explained in the theoretical framework. During that year I also started to work intensively with teachers from two of the three case studies. At the end of that year, all participating teachers responded to an open survey, detailing what they had learned from the program, how they had fared in incorporating creative mathematics in their classrooms, and what they felt was needed to better the program. After analyzing the survey and information given by teachers at seminars and in meetings, the program was developed during that year and the foundation laid for the following year.

The fourth research cycle happened in the second research year. Keeping in mind results from the previous school year (research cycle three), the program was planned with seminars closer together and more frequent collaborative meetings for teachers who requested support. During that research year, I worked with teams of teachers from all three case studies inside and outside their classrooms. I took interviews with the five mathematics teachers from the three cases and used the interviews to further develop our joint work. I also interviewed two special subject teachers. The teachers' needs, as expressed in meetings, seminars, and interviews, were instrumental in how we worked together. I realized that year how important coaching was for the PD program, and it became part of the theoretical framework for the study.

Although it would have been easy to go on another year and continue the research cycles at the school, the fifth research cycle was the last one and was focused on the summative analysis of the entire data set. The fifth research cycle is the one on which I focus the most in the results and discussion chapter, trying to provide a clear picture of the entire research process and triangulating findings from all cases. In the next section I explain the different data sources used for the research and the methods used for gathering those.

3.7 Methods of Data Gathering

This research is an educational action study. Data gathering and analysis are connected to the PD program and are a continuous process. Mertler (2020) argues that a large part of the acting part of action research entails the action of gathering data. Action research can be qualitative, quantitative, or mixed. This research adheres to qualitative methods in data gathering and analysis as they fit the nature of the research and the research questions. These methods allow for a thorough investigation into the stories that best help answer the research questions.

The true validity and trustworthiness of an action research lies with the existing data and its analysis. Authenticating data is vital because data is the main resource when showing the results in an authentic light (McNiff, 2017). I have kept all data organized by time and place and kept a log of all the work with the teachers and how and when data was gathered in those instances. In this section I explain the timeline of data gathering, how triangulation was used for data sources, the difference between primary and secondary data in this research, and all the methods used for data gathering.

3.7.1 Timeline of Data Gathering

Previous chapters have explained the PD program and how it developed alongside the research. To better explain when each type of data was gathered and the time relation, I have created Table 4. It shows at what time each type of data was gathered in the two years that the PD program took place. Some of the data was ongoing and gathered throughout the entire PD program whereas other data was gathered at specific times within the program. The timing of the data related to the different action research cycles as Mills' (2007) dialectical action research spiral was applied.

Table 4 The Gathered Data, Timing and Description

Data	Timing	Description
Videos, fieldnotes and artifacts from seminars	Sept 2018 – May 2020	All eight seminars are videotaped, as well as data gathered from teachers during the seminars that is related to their views and the projects they work on.
Reflective journals	Sept 2018 – May 2020	All the teachers and the facilitator use reflective journals to write about their learning process and how the PD program is influencing their practices.
Surveys	After seminar 4 (May 2019)	The facilitator provides the teachers with an open survey, asking the teachers to describe their views and changes thereof. This includes information about whether the teachers find the PD useful for their classroom practice, reflections on what would further support them, how they see student learning, and their vision for their continued work.
Action research	Ongoing, but reported in May 2019	The teachers choose their topic and gather data that is required for their topic. Data is gathered from the teachers who chose a topic related to the PD program on creative mathematics.
Interviews	Nov 2019 – May 2020	Interviews are conducted with the teachers who report the most use of the PD in their classroom. These are the five teachers from three teams that were chosen for case studies, as well as two vocational teachers.

In Table 4 the timeline of the data gathering is shown. At the beginning of the PD program, data was gathered from the first seminar on teachers' initial ideas of creativity and creative mathematics. After that, data was gathered from all seminars throughout the PD program. Reflective journals were presented in that first seminar and used throughout the PD program for the continued learning of the teachers. The surveys happened at the end of the first research year and the interviews during the second research year.

The timing of the data was planned in accordance with Mills' (2018) dialectical action research spiral. Data on teachers' initial ideas was analyzed in the second research cycle. The surveys were analyzed in the third research cycle. Interviews were analyzed in research cycle four. Other data from seminars, reflective journals, and teachers' action research was analyzed through all the research cycles. In research cycle five, all the data was analyzed again more thoroughly.

3.7.2 Triangulation of Data Sources

For a strong action research, it is important to make sure that all reported findings and conclusions are supported by multiple credible pieces of data (Sagor & Williams, 2017). Triangulation is a process where data about the same event is gathered from multiple sources and authenticated by different sources or observers (McNiff, 2017). This concept is closely related to that of overlap methods (Mills, 2018).

Triangulation was an important method for data gathering and analysis of this research. To triangulate different data sources and validate findings, I have cross-checked different sources, situations or points to see if the same patterns keep occurring. To show how I use the data to triangulate, I have created the following matrix (Table 5), based on a model from Sagor (2017, p. 111).

Table 5 Matrix Showing the Relation between Research Questions and Data Sources

Research Question	Data Source 1	Data Source 2	Data Source 3	Data Source 4	Data Source 5
How do the teachers explain the influence of the PD program on their pedagogy and views of mathematics education?	Interviews	Teachers' action research	Videos, fieldnotes and artifacts from seminars	Surveys	Primary researcher's reflective journal
How do the teachers who implement creative mathematics in their classroom report its influence on their classroom practice and student learning?	Interviews	Teachers' action research	Videos, fieldnotes and artifacts from seminars	Surveys	Primary researcher's reflective journal
What barriers do teachers experience when working with creativity in the mathematics classroom?	Interviews	Surveys	Teachers' action research	Videos, fieldnotes and artifacts from seminars	
How can teacher experiences, as seen through the action research, add to the existing literature on how to support teachers with creativity in their mathematics classroom?	Interviews	Teachers' action research	Videos, fieldnotes and artifacts from seminars	Surveys	

Gathering multiple types of data for each research question helps shed light on the PD program and the teachers' experiences, while data from different data sources added to the complexity of the analysis and showed different perspectives of the PD program. In the surveys, teachers answered very specific questions which helped develop the program and showcase the teachers' views. The interviews were open and free flowing, allowing for the teachers' input to the program as well as giving a detailed account of their classroom practice and student learning as they progressed in the PD

program. In the seminars, teachers shared their learnings with their colleagues, explaining classroom practice in a more thorough manner and commenting directly on student learning. The teachers' action research projects focused on what the teachers experienced as important in their practice and for their students' mathematical learning. Analyzing all the data helped in investigating the topic in its complexity and gave a full picture of the teachers' experiences and learning from the PD program.

Triangulation worked in many ways for this research, as I relied on a variety of different types of data and on three different cases that included five teachers, as well as data that related to how the entire teacher body participated in the PD program. Although I did not rely on a second observer, I sought support from critical friends at the university with data analysis and checked out my assumptions with the participating teachers. Triangulation of theory is explained in the section on data analysis. In the next section I explain how the data sources are seen as primary and secondary data, how the data connects to each case, and how teachers from the whole school participated in the PD program.

3.7.3 Primary and Secondary Data

Hox and Boeije (2005) write that "Primary data are data that are collected for the specific research problem at hand, using research procedures that fit the research problem best". Secondary data, however, is gathered for a different research purpose but used for the research at hand (Hox & Boeije, 2005). For this study, the primary data refers to the data that I gathered directly from the PD program seminars and participating teachers. This data includes interviews and surveys, along with videos, fieldnotes and artifacts from PD seminars.

The data gathered by the teachers for their action research is considered secondary data as the teachers were themselves engaged with different research problems that they chose. The presentations and reports on their action research projects are however analyzed for this study to gain a better insight into how they used the PD program on creative mathematics education in their practice and what they learned from the PD process. The reflective journals are also considered secondary data as they contain information related to each person's learning and personal inquiry. Nevertheless, my own reflective journal is considered primary data as it relates to my work with the teachers.

For each case I had different data sources, allowing for a thorough analysis based on triangulating and seeing results from different

perspectives. The same was true for the analysis of how the whole school participated in the PD program. Table 6 shows what data I have for each case, along with data I gathered that is related to the analysis on how the whole school participated in the PD program. Table 6 explains which of the data is primary data that was gathered specifically for this research and which of the data is secondary and was gathered and analyzed by the teachers. The secondary data was not analyzed directly for this research but the process of the teachers gathering and analyzing their own data was important for this action research and the teachers frequently referred to their own data throughout the PD program and research.

Table 6 Primary and Secondary Data

	Anna	Jenny & Silva	Sunna and Inga	Whole School
PRIMARY DATA				
Interviews	X	X	X	X
Videos from seminars	X	X	X	X
Surveys		X	X	X
Artifacts from seminars	X	X	X	X
Researcher's reflection journal	X	X	X	
Presentations from teachers' action research	X	X		
Reports of teachers' action research	X			
SECONDARY DATA				
Data teachers gathered for action research	X	X		
Teachers' reflection journals	X	X	X	X
Teachers' data related to student learning	X	X	X	X

As seen in Table 6, the data for the five cases varies as the teachers participated in different ways in the PD program. The first two cases of Anna and of Jenny and Silvia include an action research project that the

teachers chose to do, whereas the third case of Inga and Sunna does not. Nevertheless, that case has some other strong components, with two new teachers working towards bringing creativity into their mathematics classroom.

The data that was analyzed for Anna's case includes an interview with her, excerpts from her participation at PD seminars, and notes from my reflective journal that relate to our work together. Furthermore, a report and presentation from an action research Anna did with her co-teacher was analyzed. The data I rely on for Jenny and Silvia consists of interviews, a survey, and some excerpts from the teachers' PD seminars. Data was gathered from the presentation of an action research that Jenny and Silvia carried out, as well as my own fieldnotes and reflective journal. Inga and Sunna asked to be interviewed together and their case is built on that interview, surveys, and excerpts from seminars, along with my own reflective journal and fieldnotes.

The three cases were important for the triangulation process as I had data sources within each case as well as across the cases. Furthermore, I analyzed videos, fieldnotes, surveys, and interviews related to how other teachers at the school participated in the PD program. In the following section I explain all the methods of data gathering, why these were chosen and how they are used in this research.

3.7.4 Videos, Fieldnotes and Artifacts

Videos, fieldnotes and artifacts were collected from all PD seminars. Videos are common in qualitative research as they are a way to gather data from observations, and although not everything is seen in a video it allows researchers to come back to it and look at it after the event. As I was a facilitator at the seminars, video was very helpful for data gathering, as I could not write fieldnotes when I actively engaged with the teachers. I did however write fieldnotes while teachers worked on projects in groups, when I was in a more passive role.

At the initial seminar, I gathered data on teachers' ideas of creative learning and creativity in mathematics education. I gathered this data by asking the teachers to discuss the following two questions in groups of four to five people:

- What is creativity?
- What role does creativity play in mathematics?

The groups then reported back on their discussions, providing information about the teachers' initial ideas on creativity and the role that creativity plays in mathematics. As the teachers reported back on the two questions at the same time, answers to both questions are analyzed together. The analysis of this data was used in research cycle two to shed light on the teachers' views at the beginning of the PD program and helped in analyzing their developing views as the program progressed.

Videos from all seminars are used to reflect on the collaborative learning of the teachers and to plan further seminars during the third and fourth action research cycles. The videos from seminars were particularly important for the analysis of the case studies and excerpts from the videos were used for that aspect. Artifacts from seminars include projects that the teachers work on and written data from group work. These were used in research cycle two and three to plan the next PD seminars. Together with the field notes, these artifacts are also used for the triangulation of results.

3.7.5 Reflective Journals

> *Reflection about action* drives you to change the context and conditions of what you practice, so that your practice can improve a lot more. (Hargreaves & Fullan, 2012, p. 119)

Reflective practice is of great value in professional development of teachers (Hargreaves & Fullan, 2012). Although practising is very valuable, there needs to be an opportunity to reflect, otherwise there is little room for improvement. This crucial trait should be part of every professional development venture.

Reflective journals or learning journals can provide valuable information on the workings of a classroom (Mertler, 2020). As stated, the teachers are collaborators in this research process and help gather data and tell their own story. One tool I provided the teachers with for this process is reflective journals. I gave the journals to the teachers at the first seminar, when I presented the two-year program. I told the teachers they could write whatever they wanted in the journal that was related to their learning with this program, with other PD, or even their own musings on teaching and learning.

I told the teachers that I might later ask if they wanted to share with me something from the journal but that they were never obliged to do so. I also used such a journal to reflect on my learning process and observation of the joint process. In the end, very few teachers shared anything from their

reflective journals with me, and I did not use those for the analysis. Although not used for analysis, I believe that the teachers' reflective journals served as an important tool for the teachers' self-reflection during the PD process. I, however, used my own reflective journal as a tool in research cycles three and four for the development of the PD program. I also used my own reflective journal in the triangulation process for the data analysis of the selected cases when going through the fifth research cycle.

3.7.6 Surveys

Surveys are commonly used as a method when carrying out action research. Although surveys are commonly seen as quantitative, surveys were used qualitatively for this research. The intention was to have two surveys for this research. The first survey was given to the teachers midway through the program at the fourth seminar and the second survey was intended for use after the last seminar. However, unforeseen circumstances prevented the second survey from happening. The first survey can be seen in Appendix A. The questions are open-ended and encourage teachers to focus on their own views, learning and process. Teachers were asked what they had learned during the PD, if the PD had been helpful in their classroom, what they would find more supportive in regards to the PD program, what their views were on creativity in mathematics, and if they saw a change in their own views.

The purpose of the surveys was two-fold. The first reason was to evaluate the PD program and change it according to the teachers' needs. This analysis falls into the iterative action research cycle. Results from the first survey were used in research cycle three to develop the PD program based on teachers' feedback. The second reason for analyzing the surveys was to get descriptive data from the teachers. These results are incorporated into the triangulated data that sheds light on both the process of the PD program and the gradual transformation for the teachers, and is used for analysis in research cycle five.

3.7.7 Teachers' Action Research

At Vinaskóli all the teachers do action research each year. This is part of the PD at Vinaskóli and the teachers choose their own topic, gather their own data, and present their own results. The teachers are assisted in this by school management and often cooperate with their co-teacher. During the two-year research process of the PD program, two teams of teachers chose to conduct an action research project on mathematics learning. Teachers

from these two teams became two out of three cases that were chosen for the case studies.

Jenny and Silvia chose to analyze their own teaching process during one school year, looked at students' feelings and learning, and explored their own values in connection to mathematics education. Jenny and Silvia participated in the PD seminars and met with me four times during the year they did this action research. They were simultaneously involved in a PD that focused on using open problems in mathematics.

Anna also did research with her co-teacher. They researched how to increase interest and understanding in mathematics for their class of 3rd graders. The teachers' class had scored below average on a standardized test addressing numbers and operation. I worked with the teachers over the course of three months, planning and carrying out intense workshops with their class of 3rd graders. The teachers gathered data from the workshops and compared their end-of-year test results to the results of the previous standardized test.

For these action research projects, the teachers themselves gather their own data and come up with their own answers to their chosen research questions. I did not partake in the action research process with the first team of teachers but was involved in the workshops with the second team. However, the teachers chose themselves to do an action study: they decided the topic and gathered most of the data. These results were presented to other teachers at the end of the school year. For this research, the teachers' presentations and reports are used as data. This data includes the Power Point slides from their presentations and my own field notes from their presentations. These teams of teachers were chosen for the case studies. Analysis of the data from the teachers' action research projects helps in reflecting on their work, learning, and process in research cycle five.

3.7.8 Interviews

Interviews comprise the core data for this research. Interviews are a common way to gather data in qualitative case studies and in action research. I have chosen to use semi-structured interviews as they fit the research questions. Semi-structured interviews are usually based on a fairly structured question guide that is used in a flexible manner (Merriam & Tisdell, 2016). Although having a specific focus, the order or wording is not predetermined, and the researcher is open to listening to the participants and gathering information and new ideas on the topic.

In conducting an interview, the types of questions that are asked depend on the type of data you are trying to gather. Types of questions include the following topics: experience and behaviour; opinions and values; feelings; knowledge; sensory issues; and background (Merriam & Tisdell, 2016). For the interviews conducted for this research, most of the questions fall under experience and behaviour or opinions and values. This is because I am looking for information and accounts of the teachers' experience and behaviour within the PD program as well as their own views. I interviewed the five teachers included in the case studies and two vocational teachers.

When meeting with the teachers for interviews, I explained that the goal was twofold. I told them that I was gathering data for my doctorate action research and that it would be recorded. I explained that I would be the one listening to it and transcribing it, and that to protect their anonymity a pseudonym would be given to all participants to use in the analysis and all reports. I let them know that the school would be harder to conceal and most of them said they were not concerned by this as the nature of the topic was not sensitive. I further explained to them that the purpose of the interview included seeing how we could further work with creative mathematics and how I might better support that cooperative process. The interviews were taken in research cycle four.

When I did the interviews, I knew all the teachers and had worked with them all for at least two years at Vinaskóli. I had further been working with these teachers and their teams as part of the PD program inside and outside the classroom at the time of the interviews. I usually interviewed one person at a time, but in one instance the team of teachers asked if they could come together as that was convenient for them, so I conducted one interview with them both.

I conducted most interviews in a room that is used for meditation at Vinaskóli because I wanted to have a calm and inviting setting. The environment and an attentive listener are important for the interview process (Kvale & Brinkmann, 2015). My primary focus during the interviews was to be present and listen to what the teachers shared. I was mindful that it is important to listen more than talk during an interview and found that very helpful for this process.

The interview guide is found translated in Appendix B. I was open to the process of the interview and added follow-up questions and other relevant questions as I saw fit. Although not included in the interview guide, I usually asked the teachers to describe their background briefly if they did not do so

without prompting. I concluded each interview by asking if the teachers wanted to add anything. The information was used to improve and develop the PD program in research cycle four, but the interviews also provided crucial data for the fifth research cycle when bringing all the results together.

3.7.9 Summary

As explained in this section, the data gathering was an ongoing process intertwined with the PD program and connected to the five action research cycles used within Mills' (2018) dialectical action research spiral. For the trustworthiness and validity of this research it was important to gather varied data focused on different aspects of the PD program.

Videos, fieldnotes and artifacts were gathered from PD seminars and used for research cycles two to four and for the ongoing development of the program. Reflective journals tied in with the PD process for each person and surveys helped further develop the PD program in research cycle three. Teachers' action research projects were the teachers' way to explore their own and student learning while the PD program progressed, and interviews were vital for research cycle four in the second year of research. All the data was analyzed in research cycle five to provide a full picture of the PD program and the teachers' experiences of its influence on their views, pedagogy, and student learning. In the next section I give a detailed description of data analysis.

3.8 Methods of Data Analysis

> Every action research project is a story of what transpired during the course of the research. As in the world of literature, each story has its own theme, plot, and set of characters. (Sagor & Williams, 2017, p. 128)

McNiff (2014) notes that in action research people speak for themselves. The progression of an action research story is transformational and aims to communicate underlying connections between the characters in the story, the author and the reader, but also between experience and meaning (McNiff, 2014).

For this research the data analysis reveals a story, and in some cases multiple stories. The overarching story is about how the PD program unfolded, how it affected the teachers and how they changed from

participating in the PD program. Within this story there is a community of storytellers speaking for themselves through the data. My role is to write down these stories and bring them together to provide meaning and insight.

The goal of any PD is to influence teachers towards creating better learning opportunities for students (Desimone, 2009). Analyzing the data, I saw that it was my role to reveal the story of how the teachers changed their teaching to facilitate those learning opportunities for their students. I write the results in first person to be true to the scene that I was part of, but I use the teachers' words as much as possible when sharing their stories. The cyclical nature of the action research allowed this process to be dynamic and fluid, with the focus on supporting the teachers to bring creative mathematics into their classrooms.

In this section I detail how the analysis unfolded for this study. I clarify why I chose a manual analysis and I illustrate how coding and categorizing is used to develop themes that can add to existing theory on creative mathematics education. I then explain how the analytical process unfolded in relation to the PD program and how the four action research cycles happened through the use of Mills' (2018) dialectical action research spiral.

3.8.1 Choosing a Manual Analysis

Computer assisted qualitative data analysis software (CAQDAS) has become increasingly popular with advances in technology. Many use these programs to help find codes, as well as for categorization and other analysis. CAQDAS offers a variety of options to help data preparation, management and manipulation, including tools to analyze text and videos; easy ways to store codes and categories; and visual models (Merriam & Tisdell, 2016). These programs can be helpful and save time with the manual aspects of the analysis.

Saldaña (2015) argues that there is still some value to coding manually. He states that "there is something about manipulating qualitative data on paper and writing codes in pencil that gives you more control over and ownership of the work" (Saldaña, 2015, p. 22). He states that there is something to be said for a large desk space with multiple pages and strips of paper and that it can help the analyzer to see the smaller pieces of a larger puzzle. He also argues that for newer researchers, it can be overwhelming to learn both how to code and how to use CAQDAS at the same time.

Merriam (2016) writes that although some people like to directly handle the data and find that computer software can create an uncomfortable distance, CAQDAS can make the coding less tedious and provides new avenues for analysis. She notes that making the more tedious aspects of qualitative analysis easier can enable the researcher to focus more on the creative aspects of the analytical process, because while CAQDAS can help with the process, computer programs do not make any decisions. The researcher is still in charge of determining codes and deciding how to work with them. Nevertheless, the program offers an organized filing system for the data and analysis where data is sorted into categories, filed, and easily retrieved, saving time and effort.

I decided to start out with a manual approach and analyzed the first interview that way (Saldaña, 2015). This process was helpful and brought me closer to the data. However, after that I decided to try out CAQDAS and downloaded the computer program atlas.ti. I started using it and realized that although it probably had many positive qualities, I wanted to be physically closer to the data for this research. I wanted to manually code with colours and I wanted to hold the different types of data for each case. I therefore chose to use manual approaches of analysis for this research.

3.8.2 From Codes to Theory

> Coding is nothing more than assigning some sort of shorthand designation to various aspects of your data so that you can easily retrieve specific pieces of the data. (Merriam & Tisdell, 2016, p. 199)

Coding and categorizing is particularly useful for finding significant features and elements of the experience and for the perception of those involved in a study (Stringer, 2014). Procedures for this form of analysis involve reviewing the data, unitizing the data, coding, categorizing, identifying themes, and developing a report framework (Stringer, 2014). The coding includes reading the data slowly and dividing it into topics from different segments within the data (Efron & Ravid, 2020). This can be done by focusing on events or what is going on, and how these are seen by participants; by behaviours, including leading actions and response; by perspectives, such as how the participants perceive the issue and how their opinions differ or align; by relationships, and how these are seen by participants; and by strategies, including how these are used and seen by participants (Efron & Ravid, 2020).

Coding is a cyclical act that entails arranging data in a systematic order to see how it fits within a system or classification (Saldaña, 2015). This then allows you to organize and group similarly coded data into categories, and is the initial step towards a rigorous, evocative analysis and interpretation in qualitative research. The connections between codes, categories, themes, and theories are shown in a streamlined way in Figure 10. Coding is the first stage and categories emerge from that process and are later developed into themes and theory. However, qualitative research is not an algorithmic process and sometimes themes will merge at earlier stages. It is nonetheless important to follow the cyclical process of coding and then move into categorizing.

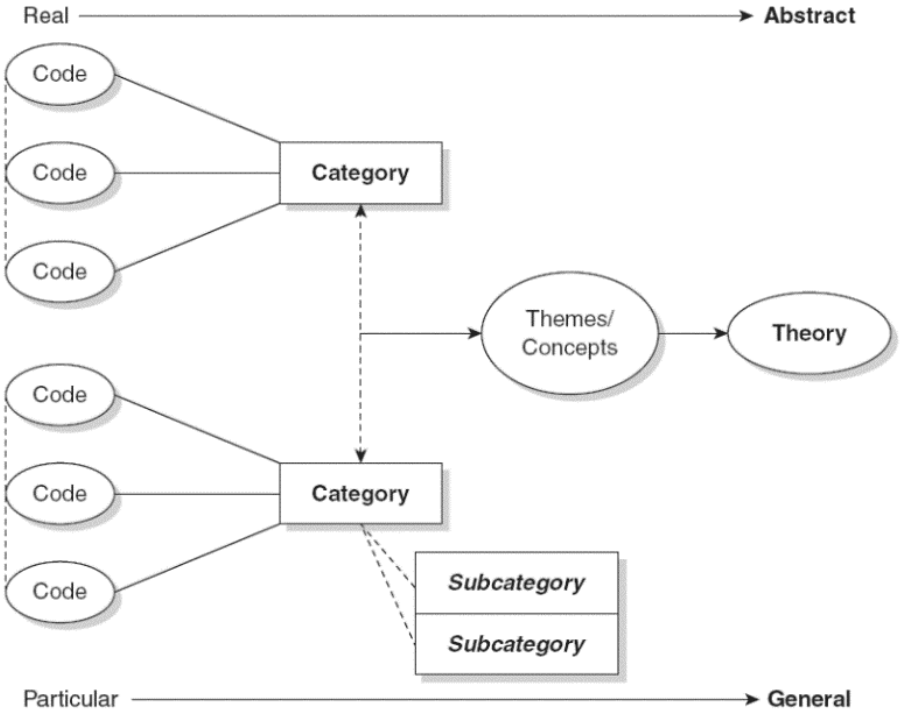

Figure 10 A Streamlined Codes-to-Theory Model for Qualitative Inquiry (Saldaña, 2009, p. 12)

There are two options for categorizing: working with predetermined categories or emerging categories (Efron & Ravid, 2020). For this research I chose emerging categories, as I wanted to work with the data from the ground up. This process involves developing codes and organizing them into

categories (Efron & Ravid, 2020). Figure 11 shows how this process of emerging categories is carried out.

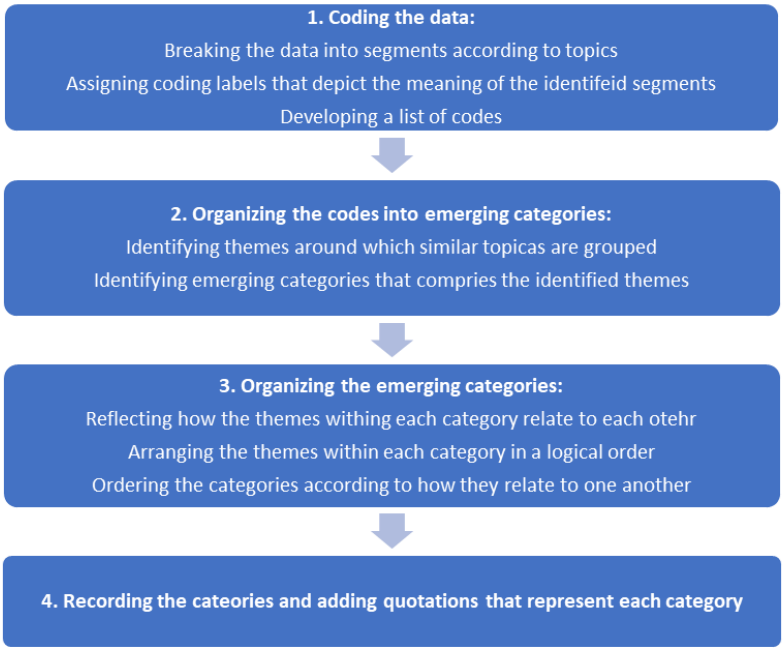

Figure 11 The Process of Using Emerging Categories to Analyze Data (Efron & Ravid, 2020, p. 179)

Constructing categories is highly inductive (Merriam & Tisdell, 2016). Starting with either bits of data or the codes, you cluster them together and name the cluster. This can then later be analyzed with other data and become a code. Naming these is an intuitive process, but I kept in mind the literature and the purpose of the study, while listening to what the data (or teachers) was telling me when finding and naming categories.

There are several layers of data analysis in a qualitative study (Merriam & Tisdell, 2016). Categories describe data but can also interpret it on some level. The process of moving from the observable to the conceptual involves moving from categories to themes, and from themes to theory. This last step in analysis is fraught with ambiguity, as it deals with connecting categories in a meaningful way

3.8.3 Analytical Process

> Data analysis is the process of making sense out of the data. And making sense out of data involves consolidating, reducing, and interpreting what people have said and what the researcher has seen and read – it is the process of making meaning. (Merriam & Tisdell, 2016, p. 202)

Data analysis is a complex process that involves moving back and forth between concrete data bits and abstract concepts (Merriam & Tisdell, 2016). It relies on both inductive and deductive reasoning, along with description and interpretation, and is the process of answering the research questions (Merriam & Tisdell, 2016). For qualitative analysis there is generally a common theme of an inductive process (Efron & Ravid, 2020). The analysis moves from parts to a whole, or from specifics to a general understanding.

Data analysis in action research begins as the data is collected and emerging preliminary understandings help shape, revise and refine investigation throughout the collection process (Efron & Ravid, 2020). This is true for qualitative research in general but particularly fits with action research and the cyclical process of research and action. During the PD program, data analysis was used to improve the program and change the learning opportunities for the teachers in a way that they experienced as beneficial for their students.

The first action research cycle investigates theories rather than data and began before the first PD seminar. However, the theoretical framework developed through using Mills' (2018) dialectical action research spiral and while data was gathered and analyzed in other research cycles. In that way, some theory was added to the theoretical framework as the research and PD program progressed. This is particularly true for literature on coaching and collaborative professionalism, as it became evident as the program progressed that coaching was fundamental for the teachers in bringing creative mathematics into their classrooms.

The second research cycle focused on teachers' initial ideas of creativity and creative mathematics and in that action research cycle data from the first seminar was analyzed to shed light on the teachers' ideas at the beginning of the program. The teachers' ideas were important in developing the PD program and in committing to a joint vision for the PD, which is a vital step in the Professional Development Design Framework as

explained in the theoretical framework (Loucks-Horsley et al., 2010). The joint vision we committed ourselves to at the initial seminar was to work towards fostering creativity in mathematics education and the data from the initial seminar was explored to help that vision.

The third research cycle happened in the first research year where I continuously analyzed data from seminars, my own reflective journal, and the surveys. This analysis was preliminary and mostly focused on reading and reviewing the data in order to further develop the PD program. Results were used to identify how the teachers experienced the PD program, what support they said they needed to better focus on creativity in their mathematics classrooms, and in what way they found the seminars most beneficial. The fourth action research cycle was similar to action research cycle three but included interviews taken at different times, which helped to better develop the PD program in a way that was useful for the teachers.

Research cycle five is the core focus of the results and discussion chapter. For the analytical process of research cycle five, all the primary data was used. The data was organized as it related to each case and the whole school PD program, while each case was analyzed as a comprehensive entity. The different data points provided thorough information about each case and pinpointed which contextual variables had a bearing on the case. After analyzing each case, a lateral cross-case analysis was done, building abstraction across cases.

Following the analysis of the cases, an analysis was carried out on data relating to how the whole school participated in the PD program. The analytical process of data from the entire teacher body helped in generating knowledge that sheds light on the research questions in a broad way and attempts to build some general explanations on creative mathematics learning, with potential value for mathematics teachers, teacher educators, those in charge of PD, and policy makers.

All the interviews and videos were transcribed using the computer software Express Scribe. After that, the transcripts and other data were coded and categorized with pencil colours, writing notes on margins. The coding and categorizing helped with identifying emerging themes as well as key experiences or critical moments in the teachers' process. When analyzing the entirety of data, two different methods were used. The first method was coding and categorizing to identify themes related to the research objectives. The second method was based on identifying critical incidents or key experiences to illuminate the experiences and transformation of the teachers.

Trend analysis was used to identify critical or compelling events. Understanding such events and asking why these events happen, along with how they relate to the place and time, can be very helpful in reflecting on the teachers' learning process and classroom change (Sagor & Williams, 2017). Such events include change in the priorities of the teachers or the performance or achievement of either the teachers themselves or their students. This analysis is based on reviewing the data, identifying key experiences, identifying the main features of such experiences and the elements that comprise them, and subsequently identifying themes from these features.

When analyzing and reporting the results from the interviews and other data from the teachers, I rely substantially on their own words. Using this approach helps capture the meanings inherent in people's experiences (Stringer, 2014). Stringer (2014) notes that those involved in the data analysis process must "bracket" their own understanding, intuitions or interpretations as much as possible to focus on the meaning inherent in the words of participants, although Sagor (2017) argues that our own intuition can provide insight for hypotheses. However, a thick description with a vivid narrative quoting participants' voices can provide trustworthiness, honesty and depth into the results (Efron & Ravid, 2020).

Interpretation of data can happen at multiple levels, including surface-level factual perspective and a deep-level analytical perspective (McNiff, 2017). Surface-level interpretation includes information on what transpired, who participated, where it took place, which aspect each person did, how they did it and why. A deep-level analytical perspective means engaging in multiple levels of reflection, analysis, re-reflection, and interpretations. For a deep-level analytical perspective it is important to respect the shifting meanings and patterns in the data. However, in the end there needs to be a decision of which interpretation to choose, especially if the goal is to take a specific course of action.

As explained previously, I relied on triangulation of data sources to ensure that I would paint a full picture of the PD program and the teachers' experiences. In the fifth research cycle I began by thoroughly analyzing all the data as it pertained to each case, such as interviews, surveys, and data from the action research. I then analyzed data from all the seminars and organized how it pertained to each case. This allowed me to use triangulation of data sources within each case. After completing the analysis for each case, I did a lateral analysis which examined themes that

were important across the cases, which allowed for further triangulation of results.

After completing analysis of the case studies in research cycle five, I did a thorough in-depth analysis of all the data related to the whole school. This data included all the data from seminars and surveys as well as some interviews. I used the same methods for this analysis with transcribing, colour coding, and finding categories to develop themes. Some themes were similar to the case studies and others were specific to special subject teachers and other aspects of learning. Analyzing data from all participating teachers was important to further triangulate the results and provide alternative perspectives on the PD program and research questions.

Another form of triangulation is based on contextualizing the findings with existing theory (McNiff, 2017). An important component in academic research is generating knowledge and adding to the body of theory that already exists (Herr & Anderson, 2015). By interpreting assertations and explanations from the findings within the context of theories and previous research, I can better support the depth of the analysis. This focus can be seen in the results, where themes are connected to and compared to previous research.

Although the research had goals related to the local research site, it also had the goal of generating theory that is of value beyond the setting. Given the nature of the educational action research, I was mindful to let the stories found in the analysis be the main focus of the results and generators of theory. However, the results from the teachers in the research were reflected on within the theoretical framework to see how they supported or added to existing theory on creative mathematics education, how teachers could be supported to foster creativity in their mathematics classroom, and what can be learned from the active development of a PD program of creative mathematics education.

3.8.4 Summary

In this section I have explained the data analysis. I account for the choice of a manual analysis and detail how coding and categorizing are used to develop themes and theory. The analytical process is described, along with how the analysis relates to the PD program and how Mills' (2018) dialectical action research spiral is used within the research. The analysis was partially ongoing but also took place after the two years of the PD program had been completed.

The research questions for this study look at how the teachers who take part in the PD program on creative mathematics report its influence on their pedagogy, their views of mathematics education, classroom practice and student learning. The cases are instrumental in answering the research questions. By analyzing the three different cases and the stories contained within them, a lateral analysis became possible. The cases were all unique but there were some strong commonalities that I emphasize, as they help answer the research questions. The teachers explain the barriers they experience when working with creativity in the mathematics classroom.

However, since the PD program and research happened within a whole school, valuable information was derived from the data that pertains to the whole school process. The data includes videos and fieldnotes from the creative mathematics seminars, surveys completed by teachers, and interviews with teachers who were not included in the case studies. Together with the case studies, the results pertaining to the whole school provide in-depth answers to the research questions. In the next section I explain the ethical considerations of this study.

3.9 Ethics

Ethical considerations are important for all research (Mills, 2018). Research studies are built on trust between the researcher and the participants. The two overarching rules are that participants should not be harmed in any way and that all participants should give informed consent to be in the research. Given the emancipatory nature of action research, the responsibility for ethical conduct and transparency is particularly important.

All the teachers and the principal at Vinaskóli gave written consent to participate in the research. In addition, the teachers who were interviewed also gave verbal consent to be interviewed and audiotaped. Children are a particularly sensitive group and special attention should be paid to their participation in research. Although the process of the PD program unfolds partially in the classroom, no direct data was analyzed from students for this research. Therefore, consent was not needed from students or their parents.

All the data gathered was kept confidential and pseudonyms are used for all the teachers who participated. The teachers were informed that all results are anonymous but were also advised that the school is more challenging to conceal, since the school community in Iceland is small. Although I have generally not dealt with information of a sensitive nature, I have been careful to do my best to hide the identity of the participating

teachers. No personal information on students is included. To further adhere to all ethical guidelines, The School and Leisure Department of Reykjavík and The Icelandic Data Protection Authority have been informed of this research.

3.10 Rigour

Research is always a scientific endeavor that must meet standards of sound practice. The basis of quality for traditional experimental research lies in concepts of validity and reliability (Mertler, 2020). Although a qualitative research process is subjective, it is not any less rigorous and relies on methods that ensure its findings are trustworthy and valid (Efron & Ravid, 2020).

The goals of action research are to generate new knowledge, achieve action-oriented outcomes, educate researchers and participants, and generate results that are relevant to the local setting using both sound and appropriate research methodology (Herr & Anderson, 2015). Herr and Anderson (2015) developed the goals and criteria listed in Table 7 to describe goals, quality and validity for action research.

Table 7 Anderson and Herr's Goals of Action Research and Validity Criteria (p. 67)

Goals of Action Research	Quality/Validity Criteria
1. The generation of new knowledge	Dialogic and process validity
2. The achievement of action-oriented outcomes	Outcome validity
3. The education of both researcher and participants	Catalytic validity
4. Results that are relevant to the local setting	Democratic validity
5. A sound and appropriate research methodology	Process validity

Outcome validity speaks to the extent to which actions lead to a resolution of the problem that is studied. Process validity asks to what extent problems are framed and solved and how these allow for ongoing learning of the individual or systems. Outcome validity is based on the process validity, which deals with making sure the evidence sustains assertions and that the relationships with the participants are of quality; this then connects with democratic validity that looks at the collaboration

of the process and how multiple perspectives are taken into account. Catalytic validity looks at the ability of researchers and participants to deepen their understanding of the social reality that is studied and how they move towards action to change it. Dialogic validity focuses on peer review and is particularly important for an action research dissertation.

For this research, the outcome validity focuses on how effective the acting is within the PD program in influencing teachers so they can better support creativity in mathematics. The process validity is based on the collaborative process with the teachers on site and in the research. This connects to how and what data was gathered and the process of analysis, with the goal of deepening the understanding, locally and in a broader context, of how creative mathematics can be fostered in a learning situation.

In this action research, I rely on ongoing collaboration with the teachers as well as my academic advisors, co-workers at the University and fellow students. The dissertation goes through repeated cycles of analysis and review. This was particularly useful for developing the PD program during the two years of research. By analyzing data from seminars, surveys and interviews, the teacher's voices are used to improve the program and collectively move towards better learning. This is connected to goal-setting to implement the action that is deemed helpful by those involved.

Although reliability is a common concept when describing the accuracy of research results, I have chosen to use the word trustworthiness as I find it better captures the essence of what is important to look at for a qualitative action research. Many action researchers use rigour in a broad sense within the whole research process, including reflection on bias (Mertler, 2020).

Tools to use for the trustworthiness of the qualitative inquiry relate to credibility, transferability, dependability and confirmability (Mills, 2018). These can include prolonged participation, persistent observation, peer debriefing, triangulation, varied and descriptive data, coherence, reflexivity and accuracy in writing (Mills, 2018). Action researchers use many of the tools for qualitative research, but due to the unique positionality of action researchers further measures are sometimes required (Herr & Anderson, 2015).

It is important to take disconfirming data seriously to ensure that assumptions are correct (McNiff, 2017). Reasonable limitations and alternative interpretations of the data should be recognized and addressed (Sagor & Williams, 2017). This practice also aids in methodological rigour

and transparency. To ensure the trustworthiness of my findings, I have searched for discrepancies and counter-evidence that could have refuted my assertions or provided alternative interpretations.

For this educational action research, prolonged participation was crucial for trustworthiness. I have explained how triangulation was used for data gathering and analysis and I rely heavily on participants' words when describing findings. I have explained my position within the research and although I am part of the research process, I am very clear that it is a collaborative process and that I work towards a joint learning process with the teachers who participate. In the next section I discuss the limitations and delimitations of the research.

3.11 Limitations and Delimitations

> Producing evidence of other people's learning is more problematic as it means producing instances of them saying that they have influenced their own learning or that someone else, including you, have influenced their learning: your evidence would be more robust if you could get their testimony that this actually was the case. This means asking the people themselves to say what they have learned and the significance of their learning. Their responses would stand as evidence of their learning. (McNiff, 2017, p. 87)

For this study, the research questions look at the influence of the PD program on the teachers' pedagogy and classroom practices. Answers are sought that reflect how teachers explain the influence of the PD on their views, teaching and student learning. The teachers and their stories are the main tool and evidence. The teachers are not observed before and after in their class, but their account of their learning and their process is used as evidence. Some might say that this evidence is subjective and from a traditional positivist research standpoint it certainly is. However, from a qualitative standpoint subjective does not have to be negative.

In this case, I have particularly chosen to do the research in such a way for two reasons. First, I believe in the value of the teachers' accounts and their visions of their learning and of what they see happening with student learning. This is imperative to show a detailed, thorough, and comprehensive description of how the teachers experience the change in their views, pedagogy, and teaching practices. The second reason is the collaborative nature of this action research. I wanted to work with the

teachers to improve teaching and learning, rather than as an observer trying to see if the teachers do what I want. This process could have been hindered by pre- and post- observation and the judgment that follows that process.

In a collaborative insider research study such as this, it is important to look at who the stakeholders are in each case (Herr & Anderson, 2015). In this research the teachers are certainly stakeholders who work towards creative learning in mathematics. Students are stakeholders as well, and it would have been interesting to include students' voices in this research. I absolutely believe that the student voice is important in action research and that much can be learned from students about their own learning. However, for this research the focus is on the teachers and their learning, views, and transformation. This same process for students is beyond the scope of this research, although it is important as a research topic for a later date.

Personal bias is another point that has been addressed (Herr & Anderson, 2015). We all enter the research process with a perspective drawn from our own unique experiences. I have previously acknowledged these perspectives or biases to build critical reflexivity into the research process. I have used my own reflective journal and field notes for self-reflexivity. I also use a variety of tools that have been chosen to ensure the analysis process is wholesome, honest, and clear.

Unlike quantitative research, although qualitative research results are not generalizable in a statistical sense, there are other ways to approach the subject of generalizability in terms of an action research (Herr & Anderson, 2015). For this research, the term generalizability is seen in the potential the results hold to create transferable knowledge of how to support teachers to promote creativity in their mathematics classrooms.

This extended research project takes place within a PD program that is intended to further the teachers' knowledge and ability with creative mathematics. Researching such a PD program helps create a better understanding of how such a PD program can be of use to teachers. It further adds to understanding on what barriers need to be overcome for teachers to work with creativity in their mathematics classroom. The transferability lies in furthering knowledge on how to promote creative mathematics education and holds implications for teachers, students, and research.

Delimitations of this research are based on the choices of the case studies. Although the PD program happened within a compulsory school that teaches children from ages 6 to 16 and all the teachers at the school participated in the PD program, the teachers chosen for case studies all taught the first four grades. This allowed me to do a lateral cross-analysis

with teachers who taught a somewhat similar student group and provided in-depth answers for this particular age. Data from other participating teachers was also gathered and analyzed but provided results in broader strokes. This means that the core results are based on a younger student group. A follow-up study on other age groups would be interesting, especially in regard to a model for creative learning that was developed based on the results.

3.12 Summary of Methodology and Methods

The methodology and methods of the educational action research on the PD program of creative mathematics education have now been explained in detail. As the program was a collaborative construct at a school, the methodology of educational action research fits well. The research happened at a school where the teachers were already working in creative ways in other subjects and actively participating in regular PD. By bringing in this particular PD program, I was able to support the teachers to foster creativity in their mathematics classrooms.

The PD program was originally constructed based on theories from the theoretical framework, but the program developed with the teachers at the school. The teachers participated in collaborative seminars and implemented creativity in their classrooms between seminars. Teachers who requested support for implementing their learnings into their teaching received coaching from me. Case studies were chosen based on the teachers who I worked with the most.

Within the methodology of action research, there are several methods that can be used for data gathering and analysis. In this chapter I have explained how the data was gathered, how triangulation was used for trustworthiness and how the analytical process unfolded using Mills' (2018) dialectical action research cycle. This process led to in-depth answers about how the teachers reported the PD program had influenced their pedagogy and classroom practices.

I have explained the importance of ethics and rigour for this research and how validity and trustworthiness were ensured. An account has been made of the limitations and delimitations of the study. In the following chapter the in-depth results of the data analysis are presented, as well as a discussion of how the results connect to the theoretical framework. The chapter explains how the thorough investigation into the learning process of the teachers has led to stories of transformation that can inform future practice.

4 Results and Discussion

Having explained in detail the methodology for this study, we now look at how the analysis unfolded and the relevant results. This process proved to be an interesting journey of discovery and learning, and throughout this chapter I try to make it clear how the results were derived using Mills' (2018) dialectical action research cycle as well as discussing the meaning they hold in a wider context of theory and practice. The first action research cycle focuses on exploring theory; it began before the PD program and developed with the ongoing analysis throughout the other action research cycles. Therefore, the discussion of the results in relation to theory is included in this chapter rather than in a separate chapter. This is common in action research and allows for a more connected analysis and theorizing (McNiff, 2014).

The primary focus of this chapter is to describe the results from an in-depth analysis of case studies and the ways in which the whole school participated in a PD program on creative mathematics learning. The goal is to explain the learning of the teachers within the PD program and provide answers to the following research questions:

1. How do teachers who take part in a PD program on creative mathematics education report how it influences their pedagogy and classroom practice?

2. What can be learned from the active development of a PD program of creative mathematics education?

With the sub-questions:

A. How do the teachers explain the influence of the PD program on their pedagogy and views of mathematics education?

B. How do the teachers who implement creative mathematics in their classroom report its influence on their classroom practice and student learning?

C. What barriers do teachers experience when working with creativity in the mathematics classroom?

D. How can teacher experiences, as seen through the action research, add to the existing literature on how to support teachers with creativity in their mathematics classroom?

This chapter begins with a section on the teachers' initial ideas that were expressed in the first PD seminar and analyzed from group discussions in action research cycle two. Following that are three sections that detail the results from the individual case studies that were analyzed in depth, and which focus on five teachers who belong to three different teaching teams. The case studies are based on analysis that took place in research cycles three to five. The focus is on how the teachers explain the influence of the PD program on their pedagogy and classroom practice and our collaborative journey throughout the PD program as seen through action research cycles three and four.

After presentation of the results from the three case studies, there is a section focused on the lateral analysis of the cases from the fifth action research cycle. Next, there is a section on the value of the whole school program, which is based on results from data related to how the entire teacher body participated in the PD program. These results are based on action research cycles three to five and provide broader information on the PD program. The summative results from the case studies and the whole school PD program were developed into a model for creative mathematics learning that is presented in the last section. The model connects the most prevalent findings from the study in a novel way. Together, the results show how a whole school PD program on creative mathematics education can work to influence teachers' pedagogy and practice in a positive way.

4.1 Initial Ideas of the Teachers

At the beginning of the first PD seminar, I asked the teachers to discuss in groups what creativity is and its role in mathematics. This inquiry into the teachers' ideas in the second action research cycle helped in the development of the PD program which, as explained in the theoretical framework, was based on committing to a joint vision and standard for the PD, as Loucks-Horsley's (2010) design framework presumes. This is in line with Jaworski's (2011) socio-cultural model for PD, which assumes that mathematics teachers are active participants in their own PD. Incorporating and building upon teachers' knowledge and beliefs when embarking upon PD is important (Desimone, 2015; Jaworski, 2011; Loucks-Horsley et al., 2010; Zepeda, 2019). Therefore, it was seen as important to start by

hearing how the teachers at Vinaskóli viewed creativity and creative mathematics.

In that first seminar I began by asking the teachers if they had previously thought about mathematics and creativity together. Although most of the teachers said no, some claimed they had considered creativity and mathematics jointly. When they discussed this further in groups, the teachers were able to see the importance of creativity in mathematics. The teachers had their own ideas about creativity and mathematics learning and stated that they were open to new learning.

During the discussions, the teachers shared interesting ideas about creativity. They brought up how creativity is the driving force of life, how important it is to have an open mind for creativity, and how important it can be to put creative thoughts or ideas into words or action. The teachers brought up the importance of flow for creativity and reflected on the creative process, the role of play in creativity, and how research plays a part in creativity.

The group discussions revealed themes that emerged from the analysis of the second action research cycle. Some of the groups brought up similar ideas, although some groups also added points not noted by other groups. Four themes emerged from the initial discussions of the teachers about what creativity is and what role creativity plays in mathematics, namely mathematics all around, play in mathematics learning, developing solutions, and problem solving. These themes are addressed and discussed in four different sub-sections.

4.1.1 Mathematics All Around

When discussing creativity in mathematics, many of the groups noted that mathematics is everywhere and relates to many areas of our life. Different groups noted that mathematics can be connected to art, engineering, architecture, natural sciences, home economics and students' daily life. The teachers felt it was important to connect mathematics to students' reality, experience, and events. They saw how mathematics can be a tool to use for creativity and how it is connected to experiments and research.

A common theme for the teachers was art and one group particularly noted creative artists, such as Björk Guðmundsdóttir, with her biophilia educational project (https://biophiliaeducational.org/) and how she connects mathematics to music and rhythm. Another group wrote that creativity and mathematics is about "creating a new awareness and ideas, and making shapes, patterns, and numbers in the wonderful mathematical

form". Some of the teachers even went as far as talking about the mental impressions of creativity and the overall beauty of mathematics.

These ideas of mathematics as parts of other subjects can be mirrored in the literature around creative mathematics. This focus can be seen in what has been written about creativity in general and further resonates with mathematical modelling, STEM education and more. Some of the art or individual subject teachers brought in a different view by noticing music, art, or feeling. Goldin (2017) touched on a similar theme when he wrote about the emotional aspect of mathematical creativity. It was interesting overall to see how widely the teachers noticed creativity in mathematics, despite not having given it much previous thought.

4.1.2 Play in Mathematics Learning

In the discussions, a few groups mentioned play as an important aspect for connecting creativity and mathematics. The groups discussed different techniques they used, such as games, learning mathematics outdoors, using stories, and learning through play. The teachers were often focused on how mathematics learning could and should be fun.

Some groups noted how they wanted to use creativity to encourage children's imagination, picturing a story. For example, one teacher said: "As soon as you get them [students] with you into something like this [play, story or imagination] then you have caught them," implying that play and fun are crucial aspects for students' motivation. Another teacher noted how creativity can help students collaborate and said: "Creativity unites, because different strengths can blossom. Creativity is a way to combine different strengths of students."

Other groups of teachers brought up concepts such as expression, stimulation, connections, and perspectives, emphasizing how important it is to allow students to express themselves and see mathematical connections from different perspectives. This focus on fun, expression and collaboration was seen through the group discussion and is mirrored in both the writings of Csikszentmihalyi (1997) on flow and in Amabile's (1989, 1996, 2013) musings on the role of motivation for creativity.

4.1.3 Developing Solutions

The discussion of many groups was focused on how students need to develop their own solutions to mathematical problems. The teachers emphasized how important it was to allow multiple ways to reach a solution to a problem. One teacher said: "Children create their own ways,

whether for traditional mathematics or other. It is something they create inside of themselves." Another teacher was clear on the learning process for students and expressed it by saying, "You are going to get over this obstacle of your own accord and find your own way to do so."

Some of the groups discussed the importance of a creative teacher who is open to both new ideas and new modes of teaching. Some teachers shared experiences from their own mathematical learning as students, where they felt that they had not been allowed to question the reasons behind the mathematics they were learning. The teachers felt that this type of learning had hindered their own understanding of the mathematics they were expected to learn. One teacher explained this by stating that if she, as a student, had asked why something was the way it was in mathematics, she got the answer "It just is". The other teachers generally agreed with this sentiment and felt that this was not a creative way to learn mathematics.

Though the teachers did not refer to learning with the terms conceptual and procedural learning, they are in fact describing the difference between those two modes of learning mathematics. The teachers describe how they learned mathematics through procedures they were expected to follow. The teachers are, however, clear that this is not a creative way to learn mathematics and that they prefer a more conceptual mode of learning where students are given an opportunity to understand mathematical reasoning behind their learning.

This same focus can be seen in the literature that describes how moving from procedural-based learning to one that is concept-based is a cornerstone for creative mathematics learning (Luria et al., 2017). It is evident that the teachers were aware of the importance of concept-based learning, which aligns with the theories of creative mathematics education.

4.2 Problem Solving

Closely related to the idea of developing solutions is the concept of problem- solving, which was particularly noted by a few of the groups. Figure 12 is an example of data where one group has written up their musings on creativity (Hvað er sköpun) and creative mathematics (skapandi stærðfræði). In that image, the group has written that mathematics is everywhere and that there are many paths to a solution. The group noted Björk's Biophilia (https://biophiliaeducational.org/) and the importance of the teacher being open to new ways in their classroom. The group wrote down "open problems" on paper and made visual connections to point to many ways to a solution to explain the connection between the two ideas.

Furthermore, the group noted a connection to music and numbers, as well as the practical aspect of creativity. In addition to their musings, they created their own art to make their page more attractive to the audience.

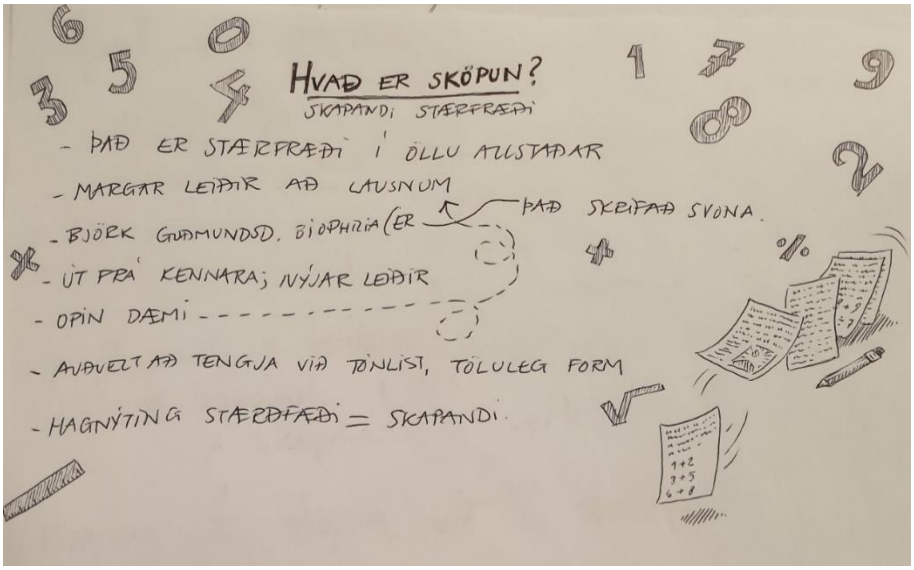

Figure 12 Image from One Group on Ideas of Creativity and Creative Mathematics

Although using different terms, the other groups captured similar ideas on problem-solving as well. The terms they used included open problems, mind puzzles, and solving problems. It was clear that all groups thought that problem-solving was an important aspect of creative mathematics learning. Problem-solving is seen in the literature as an effective way to encourage creativity in mathematics (Beghetto, 2017; Nadjafikhah et al., 2012; Sheffield, 2013; Sriraman et al., 2011; Sternberg, 2017). Therefore, it was interesting to see how many of the teachers looked at problem-solving without prompting or instruction when reflecting on creativity in mathematics.

4.2.1 Summary

Beginning the PD program by inquiring into the teachers' ideas about creativity and creative mathematics was important for the development of the PD program. Although most of the teachers stated that they had not given the concept of creative mathematics any thought previously, many of the ideas they expressed at the initial PD seminar align with theories of creative mathematics learning. This includes their ideas around art in

mathematics, problem-solving, the role of play and motivation, and conceptual learning of mathematics (Amabile, 1989, 1996, 2013; Beghetto, 2017; Luria et al., 2017; Nadjafikhah et al., 2012; Sheffield, 2013; Sriraman et al., 2011; Sternberg, 2017).

A crucial step in the development of the PD program was discussion of the teachers' initial ideas, followed by presenting the teachers to theories of creativity and creative mathematics learning as presented in the theoretical framework. Through the second action research cycle, joint vision and standards were set and the PD program began at Vinaskóli. The teachers and I embarked on a learning journey of discovery and creativity. The next three sections present the results from the case studies that were developed through action research cycles three to five.

4.3 Anna's Story

> First of all, I find mathematics very enjoyable. I believe that mathematics is by nature fun because it is entertaining to solve puzzles and to think about sizes and the mathematical reality. (Anna, November 2019)

Anna has been a teacher in compulsory school for five years. She usually teaches children in grades two or three. She shares that she really likes mathematics and thinks that it is inherently fun to solve problems and think about "the reality of mathematics". In addition to her classroom teaching, she has worked as a tutor for older students. She says that she previously based her teaching mostly on her intuition and the textbooks provided by the school. In her interview, Anna says that she appreciates the opportunity to look back on how she started out and what she is doing now. She says that when she began teaching, she did not have much to base her teaching on and relied on the textbooks that her school has been using. She says that in the beginning her teaching was based on the idea "Now we are working in this textbook".

During the first research year, Anna and her co-teacher taught 35 students in third grade. This was the second year she and her co-teacher taught these students. Anna and her co-teacher did creative mathematics workshops for three months with their students, with my participation and coaching. During that school year, Anna and her co-teacher conducted an action research project where they investigated their own implementation of these creative workshops that were based on the PD program. They examined the process of the workshops and student learning. They

presented their research at the end of the first research year. The workshop and Anna's action research took place in the third action research cycle and were instrumental in how I chose to work with other teachers.

In the second research year, Anna taught a different class with 42 students. Her students that year were in second grade, and she had a different co-teacher. I did not get a chance to work with Anna as intently that school year but interviewed her in November and met with her and her co-teacher a couple of times. She and her co-teacher planned weekly student workshops that year on their own, where she used some of her learnings from the first research year. Therefore, Anna's story develops through action research cycles three and four and is brought together through the analysis of action research cycle five.

My intensive work with Anna and her co-teacher in the first research year began after they approached me. Their students had previously scored low on a common evaluation test based on numeric problems that tested the four basic mathematical operations. Following the poor results, I was asked to come to an "emergency meeting", as Anna called it, which was held with the principal, special educator, Anna, her co-teacher, and me. This meeting was to discuss these results and to decide what steps to take to support the students in the class.

It was decided that creative mathematics workshops would be held for three months with the students. The student workshops were planned twice a week for two hours, at a time when the class usually had an additional teacher, but in this case I would be that teacher. The workshops began in December of the first research year, and it was the first time that Anna and her co-teacher implemented their learning from the PD seminars on creative mathematics.

Anna, her co-teacher and I held weekly meetings to plan the workshops and reflect on student learning. At the meetings we discussed the goals for the students and planned the activities that we would use with them to meet their learning goals. In this process I was open for ideas from Anna and her co-teacher, but I brought in my own thoughts which were based on theories of mathematics learning and my own experiences of creative mathematics teaching and learning as they connected to the PD program.

In both her interview and the presentation of the action research that she did with her co-teacher, Anna explains how she found the workshops to be beneficial for her students. Anna was clear that a longer class period and smaller group size allowed her to focus on different components that she experienced as crucial in the creative mathematics workshops we did with

her students. These components were conversation about mathematics, learning with manipulatives, and games and play. They are discussed in the following sections, as well as results on time and space and student learning. These are the themes that emerged from analyzing the data related to Anna's case through action research cycles three to five.

4.3.1 Conversation about Mathematics

Anna was clear that the collaboration on the creative student workshops provided her with an opportunity to focus on rich mathematical conversations with the students. She experienced that conversations with students about mathematics were very important. I noted in my reflective journal that this was one of the first points she shared with me after the first day of the creative mathematics workshops with the students. In her interview, she said:

> What I got out of it [the creative mathematics workshops with the class] was new insight into mathematics. What I learned from the collaboration between the three of us who were working together, was the conversation. To suddenly have time to discuss mathematics, discuss the meaning of the concepts we are using in mathematics, what lies behind these concepts and to get the children to express their thoughts about numbers and operations, as we were mostly working with operations. To get them to go deeper and understand and discuss the meaning of what we are doing.

This same focus was seen in the action research she did with her co-worker. During their presentation of that research, Anna said: "A longer, more creative project was possible when we had two consecutive hours with the class. We had time for mathematical discussion, which was new for me. That was something we gained from working with Ósk [and from participating in the PD program]."

Anna's focus on the mathematical conversations with students and the time needed to develop meaningful discussions aligns with theories of creative mathematics (Luria et al., 2017; Nadjafikhah et al., 2012; Pound & Lee, 2015). She described the fact that she was able to focus on the conversation with students as part of the learning process, as something she had wanted to do all along but had not had the opportunity to do. She believed that the ability to implement conversation in mathematics was a gift for her and particularly for her students. She commented on this after

the first workshop day and it was the first thing she brought up in the interview, while her expressions gave evidence of relief and pleasure.

4.3.2 Learning with Manipulatives

In the creative mathematics workshops, the students worked with various aspects of mathematics through real-life projects, games, and problems. Anna and her co-teacher prepared a game they called "the big store game", where each student got a fixed amount of educational play money to go shopping with and practised adding and subtracting. Another game in which the students participated was a paper shuttle competition, practising measurement and addition while working collaboratively in a team. They played various games that integrated mathematical concepts, including the base ten system and the four operations. The students worked collaboratively on various problems using manipulatives such as centicubes, measuring tape, dice, etc.

Anna described in her interview how she found that it was important for student learning to use manipulatives in the creative learning process:

> Because we had continuous time and small groups, we were able to use manipulatives in a new way. We were able to have a more hands-on classroom. Using centicubes, measuring tape and other tools helped to connect concepts to hands and senses.

Anna's emphasis on the importance of using tools to build mathematical understanding was also evident in the action research project she and her co-teacher conducted. In their presentation they shared: "It was better when the children had something in their hands, i.e. when they worked manually with tangible materials such as money, cubes, blocks, measuring tape and games."

Theories of creative mathematics report how important it is to connect to children's reality through the use of hands-on learning (Boaler, 2016; Pound & Lee, 2015). Clements (2000) further writes about concrete manipulatives for mathematics learning and the importance of sensory-concrete experiences for younger students. It was evident that Anna was able to use manipulatives more in the creative workshops when she had longer sessions and smaller groups.

4.3.3 Games and Play

Another component that Anna described as being important for student learning that was a result of the creative workshops was using games and play:

> We were able to make better connections within the mathematics learning by relying more on play and games. We suddenly had an opportunity to teach the students games. When we had taught them the ten-friends game, or some other game, they were able to use those games again. The students were then able to use the games also in the middle of the week and in play amongst themselves. It therefore strengthened what we were trying to teach through that game.

Theories on creative mathematics focus on the use of games and play in mathematics learning and the connection to student motivation (Pound & Lee, 2015). Closely related to this is the idea of fun that was brought up in the initial seminar with all the teachers. In one seminar in our second research year, Anna further commented on how her students can create meaning in mathematics in a playful way. She tells the story of one student in this regard: "I had a child this morning who braided their hair, laid the braid down over the middle of their face and said: 'I am a reflection angle'." Anna sees how it is helpful to get students motivated in learning mathematics by incorporating playfulness and games in their learning.

This idea of play and joy in learning is connected to Csikszentmihalyi's (1997) writings about flow, which is often seen in children's play. The idea of flow is closely connected to Amabile's (1989, 1996, 2013) writings about the importance of intrinsic motivation for creative learning. Craft (2011b) also identifies play as one aspect of possibility thinking which plays a crucial role in the creative learning process, as mathematics learning through play is one way to foster creativity and teachers can be instrumental in encouraging students' playfulness (Baer, 2013; Daniels, 2013; Pound & Lee, 2015; Ranjan & Gabora, 2013).

4.3.4 Time and Space

It can be seen throughout Anna's comments that she feels that time, the learning environment, and group size are important factors in her being able to provide creative learning in the mathematics classroom. Anna mentions time as a crucial aspect of learning in her interview, in our

discussions at meetings, and in the action research she conducted with her co-teacher. In their presentation of the action research project, she and her co-teacher stated:

> We particularly enjoyed being able to do a longer, creative project since the workshops were at a time where we had two continuous hours (it was worth it to set up a store). We also enjoyed having time for mathematical discussion.

Anna also states in her interview that she found it particularly useful to have a period of time that was sufficiently long and without interruptions, as well as a third teacher, which gave the option of dividing the class she shared with her co-teacher into three groups instead of two. She explained: "This meant that we could support each individual [student] better in a somewhat smaller group [of 10-12 students]." In her second year Anna was teaching a larger group and she commented on how hard it was to have the conversations she found so helpful for mathematics learning: "We are trying to go into smaller rooms with groups to present and have a conversation but it is almost impossible to have a conversation in two 22 student groups, each in one half of the classroom."

Anna's comments on time and learning space are certainly in alignment with theory that argues that creativity often is not pursued in learning because of lack of time and resources, when time is a crucial component in creative learning (Daniels, 2013; Fautley & Savage, 2007; NACCCE, 1999; Ranjan & Gabora, 2013; Sternberg & Williams, 1996). This is true for general theories of creative learning, and particularly to creativity in mathematics learning where students need to be given ample opportunities to develop their own solutions, partake in discussions and experience a flow of thought (Boaler, 2016; Nadjafikhah et al., 2012; Shriki, 2010; Sternberg, 2017).

4.3.5 Student Learning

> We felt like we took flight at the end of the mathematical workshops [we did with the students]. (Anna, November 2019)

As previously discussed, the goal of any PD must be to affect teachers' views and practices in a way that results in better student learning outcomes (Desimone, 2009). This research does not focus specifically on learning outcomes, but Anna and her co-teacher investigated that aspect in their own action research that she explains with the following words:

> To wrap up the [mathematical] workshops, we conducted a little research on student progress regarding the operations that *Talnalykill* [the evaluation test] had assessed. Our results indicated progress. We gathered data that showed progress for the children. Of course, we do not know if they [the students] would have accomplished the same [learning] if we had not done the workshops, but my evaluation is that they [the workshops] were crucial in that they jumped ahead. So that was very interesting.

Anna explained how she and her co-teacher investigated, as part of their action research project, the impact of the work they had implemented in the classroom. This investigation was based on using a written assessment to look specifically at how the students fared in solving problems connected to the four operations and the place value system. Anna and her co-teacher researched how many children reached the goals and understood problems related to the four operations and the place value system. Anna explained that although their investigation was not complex, they noticed a great improvement in students' success on the assessment compared to that from before the creative workshops. Through their inquiry, Anna and her co-teacher noticed how the students had a better grasp on three out of four basic operations. Anna noted that it was particularly interesting to see that the same depth of student understanding was not noticeable with division compared to the other three operations. She reiterated that she thought that this was due to the workshops focusing more on the other three operations and not on division. Anna explained that she saw how many of the students had grasped concepts after the workshops that they did not fully comprehend before.

Anna experienced the workshops as instrumental for student motivation and learning of basic skills and explained this by stating: "There was some mass [of knowledge] that was gathered during this time period [of the workshops] that further supported what came after [in learning]. You [teachers and students] are well prepared to continue." This emphasis on how the student workshops helped student learning and prepared them for further learning was summarized by Anna with the words: "We felt like we took flight towards the end of the mathematical workshops." Anna's experience and descriptions of the student learning process are aligned with Desimone's (2009) explanation of the process of a successful PD, shown in Figure 1, which emphasizes that the goal of any PD is to affect

teachers' views and practices in a way that results in better student learning outcomes.

Anna further discussed the connection she saw between formative learning and creative mathematics. At her school there are regular PD sessions on formative learning that the teachers implement in their classrooms. Anna has focused on implementing study buddies, where two to three students collaborate and discuss concepts related to their learning. Anna states that she experienced that this aligns with the creative mathematics work. She appreciated that during the workshop she was able to have a discussion with a smaller group of students about what mathematical concepts mean and explained that she found it useful to hear from students about their mathematical understanding. Anna states that the longer lessons allowed time for the mathematical discussion with students and notes that the normal 40-minute mathematics lessons are too short. She states:

> Having two hours changes everything. You can be so relaxed around the discussion. It becomes easy to use the formative learning when you have an extended time period, because it takes time to use this method [of conversations in the mathematics classroom]. I am very interested in using what [components] I thought were interesting from the [student] workshops, which is the conversation, using manipulatives for learning, bringing the learning of mathematics to life with play and games, and having the time for that.

Anna is clear that she wants to continue focusing on creative mathematics in her classroom, but she is very adamant about how important it is to her to have good space, a manageable group size, and manipulatives to work with.

4.3.6 Summary

It is obvious that Anna gains a clearer vision of how she would like to teach mathematics through our collaboration in the first research year. She is clear that she finds it important to have time for rich mathematical discussion with students, to use manipulatives for learning, and to provide students with opportunities for play and games. Anna explained in her interview: "These were the three things I felt we gained most from when we were doing the [student] workshops with you: the discussion, using manipulatives for learning, and play and games. These brought new depth,

life and joy into the mathematics." Here, Anna articulates the importance of joy and motivation in mathematics learning. Her reference to new depth in learning and how she has articulated the importance of discussion makes it clear that she sees value in strong conceptual learning and understanding, as opposed to what she previously referred to as "working in the book".

When asked in her interview what support would help her to work towards her goals in mathematics education, she answered:

> The longer uninterrupted time clearly helped. It helps a lot to get assistance from people, such as teachers or teacher assistants. It also helps if the school is well equipped with manipulatives that are easily available, such as centicubes, counters, measuring tapes, etc. Having more games available that are compatible with the goals that we are working with would be great. And like I said, it clearly helps to have a long time. Also, obviously it is very rewarding to be in an adult community discussing mathematics and such [at PD seminars].

When discussing what barriers might prevent Anna from working in a creative manner in her classroom in general, she brings up time, space, and group size. She also notes that behavioural problems and how broad the learning group is can be a challenge, especially in a larger group. She says that some students lack self-efficacy and that those who are quiet often do not get the support they need while she is managing the noisier ones. The ones that are fast or ahead are often not given an opportunity to grow and learn with the little time the teacher has for them.

Overall, working with Anna was a delight. There were learning opportunities for me as well, since I was not used to teaching the younger grades. Anna had a calm approach to discipline that I saw was valuable. Working with the students, I was able to further experience how the theories of creative mathematics learning apply to this age group. Anna was very clear on how it helps student learning to provide them with manipulatives, meaningful mathematical conversations and opportunities for play and games. She sees the value in having conversations with students about mathematical concepts and how the students understand the mathematics they are learning.

The largest part of my collaboration with Anna happened in the third research cycle, which was the first research cycle of actively collaborating with teachers to incorporate creative mathematics in their classroom.

Through our collaboration I realized the importance of collaborative professionalism and how important it was to embark upon a collaborative inquiry with collective responsibility and initiative through mutual dialogue. Anna and her co-teacher worked jointly towards a common meaning and purpose through collaboration with students, as described by Hargreaves and O'Connor (2018). It also became evident that coaching was a helpful method of supporting teachers who wished to focus on creativity in their mathematics classroom.

On a more practical level, I realized through the work with Anna that as a new school, Vinaskóli needed more manipulatives to allow the teachers to work creatively in the mathematics classroom. Thus, I started an inventory of the manipulatives and discussed with the teachers which manipulatives were needed for their classroom. The teachers were very grateful for this, and other teachers used these manipulatives as part of incorporating creative learning in their mathematics classroom.

It proved beneficial to begin the collaboration with Anna early in the research process, and the collaboration provided some valuable information for the ongoing PD program. The next section focuses on Jenny and Silvia, who I worked with in the two research years, but the second year became more focused as I had been through research cycle three after the intense work with Anna.

4.4 Jenny and Silvia

Jenny and Silvia are co-teachers and were new to their collaboration when the research began. Jenny and Silvia have both a bachelor's and a master's degree in education for teachers at compulsory school. Jenny and Silvia have both exclusively taught at the youngest level of compulsory school, most commonly third and fourth grades. For the two years of this research, Jenny and Silvia taught two different classes of fourth grade.

During the first research year, I met with Jenny and Silvia three times to discuss their mathematics teaching. In that year, Jenny and Silvia chose to conduct their own action research project. They researched their own process of mathematics teaching during one school year and gathered and analyzed relevant data. They presented their results at Vinaskóli at the end of the first research year. During the first year of this study, Jenny and Silvia additionally participated in a different PD program on mathematics education based on problem-solving. Data from our first year was analyzed in action research cycle three and helped decide the course of action for our collaboration for the second research year.

Based on results from action research cycle three, Jenny, Silvia and I decided to collaborate more consistently inside and outside of their classroom in the second research year. We held regular meetings that year to plan lessons, and I joined them in their classroom on a weekly basis. When I came into the classroom, we either co-taught the class or divided the class into two or three groups. During the second research year, Jenny and Silvia taught the same students Anna had previously taught in the first research year when she, her co-teacher and I did the creative mathematics workshops with their students. This means that I worked with teachers for two years who taught the same group of students.

During the second research year I interviewed both Jenny and Silvia. I interviewed Jenny in November and Silvia in February, which gave me the opportunity to develop our collaborative work throughout the fourth action research cycle. In this section I explain themes that came out of analyzing data from Jenny and Silvia in action research cycles three and four. These results were brought together in action research cycle five with a summative analysis of all the data and results. The following sub-sections focus on Jenny and Silvia's individual stories and themes pertaining to each of them individually, as well as their collaborative action research project.

4.4.1 Jenny's Story

Jenny has a background in art and creative education. She focused on fine arts when she did her master's degree as a compulsory teacher. She believes that her background in art shaped some of her views on creative mathematics. Jenny struggled with mathematics learning and low self-efficacy in school and had not pictured herself as a mathematics teacher. The thesis that she did for her master's degree focused on creative learning and she views creative thinking as a process:

> You start off, and there are always some obstacles on your route. You just must overcome these obstacles. If there are no obstacles you are not learning anything. Creativity means to just keep going forward and finding ways to get over, through, around, or somehow past those obstacles.

Jenny explains that after the research she did for her master's degree, she realized she wanted to use her experience of creative learning in a broader context than the fine arts and chose to teach in the early grades.

4.4.1.1 The Limitations of Textbooks and Fixed Lesson Plans

Jenny explains in her interview that previously she worked at a school with multiple classes within each grade level. At that school, all the teachers had to keep the same lesson plan with the same pages worked by students. She explains her feelings around this:

> There was something inside of me that screamed that this was wrong. I was always stressed but I did not feel justified with that [feeling] because I was always pushing everyone through some pages and the results were based on how fast or well you managed the lesson plans. Then you [students] took a test that was exactly the same as those pages.

Jenny goes on to say that she and her co-teacher work differently in Vinaskóli. In their action research project, according to Jenny, she and her co-teacher saw how students could work for weeks in a textbook and not learn the mathematical subject they were expected to. To explain the lack of learning from the textbook, Jenny discussed her students' work in geometry:

> When they had worked in these textbooks for a long time on certain topics, we found out that they could not explain any of it in their own words when asked. We decided to have an open-ended project towards the end of this work that had no fixed answer. In the problem, they were exploring the concepts of circumference and area, and they did not know what to do at all because they had no competence in that way of thinking. They were only able to read and work through the book, but they did not learn anything about the subject, which you would have expected in that time.

Jenny claims that she gained valuable learning from what she refers to as "textbook focused teaching" and that she now uses different approaches in her teaching.

A similar focus can be seen in how Jenny expresses herself throughout the seminars. In the fourth seminar, when the teachers were working with patterns, a discussion arose on how little time is often given to hands-on work in the classroom. In that seminar, Jenny says:

> I find patterns to be very exciting and I can visualize so many potentials. What has happened, though, is that when working with students and using a textbook such as ours, the patterns are at a time of the year when you are racing through [other learning components] and we give it [the patterns] so little attention. Therefore, I am extremely grateful for an extra boost [here in the PD seminar] and I am so excited for this now, even though I was before as well. We work so much with this textbook, and it is the same year after year – it is always the last chapter with this discussion topic [of patterns that we leave out].

Jenny's comment shows how she experiences the textbook as controlling her teaching, but also reflects on how she would like it to be different. She explains how she wants to focus on creative learning with patterns and how she finds the PD seminar helpful to remember that enthusiasm. At the final seminar, she further expresses how important she finds it to let students work with mathematics in relation to real life, as opposed to only working with mathematics from a textbook.

It is evident that Jenny wants to foster creativity in her classroom but is not always able to do so. Prior research on creative learning has reported that teachers often want to emphasize creativity in their mathematics classroom but do not always manage to do that (Shriki, 2010; Sternberg, 2017). The research literature on PD clearly shows that teachers need extensive support to change their teaching in a way that can influence student learning (Desimone, 2015; Martin, Kragler, Quantroche, & Bauserman, 2014; Zepeda, 2019). Jenny's comment on how she is grateful for the "extra boost" that she gets from the PD program on creative mathematics shows that she experiences the PD as beneficial when working towards her goals for mathematics teaching.

4.4.1.2 Self-efficacy for Mathematics Learning

> It all started when I was a student myself and told myself that I was bad at mathematics. When we had to do anything related to mathematics, whether at school or in real life, I always froze. (Jenny, November 2019)

Jenny believes that her own experience has shaped her, and she aims to help her students feel they can succeed in mathematics. She wants to

honour her students' mistakes and acknowledges that we are all at different locations in our learning. She says it is important to allow students to think, and that if they get stuck, they just need support to continue. She talks about fostering what she calls a good "mistake culture".

Jenny says that she often begins the lesson with a mathematical task that all her students engage with. For some of the students the task might work as enticement or rehashing, whereas for other students it is a learning challenge. She explains how some of the students then move on to a more advanced task or a more difficult aspect of that same task. She explains how this results in learning for all the students: "Towards the end of the lesson, the students who struggled [with the original task] have learned something and moved through it, whereas the other students [who worked on more advanced tasks] have reached a different place and achieved deeper learning."

Jenny explains that despite her view of textbook learning, she uses worksheets considerably with students who feel hopeless in their mathematics learning. She says that the worksheets can help students' self-efficacy and explains that she believes that her students can think "that by sitting and seeing that I [the student] can work like the rest of the students, I [the student] want to do a little bit more and more [with mathematics] and [work] towards more depth [in mathematics learning]." Jenny's words reflect how she values students' self-efficacy and learning.

Jenny's description of how she wishes students to view and use mistakes in their learning is consistent with theories on creative learning. Rather than focusing on one correct way or correcting mistakes harshly, honouring mistakes as part of the creative learning process is crucial in fostering creativity within the classroom (Fautley & Savage, 2007; NACCCE, 1999; Ranjan & Gabora, 2013; Sternberg & Williams, 1996). Jenny realizes that resilience and self-efficacy are important in students' mathematical learning and connects this with creativity. This same focus is seen in prior research (NACCCE, 1999; Starko, 2018; Sternberg & Williams, 1996).

4.4.1.3 Professional Development as Empowerment

When Jenny is asked how the PD program on creative mathematics has been helpful, she shares that it was reaffirming for her own views. In a survey, towards the end of research cycle three, Jenny states that the PD program "supports my emphasis of what I consider important in mathematics teaching. It is good to review and get ideas from meetings, as well as the support, ideas and encouragement with lesson preparation and from teaching from Ósk". She further elaborates in her interview:

> As confirmation, it has helped that I am not just doing something because I think so, because I know this from the research I have conducted on creative thinking. It has also helped when I talk to parents to say that I am part of this [PD program] and that it is connected to the University and that this research is going on. That this [way of teaching] is built on some academic studies.

In her interview she says that although she has some academic background in creativity, it is both practical and "empowering" for her to partake in the PD program and she feels she gets more trust when explaining to parents. This is further explained in a survey where she is asked about the effect of the PD program on her views of creative mathematics learning, where she states: "My positive views have been strengthened further and support my professionalism. It is beneficial to refer to the fact that I am participating in research based on theoretical framework."

Jenny explains in her interview how listening to presentations at the PD seminars is very helpful "because it fits so well with what I always feel when I am working with the kids. I get this feeling that this is right". She compares her current teaching to the method of using fixed lesson plans for all students and how that does not feel right. Jenny says that she finds it reaffirming when she works with her students with creative mathematics. She finds that talking to other teachers and hearing the presentations in the PD seminars strengthens her beliefs regarding mathematics education and how she wants to teach.

Jenny connects the PD program on creative mathematics to other PD programs in which she participates, such as one on formative learning. She says the PD program on formative learning has helped her become more focused when teaching mathematics. Jenny explains that she uses discussion buddies in her classroom. She asks students to discuss their own understanding of mathematical concepts and ideas with another student and then brings the discussion out to the whole class. Jenny says that she wishes to do everything she can to help each child solve problems in their own way and that the PD program on creativity in mathematics supports her to build a learning environment that focuses on problem-solving.

Although Jenny finds that the PD program on creative mathematics helps her to be clearer in her pedagogical view, she says that there are some limitations to her being able to always work with creative mathematics in her classroom. She discusses the importance of

manipulatives in student learning but has encountered behavioural problems that have resulted in her not using the manipulatives as much as she would otherwise do. She explains that with a large group, with students at different learning stages, it is difficult to meet the needs of all students. She believes that if she had the option of more teachers, fewer children in each space, and more time to prepare, she would be better able to reach her goals.

Jenny claims that she found it very beneficial when we worked together to set lesson goals and plan lessons to meet the goals. She finds it important to realize "what is really useful for these children to learn". It is interesting how clear Jenny is in her pedagogy and how much she appreciates getting the opportunity to work within a PD program that supports her own views of mathematics learning. Although she describes some barriers that hinder her in consistently implementing creative learning in her mathematics classroom, she also describes student learning and her own development as a mathematics teacher.

Hargreaves and Fullan (2012) emphasize autonomy and a commitment to continuous learning and professional upgrading as important components in the development of professionalism. Jenny's account of how she finds the PD program supports her own views and learning accords with these and other theories on professionalism and PD. The goal of any PD is to positively influence teaching and learning and any PD must be connected to teachers' practice (Desimone, 2015). Effective professional development is often content-specific and related to subject matter, and it is imperative that teachers have opportunities to use what they learn in any PD endeavour in their own classroom (Desimone, 2015; Zepeda, 2019). Jenny explains how the PD program on creative mathematics supports her vision for mathematics learning and helps her put this into practice.

4.4.2 Silvia's Story

> You develop so much as a teacher. I believe that is very important and it gives me so much. You are teaching something, and you are insecure and unsure, and you are not fully comprehending this, or you feel you are not making enough progress although you probably are doing fine, and then you find new ways and as soon as you go into some professional development everything opens up and so many possibilities are created. You start to try, and it becomes so much more rewarding. I feel it is hugely important to always

> have an open mind, to get something new in and try it out. You need to have a framework to do this more. It could be very good instruction that somehow urges you to develop further. I feel it is very important. I thrive on this.

Such is Silvia's description of how she experiences the benefits of PD. Silvia graduated from a teacher education program and has taught the youngest grades for 10 years. "I see myself as a third and fourth grade teacher," Silvia says.

4.4.2.1 The Diverse Learning Group

The class that Jenny and Silvia teach in the second research year constitutes a group of students who vary greatly in their mathematical understanding. Anna noticed this gap when she taught the class in the previous year and Jenny also discussed this diversity in ability when she reflected on teaching. All three teachers explain how this brings some challenges for their mathematics teaching. Silvia explains:

> I have always found it difficult if the group is very diverse in ability. There is a great gap between those with poor number sense and those with good number sense. There are those who are not as advanced and those who are moving with leaps and bounds. And what I experience when you have this entire scale is that you cannot support any group as [much as] you would like. There are some who need more manipulatives, others who can do the mathematics mentally.

Silvia claims that it is beneficial to have good access to tools such as manipulatives for learning mathematics, as well as the option of dividing the class into two or more groups that can work indoors and outside. She believes that offering a variety of different learning modes for students is best but is mindful that this is not always possible, and that each teacher does their best with their group at any given time.

Discussing the issue of differing mathematical abilities of students, Silvia explains that she and Jenny use what she refers to as "making problems more interesting". Silvia and Jenny will let all students work on the same mathematical problem, which can be extended for those students who need more of a challenge. She believes it is important to encourage students to learn through struggles and to complement students based on

their efforts and not the ability to finish a task that proved easy for them. She feels that this helps students reach their personal learning goals.

Silvia finds that using the "more interesting tasks" helps some of the students to reach deeper learning in a group with different learning abilities. She explains how some of her students decompose numbers in different ways to make mathematical problems simpler. In the sixth PD seminar, Silvia explains the creative learning process of one student:

> I had an example of creativity. We were working with multiplication and using study buddies. There were study buddies who did not feel this [mathematical problem] was "interesting" enough, so they wanted to work with the 16 times table. Then he [the student] realized that when he added 16 he had increased the multiplication by one. "I know that three times 16 is this and then I add 16 and then it is four times," [the student said]. And I just thought, "he realized this there."

After some discussion with other teachers at the seminar, she added: "He wanted to work on something very complicated and gained this understanding through doing that."

Both Jenny and Silvia explain how they work with the broad ability of their class. Their experience of providing students with problems that can offer different challenges is comparable to what has been described as rich tasks in mathematics (Boaler, 2016). Rich tasks have a low floor and high ceiling, allowing students of different abilities to work on the same task while all are met with an appropriate challenge. Tasks with a low floor and a high ceiling can be particularly helpful in supporting learners of different abilities without grouping them based on grades or ability. The idea that Silvia expresses – of how it is important to complement students for their effort rather than tasks completed – reflects the concept growth mindset as explained by Dweck (2016) and discussed by Boaler (2016) in regard to mathematics.

4.4.2.2 The Language in Mathematics

Silvia reflects on the importance of language in mathematics. In her interview she explains: "We are naturally always trying to gain insight into student thinking by asking: How do you think about this?" Silvia uses study buddies in the mathematics classroom. However, she explains that at times

students do not reach sufficient depth in discussion. She believes that this is related to the students not being experienced enough in discussion:

> They [the students] are not versed in using study buddies in a theoretical discussion. They do not know how to talk like that and do not have the maturity. If they had had more practice since first grade, I am sure they could do it, but it also takes lots of time.

Silvia reiterates that she tries to foster conceptual learning through mathematical discussion. “They [the students] don’t have the words yet to describe these [mathematical concepts], so you naturally have to teach them that,” Silvia says when reflecting on a geometry lesson with her students.

Silvia uses the language to strengthen her students’ learning in mathematics and explains that she listens to how her students express their learning through discussion. She explains how beneficial she finds the use of study buddies and discussion for student learning in mathematics. She works towards including more discussion in the mathematics classroom and is aware that it takes practice for the students to become versed in mathematical discussion. Silvia is cognisant that change takes time and that extended support is needed. Her reflections of the language in mathematics and the use of discussion for learning can be mirrored in theories of creative mathematics education (Boaler, 2016; Nadjafikhah et al., 2012; Pound & Lee, 2015; Sternberg, 2017).

Silvia was increasingly more vocal about her students’ learning as the seminars progressed and she started to notice creativity in their discussions as she applied herself to the PD program and to our collaboration. The change for Silvia was gradual, and she claims that with more extensive support she managed to better implement creative mathematics in her classroom. She is also clear that she finds intensive PD beneficial and that the coaching she has received within the PD program on creative mathematics has changed her teaching in a positive way. As we progressed through the PD program, Silvia appeared to become more mindful of creative learning as it happens for her students and expressed that in PD seminars.

4.4.2.3 All Professional Development Experiences Work Together

Silvia discusses how she finds that other PD programs in which she has participated support her in fostering creative mathematics in the

classroom. She has been involved in a PD program on formative learning for a few years. Silvia and Jenny further participated in a PD program on problem-solving in mathematics during the first year of this research, where the teachers offered open-ended problems to their students and discussed the outcome with other teachers. Silvia experiences the work they did with the PD program on problem-solving as creative and she finds that the two PD programs reinforce each other.

Silvia is clear that in order to succeed with PD, it is important for teachers to get good support and have ample time to try things out:

> You need the boost, and you need to try things out until they become natural. This takes time and you need great support on this route. Or that is how I experience it. I want to have amazing support to get into it and I am really open for all methods that I experience will work. It needs management and follow-up. That is my experience.

Silvia explains what she has gained from the PD program on formative learning. The program includes weekly meetings where teachers sometimes collaborate to create their own lesson plans, which Silvia appreciates. “It is so important that you put something together rather than being handed something and told what to do,” she says. She also talks about how helpful it is to work with other teachers and to reflect on ideas together. These ideas from Silvia can be mirrored in writings on effective PD and how important it is for teachers to partake in extensive PD that provides them with the opportunity for collaboration while taking ownership over their own PD (Desimone, 2015; Martin et al., 2014; Zepeda, 2019).

Silvia explains that the within the PD program on creative mathematics, she found it particularly helpful when we worked personally together on bringing ideas from the seminars into the mathematics classroom:

> I just feel that it is of great value to get a person that you trust, and who has so much knowledge to impart. You have access to her [the PD facilitator] to ask, which is very nice. This connects to what I discussed before: of having access to a person who helps you to develop further. To get support in that is so important. I find it amazing. It is so good to know of someone who comes and helps and guides you in particular areas.

Silvia explains that the seminars "provide the teachers with that conceptual framework" but finds that the support from personal meetings and in the classroom is important to implement her learning from the seminars. "That is where the real work happened," she says.

PD programs need to provide teachers with opportunities to put their learning into practice so that teaching and learning can be positively impacted (Desimone, 2015). Silvia is clear that the collaboration and coaching she has received is crucial for implementing what she has learned from the creative mathematics seminars in her classroom. This accords with theories on coaching and how helpful active and collaborative coaching can be for teachers who choose to work with a coach (Knight, 2011). In all our cooperation, I was mindful to encourage Jenny and Silvia to identify their goals, but I also listened to their concerns, asked questions, discussed pedagogical practices, and provided them with feedback related to teaching and learning. Effective PD values teachers' prior experiences and supports both their voice and their agency, while offering coaching and expert support (Knight, 2011; Zepeda, 2019).

4.4.2.4 Collaboration for Professional Development

Another aspect that is considered important for professionalism and PD is to give teachers an opportunity to collaborate with other teachers (Zepeda, 2019). Silvia states that she values working with other teachers in the PD seminars on creative mathematics. She benefits from sharing ideas and believes that discussions with other teachers support her in reflecting on her own work.

Silvia further experiences the seminars as an opportunity to improve continuity between different grade levels at Vinaskóli. "I think that it is crucial to have some continuation from first through fourth grade," she says. Silvia says that although it helps to have a curriculum from teachers from previous years, open discussions provide opportunity to discuss important details. This can be seen in her participation in conversations in seminars with other teachers and how they approach the subject of learning materials and continuity for student learning.

Silvia is also clear that continuity from playschool is important and believes that it is still important for the children to learn through play in compulsory school. She wishes to use manipulatives more frequently for mathematics learning, but finds that lack of time for lesson planning is preventative:

> There is always that classical problem of not having enough time for preparation. A well-prepared lesson is always so beneficial. You can always see it if you are well prepared. I feel that we need to use so much time for behaviour, discipline, and parents' issues, which takes away from the time to prepare. The preparation should come first and then the rest. A little bit of that happens at times.

It is obvious that Silvia works hard and cooperates with others to improve student learning. She has also implemented some of what she has learned from the creative mathematics seminars in the classroom. However, like others, she clarifies some hindrances that have a negative effect on her reaching all her goals with creative mathematics learning in mathematics. Barriers to working with creative learning in mathematics are experienced by all the teachers in the case studies. It is, however, extremely rewarding to watch the teachers put great effort into the implementation of creativity in their mathematics classrooms and make positive changes within the current school system.

Teachers can create a stronger and better professional practice together through collaborative professionalism (Hargreaves & O'Connor, 2018). The teachers share judgement and a commitment to deepen their expertise through a joint process of reflection. The collaborative process is most successful when connected with the existing culture at the workplace. Silvia is clear on how she benefits both from the collaboration from PD seminars and the cooperation with Jenny. Hargreaves and O'Connor (2018) see joint work as a crucial tenet of collaborative professionalism that helps teachers to share the responsibility of teaching and to create common meaning and purpose with the goal of supporting student learning. The next section focuses on an action research project on which Jenny and Silvia collaborated.

4.4.3 Jenny and Silvia's Action Research

The separate stories of Jenny and Silvia provide insight into their views of the PD program and how it has influenced their teaching and classroom practices. Although their individual stories show some important aspects, Jenny and Silvia work together very intently. They co-teach their group and they jointly plan most lessons. This factor is important for their case and reflects how teachers work together at Vinaskóli in general. It means that the teachers always have another person to "think with". They share the

responsibility of their class and support each other with teaching and other tasks. Both Jenny and Silvia claim that they appreciate this collaboration.

For the first year of the PD program on creative mathematics, Silvia and Jenny chose to do their own action research project whereby they analyzed their teaching over the course of one school year. They examined the different teaching methods they employed in their teaching, their joint learning process and how they experienced student learning as a result of the different teaching methods. During the school year that Jenny and Silvia did their action research project, they participated in three different PD programs. The three programs were the PD program on creative mathematics, a PD program on open-ended problems in mathematics, and an ongoing PD program on formative learning. The different teaching methods that Jenny and Silvia explored during that school year included preparing their students for standardized tests; using procedural-based textbooks; providing students with opportunities for problem-solving; and incorporating creative mathematics in their classroom.

When the PD program on creativity in mathematics began, Jenny and Silvia were co-teaching for the first time. At the beginning of the school year, they were tasked with preparing their students for a standardized national test that all students in Iceland take in fourth grade. During the preparation, Jenny and Silvia faced a few challenges. They shared a vision for formative assessment and were at the same time obliged to have students undergo a standardized test. They felt the test was too difficult for their students and struggled with how to teach mathematics creatively while preparing their students for a test that was, in their own opinion, not creative and not in line with their vision. In the presentation of their action research, Jenny and Silvia explained:

> How were we supposed to go through all these [mathematical] components [with the students] in the spirit of creative work? It [the teaching] was mostly feeding it to them and teaching resilience. A little bit of asking about how you are thinking this out, but nothing more.

Jenny and Silvia did not experience the standardized tests as creative. They explained that “creative thinking is not evaluated, just right and wrong”. Jenny and Silvia said that the standardized tests were “in opposition to what we felt was right for our students”.

After preparing for the standardized tests, they began the PD seminars and meetings on creative mathematics that provided them with "insight into a language to use in mathematics teaching and reassurance that we were on the right path". They listed important components for their students as: "Not focusing on the answer [but] defining what they [the students] were thinking: How did they [the students] find that out, how did they reach a conclusion, and why they got different answers."

Having spent a month preparing for the standardized test with worksheets, Jenny and Silvia believed that they needed to get back to the textbooks provided by the school. When the students worked quietly in the textbooks, Jenny and Silvia noticed increased stability in the classroom. Jenny and Silvia said that at the time they felt great and thought: "This was precisely what we were supposed to be doing."

However, after weeks of the students working in the textbooks, Jenny and Silvia began working with the PD program on problem-solving that they did that year. They noted that the PD program on problem-solving was in the same spirit as the creative mathematics PD program, focusing on problem- solving, multiple solutions, and concept-based learning.

Through the PD program on open-ended problems, Jenny and Silvia were provided with an open-ended problem on circumference that they decided to use towards the end of their geometry chapter. "We had been working with circumference a lot and believed the children understood [the concept]." Jenny and Silvia claimed that they were certain that the problem- solving would go well, as the students had been very studious. They found out that this was not the case:

> We realized that they [the students] were lacking a conceptual understanding of circumference. They had a difficult time with problem-solving. They [the students] had difficulty using blocks (manipulatives). They were only able to work within the context of that textbook. We saw we needed more time to work with different parameters [for learning].

Jenny and Silvia explained that their experience with the standardized tests and the mathematical textbook provided them with some insight for their classroom: "We want to aim for equilibrium between textbook work, study buddies, creative thinking, problem-solving and perseverance. We need to increase hands-on work."

In their action research, Jenny and Silvia looked at contradictions and conflicting demands in our education system:

> Standardized test: everyone in the same box the first month. School without separation: preparing individual curricula simultaneously. National curriculum: creativity is one of the pillars for education. Different demands from parents: not enough preparation/too much preparation and stress. Differing student views: ambition, futility, anxiety, and low self-efficacy.

Jenny and Silvia claimed that their vision or dream would be "Well-prepared lessons, goal-focused learning, and offering varied learning modalities: use of manipulatives, creative [learning], and problem-solving". However, they noted that in reality there is a "lack of time for preparation [so] we make do with the textbook. [There is] no time to find manipulatives, tools are not there [or] are poorly handled [by students]. We do not know how to use the smart board, [have] behavioural problems, etc.".

Jenny and Silvia recognize in their action research the difference between using procedural-based textbooks that they do not find to be creative and teaching through open-ended problems where students use manipulatives to connect their learning to concepts. They explain how limiting they found the procedural-based textbooks to be to student learning, and in their action research presentation they explained how students who worked extensively on problems in textbooks were not able to transfer their learning to open-ended problems or real-life situations. Jenny and Silvia noted that although their students had done extensive procedural work with circumference in the textbook, the students lacked the ability to apply that knowledge outside of the textbook. The students lacked the conceptual understanding to apply their learning to physical geometry.

Jenny and Silvia claimed that the results from their action research project helped them to recognize that in order to build student conceptual understanding, they needed to change the focus and move in a different direction with their students. Theories of creative mathematics learning reflect the same focus on the importance of concept-based learning as opposed to procedural-based (Luria et al., 2017). Conceptual learning is crucial in developing student creativity and helping students become active learners who can analyze and generalize.

In their action research project, Silvia and Jenny discuss some of the same ideas that have been emphasized in their individual stories. Through the PD program on creative mathematics, both of them have become clearer about their views and how they want to teach. However, they are faced with some barriers that prevent them from teaching consistently according to their beliefs. These barriers include time, class size and management, and standardized tests. These obstacles are in line with what research has shown can prevent creative learning opportunities (Bloom, 2014; Daniels, 2013; Ranjan & Gabora, 2013; Runco, 2007; Snyder et al., 2013; Sternberg, 2017).

4.4.4 Summary

Jenny and Silvia collaborated over the course of two years to foster creative mathematics learning. In the first research year I met with them three times outside the PD seminars to support them with creative mathematics education. In that first research year, Jenny and Silvia chose to conduct their own action research project whereby they became clearer on the importance of incorporating more creativity in their mathematics classroom. Through the analysis of their case in action research cycle three as well as with the data from Anna's case, we chose to collaborate more intensely in the second research year.

In their interviews, in seminars, and in the presentation of their action research, Jenny and Silvia expressed how the PD program benefited them to incorporate more creativity in their mathematics classroom. They articulated how their views were affected and strengthened in the PD seminars and how our collaboration inside and outside of the classroom supported them to implement creativity in their mathematics classroom.

The insider nature of this study has provided me with an opportunity to experience some of the changes Jenny and Silvia have shared, as well as to develop as a teacher myself. Although my focus in this thesis is on the teachers I worked with, I wrote in my own reflective journal about how my views, my pedagogy and my teaching became stronger as well. I learned more on how to work with younger students, and in my reflective journal I have explained how both the teachers and I benefited from our collaboration. I have written that "Silvia feels she is learning a lot from me and finds that I explain well to the students". On another occasion I have written that I find it "great to talk to Silvia", which shows the mutual support of the collaboration.

The collaboration with the teachers was very important for this action research, together with the iterative research based on Mills' (2018) dialectical action research spiral (Mills, 2018). Working with Jenny and Silvia through the two research years and analyzing their stories through action research cycles three to five was very valuable. Through our collaboration, I learned what they experienced as beneficial in fostering creative mathematics in their classroom. With this extended collaboration we formed a relationship that was based on collective autonomy and efficiency, as is common for collaborative professionalism (Hargreaves & O'Connor, 2018). The next section explains the third case, which is another case of a team of two teachers with whom I worked on their journey towards increasing the use of creative mathematics in their classroom.

4.5 Sunna and Inga

Sunna and Inga are co-teachers who were new to classroom teaching at the beginning of this research. For the two years that they participated in the PD program on creativity in mathematics education, they taught the same class of 44 students. During this time, the students were in second and third grades. Inga has a background in leisure studies and has previously worked in play school and at a service centre with families. Sunna used to be a physical education teacher but recently decided to make a change and become a general teacher for the younger students.

Inga and Sunna were very diligent with the PD seminars and in the second research year we decided to work together more closely. We met regularly to make lesson plans and I joined them in their classroom weekly for lessons of two consecutive hours. During those lessons, we divided the students into three groups and used creative learning stations. There were three stations, each with one teacher. Students rotated through the stations, which consisted of different activities or problem-solving experiences in mathematics.

This type of work was partially based on how they were already working with their students in mathematics and had started doing this in the first research year. I, however, had seen in action research cycle three how beneficial it was to work intently with teachers to promote creative learning, and Sunna and Inga showed interest in this type of collaborative work. This section focuses on the themes that emerged when going through the analysis of their case. Although most of the data is from action research cycle four, some of the data pertains to PD seminars in action research cycle three, and Sunna and Inga comment on their process with creative

mathematics education throughout the two research years. Inga and Sunna asked to be interviewed together so I chose to present their case as one story.

4.5.1 Becoming a Mathematics Teacher

> It was a strange feeling last year with mathematics [teaching] being new [to me], it takes a little bit of time. Naturally, this is not a complicated curriculum, but I felt it was challenging to prepare it in an interesting way and meet their [students'] development. We taught odd and even numbers last year and it is just the foundation, but I feel the biggest challenge is that you do not just go through the curriculum, moving on without anything remaining with the students. What we felt is that it is so important for them [the students] to reach a greater depth in the subject [of mathematics] in some other way. (Sunna, February 2020)

Inga and Sunna were new to classroom teaching at the start of the PD program on creative mathematics education. Since this was their first experience teaching young students, they were learning how they wanted to approach mathematics education.

Both Inga and Sunna said in their interviews that they had not been particularly strong in mathematics or science when they studied it in school themselves. Inga said:

> Because this was my first experience [teaching] and whenever you do something for the first time you have to learn how to do it. I am stronger in the Icelandic language than mathematics. I was better at that in the old days as a student. At that time I was more interested in the Icelandic language. So, it took me a little bit of time just to get into the mathematics. Really just to spark my own inner motivation as a teacher. Because that is how you best share [that enthusiasm] with the students. There must be passion.

Sunna agreed. She believed that at times you learn things better if you have had the experience of overcoming an obstacle through hard work:

> Because of feeling that mathematics is not one's strongest field, one starts to study more, and in some way I believe that you can teach better when you have had to work hard. Even if the learning material is not complicated, you think: How do I get you [the students] to think?

Inga and Sunna also agreed that they were able to use their strengths from their previous education and work experiences in their teaching. Inga explained:

> We had to work ourselves into this and find the rhythm and find the passion. We were also able to use our strengths. Because Sunna studied physical education and I focused on leisure studies, I feel that we have used our strengths in making the mathematics creative.

Inga and Sunna value helping their students learn creatively in mathematics. As the PD program and our joint work progresses, Sunna and Inga describe in more detail how they work in creative ways with their students. In the sixth seminar of the PD program, Sunna states that it is important "that they [the students] can discover for themselves". She goes on to describe a lesson of geometry:

> I had the same set-up today as yesterday. How big is a centimetre? Will you show me? Is it this big? (shows different sizes with her fingers). Then they [the students] started a discussion. Someone said it is like the centicubes. I was intending to get it ingrained and was showing them (shows centimetre size with fingers). Then I described that the table might be a cubic metre. I can take a step that is one metre. It would be fun to measure the classroom. I think it is about seven metres. So, an understanding started [with the students]. A metre is as big as a step. I do not know if this is creativity exactly, but it is making [mathematical] connections.

In Sunna's description, it is clear that she is concerned with the students' conceptual learning and understanding. It is apparent that she is wondering what constitutes creative learning in mathematics. As the PD program progressed, Sunna was increasingly more vocal in seminars about her teaching and student learning.

Inga generally participated enthusiastically in the seminars. In the second seminar she explains how she incorporates mathematical learning in many aspects of school life:

> We can use daily tasks as opportunities for mathematics teaching. Mathematics is intertwined into everything, even lunch time. More and less [as mathematical concepts]. How many do you want? Beans even. Just having a little humour in this. Getting them [the students] to think a little. Splitting a potato in half.

Inga and Sunna are developing as mathematics teachers as they participate in the PD program. They express how their background helps them to support students in their mathematics learning. They are interested in focusing on creative learning and want to reach what they refer to as deep learning. They try different methods in their teaching such as using visuals, connecting mathematics learning to other aspects of the school day, and focusing on a variety of problems and activities at the weekly stations with the students. Many of these activities included creative problem-solving or learning through the use of manipulatives.

4.5.2 Bringing Learnings from the PD Program to the Classroom

> I feel the seminars moved me. Especially in the beginning. We needed a little boost. Because it is easy to get stuck in the same, so it is very good to get a push. What is creative mathematics? It is so broad. It got me thinking about mathematics even when I was not at work. Yet I do believe that it depends on the teacher's character how much this moves you. If you are open to new ideas, it is easier to be moved by a seminar that can prove helpful. It is the same as with students, it is a combination of who you are, what you learn, who you become. (Inga, February 2020)

Inga and Sunna both express that the seminars influenced their thinking. It was particularly interesting to read Inga's account of how she was thinking about mathematics at home. This can be mirrored in Wallas' (1926) model of creativity and incubation.

Evidence from the data indicates that the seminars made an impact on the perspective Inga and Sunna held on mathematics teaching and learning.

In their interview, Sunna said: “I did not feel the seminars helped directly in the classroom. It just got us to think in different ways outside the box. Maybe it leads to us trying to find more diverse learning materials for the students.” So, without feeling a direct cause to change her behaviour in the classroom, Sunna experiences an effect on her views of mathematics education.

This same idea is expressed by Inga, who claims that although she did not get many practical ideas from the PD seminars, the work in the seminars affected her views. She explains in a survey the change of her views of creativity in mathematics: “Increased growth mindset and thinking outside the box. Mathematics is everywhere and is intertwined with daily life, patterns, shapes, etc. No one correct answer in mathematics and the importance of applying good questioning techniques for mathematics.” She says that the change in views may not have affected her teaching directly, but believes that this change in views is important in the process of making changes.

During the first research year, Inga and Sunna made some changes to their teaching but also struggled to find the time. Inga explains in the third seminar:

> In the beginning I was excited and had many ideas in my mind that I wanted to put into practice, and we did that to some extent. Then in January it fell by the wayside. We had many tests on literacy and such and have not been able to get back into this.

The explanations of how Inga and Sunna struggled to bring their learning from PD seminars into their classroom can be mirrored in theories on PD.

The goal of any PD for teachers is to affect teaching and student learning in a positive way. Desimone’s (2009) model explains how a teacher’s change in practice is the result of a change in teachers’ knowledge, ability and views (see Figure 6). From what Inga and Sunna shared, the PD seminars provided them with knowledge and affected their views of mathematics learning. However, teachers commonly need extensive support or coaching for a PD program to bring about a change in their teaching (Knight, 2011; Martin et al., 2014; Zepeda, 2019).

During the second research year, I collaborated with Inga and Sunna regularly and they explain how they appreciate our collaboration. Inga says: “Our work on Tuesdays is very important for the students. Especially getting

the third teacher in, which is key in using the stations. And we truly benefit from your strengths." She adds: "On Tuesdays it was most important to deepen the subjects we are working with, like Sunna said. If I had to name just one thing that is most important in our Tuesday work, it is that we are deepening the mathematics."

Inga and Sunna focus on bringing their learnings from seminars into the classroom. Sunna says: "You hope to turn on one or two lights that cannot be achieved by traditional mathematics [teaching]," and adds, "If there are five who were insecure today who are more secure after the lesson, I feel it is a victory." The data shows that Inga and Sunna emphasize that their students learn mathematics with deep understanding, and they use the creative mathematics stations to work towards that goal.

The views that Sunna and Inga develop on mathematics learning are aligned with theories of creative mathematics learning (Luria et al., 2017; Nadjafikhah et al., 2012; Parker, 1993; Sternberg, 2017). Their comments on deep learning and the limitations of traditional mathematics teaching can be mirrored in writings about the difference between school mathematics and the work of mathematicians (Parker, 1993; Shriki, 2010).

Inga and Sunna claim that they experience a change in their students' learning through the creative mathematics stations. They do, however, note that it "may not be measurable performance". Their ideas of the changes they experience with the creative learning are aligned with the literature in that creative learning can be challenging to measure or evaluate (Harris, 2014; Sternberg, 2017).

Inga talks in their interview about how she believes it benefits student learning to make the lessons varied, fun and interesting. She further explains: "We have emphasized teaching mathematics from many viewpoints, so that we can capture the breadth of the student group." This focus is also present in the sixth seminar, where she describes a project she created for her students:

> You have influenced us so much. Suddenly we [the teachers] have started to think differently. I enjoyed it when they [the students] were drawing into a pencil case [which was an assignment I gave them on a sheet of paper]. They were supposed to draw an eraser, a pencil, scissors. The pencil was supposed to be 14.5 cm and yellow, the scissors were 10 cm and green, and the eraser was black and 4 cm. They [the students] said today [when they were working on this project]:

> "This was a fun task." You do not always hear that from a number [of students]. That is what you want to achieve.

Inga's focus on teaching mathematics from different viewpoints and emphasizing enjoyment for students is similar to the focus from Anna on play and fun. Inga articulates that she intends to spark interest and motivation for her students, which aligns with Amabile's (1989, 1996, 2013) writings on intrinsic motivation for creative learning and is closely related to Csikszentmihalyi's (1997) writings about the role of flow for creativity.

Inga and Sunna mention in their interview that it can be challenging to get all the students involved in the mathematics learning. Inga says: "You [PD facilitator] have emphasized how important conversation and discussion is [for mathematics learning]. We have seen that it can be difficult to reach all the students, which is understandable. But I feel that it is valuable to reach the majority." Sunna adds: "There are some students who slip away from our stations." The challenge with discipline is experienced by teachers in all the cases and can be a barrier for reaching all students.

Other barriers that Inga and Sunna bring up include lack of preparation time and how diverse the learning group is in ability. They also note that teaching large groups makes creative work more challenging. Inga says: "The larger the student group is, the more important it becomes to have more teachers in the classroom. Otherwise, you [the teacher] are not able to prepare good lessons or learning stations." Sunna notes that she thinks that there should be under 15 students per teacher in this type of work and preferably between 10 and 12. She further notes that it can be really challenging to use any manipulatives with 20 or more students per teacher. Inga says: "You cannot keep track of it. When the groups are too large, everything dies that is creative. Teachers become tired, more discipline problems arise, and neither teachers nor students blossom." Sunna articulates in this context that some students can become "lost" in the bigger group but are later found in a smaller group.

The results indicate that Sunna and Inga are pleased with their process with the creative mathematics in their classroom. Inga says: "I am kind of happy that we are not afraid to try out things. Not afraid to make mistakes." Inga and Sunna use their own learning experiences to make the learning environment for their students richer. They allow mistakes and emphasize creative learning where students can conceptualize mathematical concepts. The PD program started at the same time as they

started as mathematics teachers and the results indicate that both the seminars and the coaching support Inga and Sunna to implement creative learning opportunities in their classroom.

4.5.3 Summary

The case of Sunna and Inga is different from the other cases in that they had no prior mathematics teaching experience when the PD program began. Therefore, they do not go through the same process as the other teachers of re-evaluating their teaching. However, Sunna and Inga both explain their own process of choosing to use their background and their own mathematical experience in a positive way in their classroom. Through both research years they actively work on incorporating creative learning into their mathematics classroom. They particularly note that it is beneficial to get the extra support in the second research year, although it is evident that they were already on a path of discovery with their students.

As with the other cases, I also gained from the collaboration. I have written in my own reflective journal about the creative stations I did with Sunna and Inga: “In the third grade, I work with units and dozens in real problems. I think it works well to let them [the students] develop their own ideas in mathematics. They mostly get confused if they mean to use an algorithm but get it wrong. The understanding is not there.” This reflects that my experience is similar to the experience that Inga and Sunna describe. We all found that conceptual-based learning and giving students opportunities to be creative and to develop their own methods for mathematical problems was important for student learning.

Many of the ideas expressed in the case of Inga and Sunna are similar to other cases. This includes ideas of using manipulatives and making learning both fun and enjoyable, along with many of the barriers linked to consistently working at creativity in the mathematics classroom. The next section focuses on a lateral analysis of the cases, and shows some commonalities and differences between the three individual cases.

4.6 Lateral Analysis of Case Studies

While carrying out the analysis of the cases through action research cycles three to five, some themes emerged that were common to all the teachers. This section reports on a cross analysis done in action research cycle five after the formal research was completed. The first subsection explains how the teachers’ backgrounds tied in with their experiences with the PD program. The second subsection focuses on how the PD process unfolded

for each teacher or team of teachers. The third subsection focuses on the views that the teachers developed as a result of partaking in the PD program and fostering creativity in their classroom. The fourth subsection focuses on the barriers that the teachers describe for creative learning, while the fifth subsection focused on the teachers' needs for ongoing work with creative mathematics education.

4.6.1 Background of the Teachers

The background of the teachers as it relates to their lived experiences and their views of mathematics education is important for the case studies. For this research, however, the amount of data gathered on background was limited so as not to compromise anonymity. Nevertheless, the teachers shared some background information about their education, previous work experiences, teaching experiences, and other information they felt was relevant.

The teachers selected for the case studies were teachers in grades two, three and four. Their teaching careers varied in length. Some of the teachers were starting out as class teachers when this research began, while others had taught young students up to ten years of age. The teachers in the case studies collaborated with other teachers of the younger grades at Vinaskóli for various projects and schoolwork.

The teachers' backgrounds varied. All were women in different age groups. Jenny and Silvia had finished a bachelor's and a master's degree in compulsory education at the University, Jenny with a special focus on liberal arts. Sunna had a bachelor's degree in physical education and Inga in leisure studies, but Anna did not specify her bachelor's education. These three teachers had master's degrees in teaching. All the teachers shared the belief that PD was helpful and appreciated the opportunity to work towards developing their teaching.

Many of the teachers articulated how their background had shaped their teaching and how they were able to use their strengths in the mathematics classroom. This was evident in Anna's story when she explained how she had always enjoyed mathematics. Anna's positive experience impacted her ability to pass that motivation and joy on to her students.

Sunna and Inga used their experience of having had to work hard with mathematics themselves to help their students to overcome obstacles. Their experience and participation in the PD program on creative mathematics provided them with a perspective that helped them better realize how building conceptual understanding is important, along with

procedural fluency. Jenny saw how her struggles with mathematics helped her better connect with students who struggled or experienced fear in their mathematics learning. She expressed in the third seminar how the PD program helped her in this process:

> I come from an environment where I held the view that I was bad at mathematics and I couldn't stand mathematics. This has morphed into not needing to be afraid of mathematics because I can just think about it in my own way. I find it so awesome to be able to mediate that [experience] to the kids. When I realized this, it was such freedom to come here [to this school] and be part of something like this [PD program]. Creative mathematics and [the idea] of doing what you do is just great if you are working hard to understand what is going on.

Jenny was clear that her background as an arts teacher and her focus on creative learning, in her work towards her master's degree, helped her to better connect with the ideas of creative mathematics. Inga and Sunna articulated that their background in sports and leisure studies were of value for teaching mathematics in a creative way using a variety of problem-solving opportunities, manipulatives, and mathematical discussion. It was evident that whatever the teacher's background was, it shaped them as teachers.

Although the research is not a life story or narrative research, it can be mirrored in some of Goodson's (2017) writing around the impact of such research. Connecting teachers' life history to their professionalism can be of value when researching their values and pedagogical views (Friðriksdóttir & Aðalbjarnardóttir, 2010). Although the teachers' backgrounds were not a primary focus of this research, many of the teachers related their previous experiences and learnings to the creative mathematics PD program. Most of the teachers were cognisant that their own mathematics learning had shaped them as teachers and reflected on how their experiences affected their classroom practice in mathematics and how they related to their students.

Previous research shows that many people have low self-efficacy and a fixed mindset in mathematics (Boaler, 2016). Jenny, Inga, and Sunna have used the obstacles from their own mathematics learning to better support their students and encourage a growth mindset in the classroom. Sunna shares that she "believe[s] that you can teach better when you have had to

work hard", and Jenny articulates how she values a good "mistake culture" in her mathematics classroom.

The effect of the teachers' backgrounds on their teaching aligns with Goodson's (2017) writings on how teachers' stories connect to their professionalism. PD needs to take into consideration people's backgrounds, as an important part of professionalism is for teachers to take ownership over their own PD (Loucks-Horsley et al., 2010; Zepeda, 2019). Connecting PD to teachers' reality and journey is one way for the teachers to gain that ownership. It was interesting that even if the teachers' backgrounds varied in many ways, they were able to use this to their advantage and saw their strengths and struggles to be of value.

4.6.2 The Teachers' Process

The teachers' process in the PD program varied somewhat. The teachers in the case studies worked with me outside of the PD seminars to bring creative mathematics opportunities into their classrooms. Anna and her co-teacher held creative mathematics workshops for their students during a three-month period in the first research year, with my support. The teachers in the other two cases worked with me more regularly over the course of one school year, in the second research year.

The difference in how the collaboration was set up did not affect how the teachers experienced student learning, which indicates that there can be value to both methods of support. Sunna explained why she believed the weekly creative mathematics stations to be better for the students: "I imagine that there is more [learning] that lingers on from doing something extra weekly." However, Anna shared how the extensive workshops helped student learning: "There was some mass [of knowledge] that gathered during this time period [of the workshops] that further supported what came after [in learning]. You [teachers and students] are well prepared to continue."

Most of the teachers from the case studies discussed how the PD seminars specifically influenced them. Jenny was aware that she felt the PD seminars were empowering and reassuring while Inga and Sunna felt the seminars opened their thinking to new ideas. Silvia noted that she felt that the seminars "laid the conceptual background," but noted that the coaching and collaboration with me was where the real work happened. This was parallel to how the other teachers shared their experience of that collaboration.

All of the teachers appreciated our meetings outside of the seminars and the support I gave them in the classroom, saying that this support helped them to better implement what they had learned in the PD seminars in their own classroom. They reflected on their own views on learning, on using manipulatives for learning, on the limitations of relying on textbooks, and how standardized tests came into their work. They focused on games and play, problem-solving, mathematical discussion, and conceptual learning of mathematics.

The emphasis the teachers put on our joint work outside of the PD seminars aligns with theories of coaching as a successful way to support teachers to implement changes in their classroom (Knight, 2011). Effective PD must be connected to teachers' classrooms and provide them with opportunities to put their own learning into practice (Desimone, 2015; Loucks-Horsley et al., 2010; Zepeda, 2019). The teachers explain how the PD seminars laid the foundation of the creative mathematics education but that the extended support between seminars provided them with opportunities to bring creative mathematics learning into their classroom.

4.6.3 Developing Views

In the section on the teachers' backgrounds, I discussed how some of the teachers reported how their own experience with mathematics learning affected their views on mathematics education. Their previous experience with mathematics learning shaped how they approached their teaching. They explained how they used their own experience to their advantage when supporting students. Here we look at how the teachers' views of mathematics learning developed as they participated in the PD program on creative mathematics.

During the PD seminars, all the teachers from the case studies actively participated in discussions and activities. Sunna noted in an interview that the PD seminars "got us [the teachers] to think in different ways outside the box. Maybe it leads to us trying to find more diverse learning materials for the students". Inga explained in a survey that the PD seminars affected her views and mentioned the following points: "increased growth mindset and thinking outside the box; mathematics is everywhere and is intertwined with daily life, patterns, shapes, etc.; there is no one correct answer in mathematics; and the importance of applying good questioning techniques in mathematics". Silvia discussed how important it is for her to be involved in a cooperative learning community. Her view accords with writings about collaborative professionalism (Hargreaves & O'Connor, 2018).

Although the teachers experienced that the PD seminars affected their views, teachers from all three cases explained that the coaching they received between seminars was instrumental for their implementation of creative mathematics in their classrooms. The teachers were used to collaboration within other PD programs and some described how the experience of other PD programs helped them make better use of the PD program on creative mathematics in their own classroom. This can be mirrored in theories of coaching and successful PD, where the teachers' views are affected through PD opportunities with the goal of supporting them to change their teaching (Desimone, 2009; Knight, 2011; Loucks-Horsley et al., 2010).

Jenny describes her process of trying different teaching methods and how she comes to the realization that she wants to move towards more creative learning opportunities for her students. She emphasizes that the PD program helps her become clear on how the procedural-based textbooks they were using provided the students only with limited learning. She explains in her interview:

> They [the students] seemed to be learning a great deal because they were working so diligently. They did what they were supposed to do, but then they had not learned it [the material] at all. They had only learned this form [of working that the book provided] which was the only thing that lingered. Except for a few [students], and yet maybe not. I felt this was a big shock because I know this is the form [of teaching] that is so widely used. I think there is a great difference in how you teach and then whether they learn something.

This view is related to that of many of the other teachers who reflected on the importance of developing deep conceptual understanding for their students. The teachers describe how that becomes possible through more creative modes of learning, including the use of manipulatives, discussion, play and games. Anna, Inga, and Sunna explain how the PD work influenced their teaching and how it gave them an opportunity to put these ideas into practice. The next section looks at the barriers teachers experienced when working with creativity in the mathematics classroom.

4.6.4 Barriers to Creative Learning

> Most reports on professional collaboration and professional development typically end with a three-part advocacy for better leadership, more time, and more resources. (Hargreaves & O'Connor, 2018, p. 129)

It is important to explain how the barriers that the teachers in the case studies experienced prevented them from being able to consistently work with creative mathematics in their classrooms. Examining the barriers that the teachers listed provides beneficial information on how to develop PD, schedules, and schools to promote creative mathematics learning.

The teachers from the case studies emphasized time as a barrier to consistently focusing on creativity in their mathematics classrooms. They referred to needing more time to prepare creative lessons. The teachers both claimed they had little time to prepare and that much of the time they had was spent on other tasks, such as meetings and teacher-parent cooperation. The teachers also discussed the importance of having longer class periods in mathematics to allow time for the creative learning process. They found that longer lessons provided opportunities to use manipulatives and discussions in the classroom. The teachers said they appreciated having two hours or more for a class period. Anna explained that the 40-minute mathematics lessons they were used to did not allow time to bring out manipulatives or to get the students involved in a discussion. Time is commonly discussed in literature as an important factor for creative learning (Fautley & Savage, 2007; NACCCE, 1999; Ranjan & Gabora, 2013; Sternberg & Williams, 1996).

The teachers found that group size and space were barriers to the creative learning. The teachers experienced that a smaller group allowed them to be more successful in promoting mathematical discussion and in using manipulatives as a tool for learning. This is closely connected to obstacles that the teachers said were related to discipline and teaching a group with diverse mathematical ability. The teachers explained that in a larger group it became more challenging to reach all the students and they noted that behavioural problems became more prominent. The teachers were clear that they wish to support all students properly while struggling to find the time to do so.

4.6.5 The Teachers' Needs

The teachers from the case studies all had positive responses to the PD initiative and managed to apply learnings from the PD seminars to their teaching. Looking ahead, the teachers wanted to continue to work with creative mathematics in their classroom and believed that continued and ongoing support would be beneficial.

When the teachers were asked what they needed in order to continue the work with creative mathematics, their needs mirrored the barriers or obstacles they had described. The teachers wanted to have longer time to prepare, longer class periods, smaller groups, and manipulatives to be easily accessible. As the PD program progressed, the teachers were vocal about some of their needs for their process of implementing creative mathematics in their classrooms. Our collaboration provided me with an opportunity to support them in meeting some of those needs.

Nevertheless, Silvia notes that when we began our collaboration, she and Jenny were not fully clear on their needs: "You came and said: What do you need? But we were not in a place to take full advantage of it." However, she gets a clearer view of their needs as the work progresses and says she values co-operation and having an opportunity to work with others in planning her own teaching. She explains in her interview:

> Getting time in the meetings for lesson planning [is beneficial]. You can get guidance from the meeting facilitator around being on the right path. So, you feel prepared to go into action [teaching]. If the meetings are practical, you experience that you are prepared and that you can go out and try it [the lesson plan] out. It is quite important that you set something up for yourself rather than being handed some assignment.

This can be compared to what has previously been discussed on teachers' collaborative professionalism and how important collective autonomy, collaborative inquiry and mutual dialogue is for the joint professional process (Hargreaves & Fullan, 2012). The description Silvia gives of how helpful it is to work with others to develop a lesson plan for yourself can also be mirrored in what has been written on the importance of opportunities for both internalization and externalization in learning (Moran & John-Steiner, 2003; Vygotsky, 1978). Silvia is very clear that she values collaboration and good communication but also recognizes the need to make plans that are her own.

The teachers from the case studies articulated the value in having a person who supports them in implementing creative learning in their classrooms. When asked about her needs, Jenny says laughingly: “Just that you are always with us,” and although we both know that to be impossible this expresses how helpful she finds our collaboration. She goes on to explain: “Then we would be three educated teachers and then maybe we would get support or special educators and then I would picture us really taking care of everyone.” Her statement reiterates the need for more teachers for each class, but the teachers are also clear that having support from an expert on creative mathematics is beneficial.

Silvia explains our joint classwork with students by saying:

> You have been with us and I find it especially enjoyable to hear how you approach mathematics. I find that you approach everything [with the students] in such detail. I sometimes assume they [the students] know something when they do not, but you start from the beginning.

She further reiterates: “This is what I take from you. How you talk to the children and get them completely on their level. This is what I experience, and I learn immensely from that.” It is obvious from her and many of the teachers that this personal support is of great value. She is also seeing how important it is to build conceptual learning for the students and uses our collaboration as a learning opportunity.

Inga articulates that she and Sunna “truly benefit from your [PD facilitator] strengths [with creative mathematics education]” and argues that it is helpful to have a combination of direction and joint work. Inga’s view on balance between direction and collaborative teaching with an expert is comparable to the theory of collaborative professionalism and how important it is to work jointly towards a common meaning and purpose for student learning. Her appreciation of direction also connects to the importance of good leadership for PD and the benefits of partnership coaching (Hargreaves & O'Connor, 2018; Knight, 2011).

The teachers’ experiences reflect the same ideas as the literature on what entails successful PD (Desimone, 2009, 2015; Martin et al., 2014; Zepeda, 2019). The teachers need the PD to be connected to their classroom, to be extensive and to provide them with opportunities to work with other teachers. They further emphasize the need for hands-on projects

in seminars, which is congruent with the literature about successful PD for mathematics teachers (Loucks-Horsley et al., 2010).

The PD program is structured so that it supports teachers who promote creative mathematics learning in their classrooms. The structure of the PD is aligned with what research has reported as effective PD (Desimone, 2009). The case studies and lateral analysis have been used to describe what the teachers found helpful from the PD program, what they learned and how this connects to their aims and goals for mathematics learning. The teachers further explain the barriers or obstacles they face in meeting these goals.

The teachers in the case studies explain how they benefited from the PD program, while the analysis demonstrates how the program has influenced their views and teaching and how they experience the effect of these changes on student learning. The teachers state that they would like to continue working with creative mathematics in their classrooms. They explain what they need in order to be successful with the creative mathematics learning, and which barriers need to be overcome. The teachers were diligent in their PD work and serve as good testimonies of how an effective PD can take place.

4.6.6 Summary

The themes addressed in this section emerged during the fifth action research cycle. These themes related to a lateral analysis of the three case studies and were based on the teachers' backgrounds, their process in incorporating their learnings from the PD program on creative mathematics in their classroom, how their views developed as a result, which barriers the teachers experienced to this work, and their needs for ongoing development of creativity in their mathematics classrooms.

It was evident that in all the cases the teachers were shaped by their background, and they commented on how it affected their teaching. Some used their joy of mathematics to incorporate creativity while others used their negative experiences to try to create a more positive learning environment. Although the teachers' process with the PD program and their classroom development varied somewhat, it was obvious that they were able to use the program to their benefit and they described it as supporting student learning. Any PD should focus on developing teachers' knowledge and views in a way that helps them to improve teaching with the goal of positively influencing student learning (Desimone, 2009, 2015). The teachers in the study explained how their views developed throughout

the PD program. They explained barriers to creative learning and focused on solutions when addressing their needs for ongoing fostering of creativity in their mathematics classrooms.

In the following sections we look at results on how the whole school participated in the PD program, which further helps to shed light on the PD program and provides some valuable information on the effect of a PD program on creative mathematics for an entire school. This is followed by a discussion on what themes of creative mathematics learning were observed across the cases and through the whole school analysis. The discussion of these themes adds to the existing theories of creative mathematics learning and professional development. The theories developed have value beyond this school setting.

4.7 The Value of a Whole School PD Program

Previous sections have focused on results and discussion from the case studies. Following the analysis of chosen cases as well as a lateral analysis of the cases, an analysis was done on how the whole school participated in the PD program. These results are helpful in getting a broader sense of the PD process and information on how different teachers experienced the influence of the PD program on their teaching. The results presented here are based on data from action research cycles three and four, although the results were brought together in action research cycle five and are presented summatively.

There was some variation in how much the teachers were able to apply the ideas from the PD seminars in their classrooms. This was both dependent on individual teachers and on other professional obligations that occupied teachers at different times within the school structure. Many of the teachers discussed how the PD program changed their thinking and others explained in more elaborate detail how they used creative mathematics in their classroom as a result of partaking in the PD program. Some of the special subject teachers started paying more attention to the mathematics that was already present in their teaching.

It was interesting to see how teachers were affected, based on whether they were general teachers or special subject teachers. We will first look at the general teachers who taught mathematics on a regular basis and subsequently discuss how the special subject teachers were able to apply what they learned in PD seminars to their classrooms.

4.7.1 Mathematics Teachers in the PD Program

It may come as no surprise that the teachers most influenced by the PD program were the teachers who were general teachers who taught mathematics multiple times per week. Five of them have already been profiled in the case studies. This section looks at results based on other teachers of mathematics. In general, these teachers did not participate in coaching outside of the seminars, unlike the teachers in the case studies. This resulted in some of the uncoached teachers expressing not being confident at using the creative mathematics as much in their classrooms, which is an important finding for future planning of PD ventures such as these.

Regardless, some of the uncoached teachers reported changed views or classroom practice. In a survey, one teacher expressed: “I have started to think differently about mathematics and have used open-ended problems for teaching.” However, she explains in one survey how she would like to develop this more:

> I am positive towards mathematics and enjoy it. What has hindered me is that I do not see myself as a mathematics teacher because I did not learn to be a mathematics teacher, yet I have taught mathematics for 20 years. I am always becoming more secure in not using the books and going broader with the material and including discussions. I would love to have more creative mathematics, but I do not feel confident about this. Perhaps it would be fun to get you to come in with me next winter or we could have a creative mathematics week in school, and all try to change and prepare it together. I do not know. I would like that a lot but must dare.

This teacher was not one of the teachers I worked with extensively in the research years. However, since I started teaching at Vinaskóli we have frequently discussed mathematics education. At the time of writing this thesis, we are working more closely together on her development as a mathematics teacher. Her comment demonstrated both how a PD program such as this can influence teachers and the importance for teachers to have support to connect their learning to their practice. This is mirrored in the literature on effective PD and the importance of connecting PD to classroom practice (Desimone, 2015; Loucks-Horsley et al., 2010; Martin et al., 2014; Zepeda, 2019). Another teacher said:

> What I would have liked better is not really about the PD but the constant race for time. It would have been fun to use the ideas better in my own classroom but somehow, I did not manage to organize it. So, it could be said that I would have needed to have got projects that had already been prepared to try out.

Her comment shows how much support the teachers need, how the PD has to include classroom embedded work, and how the teachers struggle with the ongoing lack of time. These results are similar to the results from the case studies regarding time as a barrier to fostering creativity in the mathematics classroom. Closely related to this is the importance of an ongoing PD program that includes coaching to support teachers to implement their PD learnings in their classrooms (Desimone, 2015; Knight, 2011; Loucks-Horsley et al., 2010; Martin et al., 2014; Zepeda, 2019).

A teacher who taught first grade also notes time as a barrier for creative learning. However, she expressed in the seminars her ideas and application of creative mathematics in the classroom. She shows interest in using discussion with the youngest students and believes that using creativity in lesson planning for the youngest students is crucial, as "they [the students] do not even know what the numbers look like yet". In the second seminar she describes how she and her co-teacher have started to implement more creativity in their mathematics classroom:

> Yes, in the first grade we were a little bit [using creative mathematics]. Last year we just had one worksheet on ten friends but now we decided, after a presentation from you, that we would split them into smaller groups where we worked on ten friends with cards, and also worksheets, though we changed them a bit. We tried to have it more creative.

Her co-teacher explains that they have previously done a coffee shop with grade one and kindergarten, which she feels is helpful for the students' understanding of the base ten system. The ideas that the teachers express related to using games and fun for learning in the younger grades, and can be mirrored in writings on creative mathematics (Pound & Lee, 2015). These ideas further connect to results from the cases on using games and play to make mathematics more meaningful and interesting for students. Closely related to these are theories of motivation and flow for creative learning (Amabile, 1989, 1996, 2013; Csikszentmihalyi, 1997).

Some of the subject teachers who did not teach mathematics on a regular basis reported that the PD program provided them with a discussion around creative learning and different ways to teach their own subject, including languages and other sciences. One teacher said in the third seminar: “We do not teach mathematics, so we were mostly discussing open-ended questions.” Another added that she felt that it was important to get the students to vocalize their learning so that “I see whether they understand what I am teaching them”.

4.7.2 Creative Mathematics for Special Subject Teachers

Of the teachers who did not teach mathematics, the arts teachers were the ones who most frequently connected the PD to their own teaching. The arts teachers experienced the PD program as helpful for creating a bridge between their subject and mathematics. The PD program provided them with insight into how to incorporate mathematics in a more meaningful way in their teaching. The next section looks at how the arts teachers explain this interesting process.

Many arts and crafts teachers are used to incorporating creativity and design in their classrooms. Their subjects are often based on creating something new or working within creative learning models. This is particularly clear for visual art, music, design and carpentry, and textiles. It became clear during the research process that many of the special subject teachers connected to the creative mathematics PD. This was especially true for those teaching arts and crafts, home economics, and physical education. This focus was evident in the seminars and in teacher surveys. Two teachers of special subjects requested to meet me outside of the seminar to discuss their teaching, and I met once with a music teacher and once with a teacher who taught both textiles and Spanish.

Seeing the effect of the creative mathematics PD program on the special subject teachers led me into an inquiry I had not originally planned. To gain better insight into the special subject teachers’ ideas on creative mathematics learning and its connection to their subject, I interviewed two teachers. These two interviews, as well as excerpts from PD seminars and surveys, are used to explain how the PD affected the special subject teachers. The two teachers I interviewed are Margret, who teaches music, and Vala, who teaches visual art. They have both taught at Vinaskóli for several years and Vala since the school opened. They both have teaching experience of over 20 years.

Margret explained to me how she views the profession of a music teacher as universal as she teaches singing, dancing, movement, and basic instruments. Margret claims that her work is very creative, including expression, experience, social aspects, history, rhythm, fine motor skills, and letting students create their own music. She recognizes that music is connected to many aspects of learning and connects mathematics to music through construction, vibration, Pythagoras, and logic.

When discussing the value of the PD program on creative mathematics, Margret expressed that the program "sparked many speculations". She further says: "I am naturally rather conscious of the connection of music and mathematics so that was not a revelation for me, but I started to think more about how this integration could happen." She then explains:

> I remember when you had a seminar on many approaches to mathematics and I felt inspired, and it was fun and interesting. I started to think about mathematics as a force in our environment and connected it socially and environmentally. I felt that this was very encouraging and inspiring.

Vala was also focused on the connection between her subject of visual arts and mathematics. She explains the PD with the words: "It has made me conscious to point out to the children that this is mathematics. We are already using mathematics in assignments, and I have not changed that, but I have become more conscious of showing [that] this [is mathematics] to them." To elaborate, Vala explains an assignment based on using reflection, translation, scaling, dimensions, and a very precise way of working. Vala shares the students' experiences while working on the assignment:

> They have found that this type of precision work can be difficult because often in visual art they are so free. But in a few assignments, I have had them work very precisely, which they find difficult and sometimes boring, but they also see that when they do measure everything up it becomes very neat and pretty.

She connects this to patterns, artists' work, and different media. She says: "There is mathematics in this. Both in drawing and measuring it all up, but it also shows them how mathematics works." When asked how she would further wish to work with creative mathematics in her classroom,

she replies: “This is completely intertwined. The mathematics is there already. I would like to point it out more.”

Margret and Vala both experience a stronger sense of the connection between mathematics and their own subject teaching by participating in the PD program. They were both previously aware of the connection between mathematics and their subject but have become increasingly mindful of focusing on mathematics in their classroom. This is also noted in a survey from one of the physical education teachers, who emphasized that she finds the idea of bringing mathematics more into her teaching intriguing and wishes to do more of that. In the seventh seminar, a home economics teacher notes: “We use lots of mathematics in home economics.” She explains how they use fractions, multiplication and other concepts when measuring and believes that she helps the younger students to understand concepts.

The findings are interesting and shed light on the value of special subject teachers participating in a PD program on creative mathematics. The special subject teachers connect the learnings from seminars to their own practice. The program helps them become more focused on mathematics in their own classrooms. They focus on interdisciplinary work and the connection between different subjects.

Margret was particularly keen on interdisciplinary work but also explained that time could be a barrier. “A creative project demands that you rest in it,” she said, and reiterated that teachers often did not have or allow the time for the rest of which she spoke. It is interesting how she noticed how the teachers’ work is creative, as well as the students’ work. She noted that time for preparation and collaborative work can be a hindrance. However, her explanation of “resting in” the creative project reflects the creative process as explained by Wallas (1926).

4.7.3 Summary

By analyzing the process of the different subject teachers and the general teachers, information was gained on how the PD program on creative mathematics was positively experienced by all of the participating teachers. In a whole school PD program, it is important to be clear that the program benefits all the teachers and their students. The teachers in the case studies, as well as other teachers, expressed the value of participating in the PD program. Some of the teachers who were not included in the case studies claimed that they would have benefited from greater individual support and more frequent meetings. These ideas connect to Knight’s

(2011) writings on partnership coaching and how beneficial coaching can be for teachers for bringing PD learnings into their practice. The needs that the teachers express align with theory on effective PD and how it is imperative that it be connected to teachers' work (Desimone, 2015; Loucks-Horsley et al., 2010; Martin et al., 2014; Zepeda, 2019).

The following section presents a model for creative mathematics learning that is based on the results from the cases, as well as from the whole school. The model is based on the most prevalent themes seen across cases and on the analysis of how the whole school participated. The model adds to existing theory of creative mathematics learning and is valuable for those planning PD, teacher educators, policy makers, and schools.

4.8 Creative Mathematics Learning Model

After analyzing three cases, individually and laterally, as well as data on how the teachers from the whole school participated in the PD program on creative mathematics, themes related to teachers' experiences emerged. Themes started to emerge in action research cycles three and four but were further developed in action research cycle five. The themes relate to mathematical thinking, the diverse learning group, standardized tests, formative assessment, use of textbooks for learning, self-efficacy, students' mistakes, connections to the teaching of special subject teachers, and student learning. Many of these themes have been discussed in previous sections.

A model for creative mathematics learning (see Figure 13) was developed, based on the most prevalent themes, with the goal of generating theory about creative mathematics education which could be applicable beyond the school setting. This model evolved as the analysis progressed and connects to the existing literature while adding a fresh perspective on creative learning. It also explains the modalities that are important for optimizing creative learning in the mathematics classroom and can serve as a framework for what is important in promoting creative mathematics learning. The model can benefit educators planning their own teaching as well as those planning PD, teacher education, or school development.

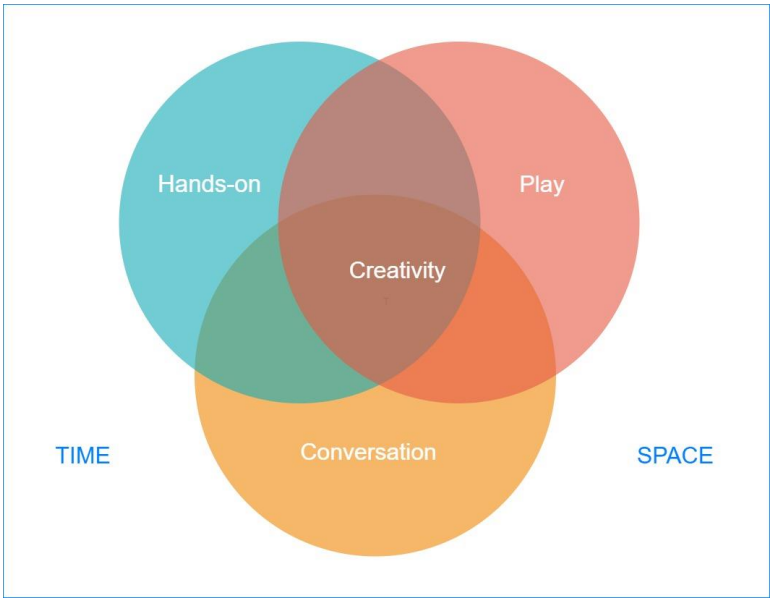

Figure 13 Creative Mathematics Learning

The model for creative mathematics learning is composed of three main modalities: conversation, hands-on learning, and play. The three modalities were reflected evidently in all the different cases and the results from the entire teacher body. Anna went as far as saying: "These were the three things I felt we gained most from when we were doing the [creative mathematics student] workshops with you. The discussion, the hands-on learning, and play and games. This brought new depth, life and joy into the mathematics." The model also accounts for the role of time and space when incorporating creative learning in the mathematics classroom. Time refers to the length of each class period as well as the time teachers needed for preparation and collaboration. Space refers to learning space in a broad sense, including physical space, learning materials, group size, and more abstract factors, such as space for mistakes and allowing students to develop their own solutions. In the next sections I will discuss each modality and explain how it connects to the literature on creative mathematics education and PD.

4.8.1 Conversation, Depth, and Learning

Mathematical discussions are one of the pillars of creative mathematics education (Boaler, 2016; Luria et al., 2017; Nadjafikhah et al., 2012; Pound & Lee, 2015; Sternberg, 2017). Prior research has identified the lack of

conversation about mathematics in many mathematics classrooms, which can result in consequent limitations to creativity (Boaler, 2016; Parker, 1993). Collaboration and mathematical discussion are seen as core factors of creative mathematics learning (Nadjafikhah et al., 2012).

The teachers who participated in the PD program on creative mathematics education discussed the use of conversation in mathematics in interviews, seminars, and surveys. The teachers are clear that conversation is important for students' mathematical learning. Some of the teachers refer to how they have used study buddies, a tool from their formative learning PD, in their implementation of creative mathematics in their classroom. Study buddies refers to two to three students discussing their mathematical learning, their understanding of concepts and problem-solving. Afterwards, the teacher encourages students to share their thoughts with the whole class. The teachers explain how the use of study buddies is one way to support mathematical conversation in the classroom. The teachers further rely on whole group discussion, especially when the groups are a moderate size such as 10 to 15 students.

Anna said that she found it important to be able to discuss with students the meaning of concepts and "what lies behind these concepts". She appreciated that within her implementation of the creative mathematics PD, she was able to provide students with an opportunity to frame their own thoughts. She said that she believed it was important "to get them [the students] to go deeper and understand and discuss the meaning of what we are doing". Anna shared that the PD program provided her with an opportunity to focus on conceptual learning through mathematical discussion.

Jenny explained in the third seminar how she uses the conversation to get students involved in conceptual learning. She explains this by describing a hypothetical discussion with a student:

> Because you [the teacher] do not need to do that much. Just how you answer [the students]. [Teacher:] I do not know, what do you think? [Student:] But this, is it correct? [Teacher:] What does your study buddy say? [Student:] I do not know.

With that, Jenny explains how she encourages her students' mathematical thinking by using open-ended questions and study buddies. Inga writes in a survey that she has learned the use of "good questioning techniques for mathematics" from the PD seminars and applies the techniques in her

classroom. The same was true for the subject teachers who discussed open questions for creative learning in the third seminar.

Inga explains in her interview and seminars how she uses mathematical conversations with students to support them in reaching a deeper understanding in mathematics, which she believes to be crucial for creative mathematics learning. She states: “If I had to name just one thing that is most important in our Tuesday work, it is that we are deepening the mathematics.” Inga brings mathematical discussion into her classroom as well as into other school situations, such as lunch time. She explains the value of using everyday circumstances for mathematical conversations in the second seminar: “To help them [the students] become conscious that we also do mathematics at lunch time. To [help the students] understand the concepts.”

Silvia is clear that mathematical discussion is vital and says in the third seminar: “One really needs to put this [mathematical discussion] into the lesson plan. Stop and talk. It would actually need to be on the list in front of you [the teacher].” She explains in her interview how she uses classroom discussion to help students learn the language of mathematics. She believes that the students need to learn the mathematical vocabulary in order to discuss mathematical concepts freely.

Silvia describes her experience when she was substituting in grade five in the second research year, with students who she had previously taught in the first research year. She is supporting the students with using mathematical terms for geometry and shares her experience: “Then I went, as a substitute teacher, into grade five and they were going through shapes and were supposed to describe different shapes and they have stopped saying box,” Silvia said, pleased, and added: “There was a square, a rectangle and a trapezoid. We try to use the correct mathematical concepts. What is perpendicular, parallel? To get that [conceptual discussion] in [with the students].” Silvia articulates that it is her role as the teacher to get the students involved in a theoretical discussion and notes that it takes time for the students to learn how to do that. She uses both whole class discussion and study buddies to help her students become accustomed to conversations in mathematics.

The theme of conversation is reflected clearly throughout the cases and was brought up by other teachers as well. The first-grade teachers wish to use conversation more meaningfully in their teaching. Some of the special subject teachers use discussion with students to connect mathematics to

their subject. One teacher who taught mathematics in the older grades said in the sixth seminar:

> We were discussing how creativity is in reality this analytical thinking and what you [as a teacher] need to explain and teach. We [the teachers] are in a good place because we are always adding to our knowledge [through PD] and we are always explaining and teaching how the study buddies are good for this creative work and creative thought: to [let students] talk together.

Her comment shows how she experiences the study buddies as a useful tool for mathematical discussion and mathematical discussion as important for creativity and creative thinking.

The literature is clear that mathematical conversation lies at the heart of creative learning and can help students move from procedural-based learning to concept-based learning (Luria et al., 2017). The teachers who participated in the PD program on creative mathematics wish to promote rich mathematical discourse as part of their teaching and classroom culture. They explain how mathematical discussion supports students to develop stronger conceptual understanding and fosters creativity in their mathematics classroom. The teachers are clear on which situations support them to create a learning environment which entails rich mathematical discussions among students. The teachers reiterate that having ample time, space and a moderate group size supports them with this important aspect of student learning. They know that their students can blossom if provided with ample opportunities to practise discussions in their mathematics learning.

4.8.2 Hands-on Learning

Hands-on learning refers to having some physical material to work with in a classroom. In mathematics this can be various types of counters, blocks, measuring tools, abacus and other manipulatives that support students in their learning. Hands-on learning can also refer to working with patterns, art, or other visual mathematics.

In creative learning, hands-on work can help students get closer to mathematical concepts and make the mathematics real for them (Pound & Lee, 2015). This is what the teachers experienced in their classrooms. Anna said that she noticed how hands-on learning “helped to connect concepts

to hands and senses". Jenny and Silvia learnt through their action research project that working quietly from a procedural-based textbook did not yield conceptual learning that the students could apply to the physical world when working with geometry.

Inga and Sunna discussed how working hands-on was beneficial both to the teachers in the seminars and to their students in their classroom. This aligns with theories that state that teachers need to have had creative experiences in mathematics themselves to foster creativity in their mathematics classroom (Haavold & Birkeland, 2017; Nadjafikhah et al., 2012; Sheffield, 2013; Shriki, 2010; Sternberg, 2017). Sunna said: "I felt that doing the hands-on projects [in the seminar] was the most helpful. It was what lingered most." Inga agreed and said: "I agree. It moved me most when we did hands-on work." The experiences of Sunna and Inga align with theories of PD for mathematics teachers and the importance of giving the teachers the opportunity for hands-on work in PD programs (Loucks-Horsley et al., 2010).

The same focus on hands-on learning is reported by other teachers who participated in the PD program. The first-grade teachers explain in the sixth seminar that they believe that creativity and hands-on learning is vital to their students' learning as "they [the students] do not even know what the numbers look like yet." With this, they are explaining how they find creativity and the use of manipulatives crucial for young learners who are building number sense and can conceptualize through hands-on learning.

The home economics teacher shares in the seventh seminar that her students work with mathematics and measuring tools: "They [students] are naturally mostly using fractions. They need to measure half, a quarter, and sometimes one eighth. Or they need to double a recipe." In this, the home economics teacher is aware of how she uses mathematics in her classroom and how students learn mathematical concepts in her classroom by using tools for cooking and baking.

The idea of using manipulatives or providing students with opportunities for hands-on learning aligns with the literature on creative mathematics and writings on PD for mathematics education (Boaler, 2016; Loucks-Horsley et al., 2010; Pound & Lee, 2015). The teachers who most discussed hands-on learning were the ones who taught the younger grades. Clements (2000) theorizes the use of concrete manipulatives for mathematics learning and the importance of sensory-concrete experiences for younger students. Young learners have often not progressed to the stage of abstract thinking. Further, some teachers discussed the value of hands-on learning

for themselves. This raises the question of the importance of hands-on learning or the use of manipulatives for older learners who have the ability to do abstract thinking.

4.8.3 Play, Games, Fun, and Motivation

Play, games and fun are often not what people associate with mathematics learning. Many people have feelings of discouragement, fear, or aversion to mathematics from their own learning experiences (Boaler, 2016). This has resulted in countless people developing very low self-efficacy for their own mathematics learning and choosing to not pursue any learnings related to mathematics later in life (Boaler, 2016).

To move away from the development of negative experiences in mathematics learning, it helps to work towards a growth mindset for students (Boaler, 2016). Jenny shares about low self-efficacy and negative emotions related to mathematics and explains how she wants her students to experience success and overcome their mathematical fear. She wishes that students can overcome obstacles in their learning through more creative learning modalities. She also discusses student mistakes in mathematics and how to create a "good mistake culture" in her classroom. Inga is also clear on the importance of a growth mindset for mathematics learning and how her participation in the PD seminars has strengthened her view on that.

Empowering students to experience joy in mathematics is a way to move away from negative emotions related to mathematics (Boaler, 2016; Pound & Lee, 2015). Anna talks about how she uses play and games for mathematical learning and to bring life and joy into her mathematics classroom. This same focus is reflected in the literature on how fun, interactive ways to teach can support students' creative mathematical learning and how puzzles and games can be used for this purpose (Pound & Lee, 2015).

Silvia discusses in the third seminar how her students use a play they have chosen to practise and perform as a learning experience for mathematics:

> Some girls in our class decided to do a play for Öskudagur [the carnival]. It grows to include an audience. Tickets are needed. Play money should be used for arriving and paying. How many chairs are needed? A lot of this [type of work and discussion]. How could we [as teachers] do more with this? I find their

> [students'] ponderings of mathematics for this extremely creative. They are wondering how many will arrive. They truly are wondering about all these [mathematical] concepts.

Silvia notices how creative her students' process is with a task they choose to do. The students are having fun but are also able to work with mathematical concepts and apply number sense and operation fluency to a real-life situation. She adds: "When they [the students who planned the play] are organizing, they are discussing and wondering and making decisions." Silvia's account shows how the mathematical conversation ties in with the playfulness of students.

In the sixth seminar, Sunna shares how she uses her experience as a physical educator to reach students and connects some movement games to mathematics. She says: "There is a whole lot that is creativity with mathematics in the first grade. Like when you are teaching them Dimmalimm [movement game], counting and running. Counting the steps. They run but they do not count the ten steps." With that comment, Sunna explains how a number game that entails running and counting to ten helps students who are developing number sense. Initially, students run and do not count the steps, but with time they learn to count each step.

This description is similar to how the first-grade teachers describe their coffee shop with kindergarten and first grade. Anna shared a similar learning experience with what she and her co-teacher called "the big store game." Many of the teachers discuss in the seminars the value of "connecting [the mathematics learning] to real life", as one middle-level teacher expresses it. Both play and connecting mathematics to real life were themes in the teachers' initial ideas of creativity and mathematics. The importance of playfulness in mathematics was present throughout the seminars and in the work with the teachers between seminars.

The teachers explain how important they find it for their students to experience their learning as fun and how that encourages motivation. Amabile (2013) writes about motivation as an important factor for learning: task motivation is an important component in her componential model for creativity. Motivation can be helped through fun, play and games. Csikszentmihalyi (1997) discusses the connection between motivation and creativity flow. In flow, the person is enmeshed in what they are doing and not controlled by external motivation. Flow is often seen in children's play (Csikszentmihalyi, 1997).

The teachers found that fun and play were important for student motivation. Inga described the importance of students' enjoyment for motivation and believes that games and play are important when teaching a group that has mixed mathematical ability. Inga says: "It [using play] meets the broader [learning] group." The teachers experience play, games, and fun as helpful for student motivation, learning, and for reaching the entire range of the student group.

4.8.4 Time

Time is important for creative mathematics learning (Nadjafikhah et al., 2012; Shriki, 2010; Sriraman et al., 2011; Sternberg, 2017). Giving students time to work on more extensive projects and space to collaborate provides them with the opportunity to come up with creative solutions (Shriki, 2010; Sriraman et al., 2011; Sternberg, 2017). Teachers and schools can provide an environment that provides time and space for developing creativity and offer students the opportunity to experience a flow of thought (Nadjafikhah et al., 2012). This flow of thought can be accomplished by creating a learning environment where students are given the chance to reflect on their emerging ideas and concepts (Nadjafikhah et al., 2012). It is reflected in the literature that students need ample time in order to be creative in their mathematics learning (Nadjafikhah et al., 2012; Shriki, 2010; Sriraman et al., 2011). When provided with that time, students can analyze, research, discuss, problem-solve, and generalize in a broad way (Luria et al., 2017; Nadjafikhah et al., 2012; Sternberg, 2017).

The teachers in the study were very clear on the importance of time for their implementation of creative mathematics in their classroom. They explained that they needed good time to prepare, they needed longer class periods, they needed to give their students time to work on different projects, time to learn new ways, time to think, and so on. That is why the word 'time' has been put in the space surrounding the modalities in the model for creative mathematics learning.

Anna explained how having two-hour-long class periods was important for creative mathematics learning. She said: "You can be so relaxed around the discussion." She further explained: "To suddenly have time to discuss mathematics, discuss the meaning of the concepts we are using in mathematics, what lies behind these concepts and to get the children to express their thoughts about numbers and operations." She stated:

> I am very interested in using what [components] I thought were interesting from the [student] workshops, which are the conversation, using manipulatives for learning, and bringing the mathematics learnings to life with play and games. And to have the time for that.

Anna's descriptions show that she experiences the length of class period as crucial for developing creativity in her classroom. She connects the modalities that have been emphasized in the model for creative mathematics learning to the need for adequate time in her classroom.

Jenny and Silvia discussed the importance of having enough time to prepare in their action research. Silvia explained in her interview:

> There is always that classical problem of not having enough time for preparation. A well-prepared lesson is always so beneficial. You can always see it if you are well prepared. I feel that we need to use so much time for behaviour, discipline, and parents' issues, which takes away from the time to prepare. The preparation should come first and then the rest. That is a little like what happens at times.

Inga and Sunna put the same focus on preparation time as other teachers did. One of the teachers who taught at the middle level said in a survey:

> What I would have liked better is not really about the PD but the constant race for time. It would have been fun to use the ideas better in my own classroom, but somehow I did not manage to organize it. So, it could be said that I would have needed to get fully developed projects to try out.

The comments on getting projects to try out can of course save teachers time and give them more time to prepare in other ways. However, some of the teachers are also clear on how important it is to prepare your own lessons. Silvia explains: "It is so important that you put something together rather than being handed something and told what to do." Preparing your own lessons is one way for teachers to actively participate in their own PD and can provide them with an opportunity to take ownership, which is considered an important component for effective PD (Loucks-Horsley et al., 2010; Zepeda, 2019).

However, teachers sometimes struggle to find the time to prepare creative mathematics lessons. Jenny and Silvia discuss how they sometimes rely on the use of textbooks because there is a "lack of time for preparation [so] we make do with the textbook". In their action research, they explained the limitations of the procedural-based textbooks they used. However, a teacher from the first grade made a comment in the third seminar on how they had been using some assignments from a different textbook that is currently out of print in Iceland: "I find there are lots of ideas and creative problems [in that textbook]," which shows how the teachers experience different textbooks as providing different experiences of learning.

The literature on PD shows that teachers need extended time and support to change their teaching (Desimone, 2015; Loucks-Horsley et al., 2010; Zepeda, 2019). Teachers noted the importance of this support, as Silvia explained in her interview:

> You need the boost, and you need to try things out until they become natural. This takes time and you need great support on this journey. Or that is how I experience it. I want to have amazing support to get into it and I am really open for any methods I experience working. It needs management and follow-up. That is my experience.

Silvia commented on how important she felt collaboration and continuity were for learning. She claimed that she learned from collaborative work with other teachers. This is in line with Hargreaves and O'Connor's (2018) writings about collaborative professionalism, mutual dialogue, collaborative inquiry, and common meaning and purpose. All the teachers in the case studies expressed the value of the personal support I provided them in developing lessons that facilitated creative mathematics learning in their classroom. These results align with writings on coaching and how important collaborative coaching is for PD (Knight, 2011). It is however clear that coaching and collaboration require time and a joint commitment by the teachers and coach involved.

4.8.5 Learning Space

Space is another aspect surrounding the modalities in the model for creative mathematics learning. As with time, space proved to be an important concept in the data that was tied into many aspects of the learning. Learning space can refer to the physical space where learning

takes place or to the space given to learning within a certain situation. Here, we discuss learning space as both time and space are used to describe the classroom, available materials, group size and the more abstract space of honouring mistakes and allowing students to develop their own solutions.

The physical space was brought up by many of the teachers as an important factor for student learning. They were very clear that smaller groups helped them with implementing creative mathematics in the classroom and that they needed to be able to use different learning areas for the groups. Some teachers found that if the classroom did not offer areas that could be used for different groups, it hindered them in using discussion, hands-on learning, or other creative modes of learning. Anna explains the challenges when she shares a large class with her co-teacher in the second research year. She shares: “When the group is large. When you have 21 children or 42 as we do for two [teachers] in one space. It makes using manipulatives more difficult.” She reiterated how challenging it is “to not be able to divide the space and have a [mathematical] discussion in two separate areas”.

Anna explains that when she has the option of smaller groups, she is able to promote conceptual discussion: “Earlier today, I had them discuss what addition means. They discussed as study buddies first and then I drew names [to ask different students to share with the group] and then so much comes from them [the students].” This shows how the learning space is important for mathematical discussion and how important group size is for creative work in mathematics.

Being able to work with a smaller group within the class proved to be crucial for the teachers and was noted by all the teachers in the case studies. They found that groups of 10 to 15 students were more manageable and that it was easier to use hands-on learning, fun, games, discussion, and creative learning with those than with groups of 20 or more. Sunna shared: “I feel there should be less than 15 students per teacher when we are doing this deeper work [in mathematics]. That there are just 10-12.” Inga agrees: “Yes I find 10-12 really to be a maximum.”

Another aspect of the learning space that the teachers valued was creating the space for mistakes. Jenny said: “I find it good to promote good mistake culture. That mistakes are good and that we are all in a different place. That is number one, two and three to admit.” It is interesting how the teachers are clear on this attitude regarding mistakes for their students as well as for themselves, as Inga says: “I am kind of happy that we are not

afraid to try out things. Not afraid to make mistakes." The teachers' ideas of mistakes align with new research on brain development and growth mindset (Boaler, 2016).

Closely related to honouring mistakes is the attitude towards students developing their own solutions. In the initial seminar, the teachers expressed how they believed that creativity could be fostered in mathematics through problem-solving and allowing students to explore their own methods for solving mathematical problems. This was also clear in the cases and in the analysis of the entire teacher body. Jenny and Silvia were particularly clear that they wanted to create learning opportunities and space for students for creative problem-solving. The same focus was seen in other cases where the teachers emphasized the importance of using different modes of learning to help students develop conceptual understanding in mathematics.

4.8.6 Summary

The model for creative mathematics learning is an important aspect of the results of this study. The model was developed based on data from all the participating teachers, although the case studies were instrumental for its development. The model assumes three components that are important for teachers who wish to foster creativity in their mathematics classroom. These components are conversations, hands-on learning, and play and games. These were all components that the teachers experienced as important for fostering creativity in their mathematics classroom. Time and learning space were considered important by the teachers to be able to use the other components successfully.

The results show that conversation about mathematics was a way to allow the students to explore their own thoughts as well as to learn from others. Allowing students to discuss their mathematical ideas, concepts and solutions fostered an atmosphere of creativity. This is congruent with the literature on creative mathematics learning (Nadjafikhah et al., 2012; Shriki, 2010; Sriraman, 2009). The teachers experienced benefits from hands-on learning and from allowing the students to use manipulatives, draw and experiment. Play and games were a helpful way to support students' motivation and engage them in different ways in a mathematical context. This is closely related to flow, which is an important aspect of creativity (Csikszentmihalyi, 1997).

By addressing barriers and needs with the teachers, it became obvious that time was an important factor for fostering creativity. The teachers

particularly commented on the benefit of a longer class period for mathematics as well as having time to prepare good lessons. This is in accordance with the literature that emphasizes the importance of allowing enough time for the creative process (Nadjafikhah et al., 2012; Sriraman, 2009). Another aspect the teachers experienced as important was that of learning space. This included group size, the physical classroom, and being equipped with manipulatives, as well as creating a supportive learning environment.

The components addressed in the model for creative mathematics learning can be brought together to support student creativity in the mathematics classroom. The model aligns with theories of creativity and creative mathematics education. Moreover, the model assumes a novel way of bringing together components that the teachers in the study found helpful for promoting creative learning in their mathematics classroom.

4.9 Summary of Results and Discussions

This chapter has reported on the results from the case studies and other participating teachers from Vinaskóli. The results explain how the PD program influenced teachers' views and pedagogy. Teachers' reports of how their participation in the PD program on creative mathematics influenced their classroom practice and student learning were analyzed and discussed. Themes that emerged from the analysis were presented. These themes were discussed and connected to the theories that were presented as part of the theoretical framework.

The plan of the chapter was based on different aspects of the results as they fit how the themes were developed. The first section reported on the teachers' initial ideas as expressed at the first PD seminar. It was evident that the teachers were able to recognize creativity in mathematics education early on and they brought up some of the ideas that were already in the theoretical framework. This was particularly true for their comments on problem-solving, mathematics all around and play. Analyzing these ideas in the second action research cycle helped set common goals and a vision for the PD, as assumed in Loucks-Horsley's (2010) design framework.

The following sections report on the case studies and the themes that emerged from analyzing the data from the case studies in action research cycles three to five. The teachers in the case studies worked with me intently to incorporate creative learning in their classroom and expressed how beneficial the collaboration was for their teaching. This is congruent

with coaching as a way to support teachers in professional development (Knight, 2011). The teachers expressed finding the PD empowering, and described how they developed as teachers as a result of the PD seminars as well as from our collaboration.

Themes emerged from the case studies that related to the language and use of discussion and the teachers explained how they were able to support the diverse learning group through games, play, and hands-on learning. The teachers explained how they used their own background with mathematics learning in a positive way in their classroom. They emphasized self-efficacy and the use of other PD they were involved in, such as formative learning, with the creative mathematics. These themes relate both to the theoretical background of a creative learning environment and to ways to foster creativity in mathematics education. Mathematical discussion, play and hands-on learning can be effective ways to incorporate creativity in the mathematics classroom (Boaler, 2016; Pound & Lee, 2015). The ways in which the teachers reflected on and made use of their background can be mirrored in writings about life stories and the development of pedagogical views (Friðriksdóttir & Aðalbjarnardóttir, 2010; Goodson, 2017).

Through a lateral analysis of the cases, in action research cycle five, more became evident about how a PD program on creative mathematics education can support teachers' process and how their views are developed as they participate in PD seminars based on theory and practice. The results from action research cycles three to five on how the whole school participated in the PD program helped explain how a PD program such as this can support teachers and how the PD seminars benefited teachers in their classrooms. It is clear that the teachers who received coaching as part of the PD program were the ones who reported the most on being able to use their learning from the PD seminars in their classrooms. Collaborative professionalism and coaching were added to the theoretical framework as a result of the work I did with teachers in the first research year and through analyzing the results in action research cycle three (Hargreaves & O'Connor, 2018; Knight, 2011).

The goal of the PD program was to support the teachers to focus on creativity. By going through five action research cycles, results have been developed that provide information on how to better support teachers with creativity in their mathematics classroom and how a PD program with the goal of supporting such learning can be developed. This is crucial for the development of any PD (Desimone, 2009, 2015). The results from the cases and the whole school shed light on themes that hold the potential to add to

the conceptual understanding of creative mathematics learning. Some of the most prevalent themes were developed into a model for creative mathematics learning. The model assumes components that helped the teachers to incorporate creative mathematics in their classrooms and has implications beyond the research setting.

The following chapter summarizes findings and explains how the results can be of value for teachers, schools, and those developing PD. The chapter addresses how this research adds to the existing body of literature, both on creative mathematics education and PD, and the value it holds for future research in the field.

5 Conclusion

In the national curriculum for Icelandic compulsory schools, creativity is considered one of the guiding pillars of education and should be fostered in all subjects (Ministry of Education, 2013). This research clearly shows that a well-developed PD program on creative mathematics education can influence teachers' pedagogy and classroom practice. This is particularly important since previous research indicates that mathematics teachers often lack the ability to foster creativity in the mathematics classroom (Shriki, 2010).

Within action research, such as this study, much can be understood about the process of a PD program and many ideas are learned along the way, albeit with room for improvement and bettering for all involved. The use of Mills' (2018) dialectical action research spiral for this study allowed the PD program to be developed as it unfolded. Input from the teachers was used to improve the program and changes were implemented throughout the two-year process and by going through five action research cycles.

The teachers who participated in the collaborative whole-school PD program reported that the PD program influenced their pedagogy and classroom practice in a positive way. The teachers describe how they became able to use different methods to support their students in mathematical creativity. These methods included conversations about mathematics, hands-on learning, play and games. The teachers focused on self-efficacy, formative learning and giving the students options of developing their own conceptual learning.

By researching the active development of the PD program, it became clear that a PD program on creative mathematics with in-service teachers can influence their pedagogy and views of mathematics education. The teachers reported that they noticed positive changes for their students through the program and by applying their learnings in their classroom. The cyclical action research revealed that the teachers who were most able to bring creativity into their classrooms were teachers who partook in a combination of PD seminars and coaching. The PD seminars were based on presentations, discussions and the teachers working in groups on creative mathematical tasks. The coaching was based on collaborative

professionalism and included collaborative inquiry, collective responsibility and initiative, and joint work focused on student learning.

In this chapter I explain the conclusions of this study and address how a PD program on creative mathematics education could be developed in the future. Implications of the research for future research and practice are discussed. The first section is focused on how the teachers explained the influence of the PD program on their pedagogy and classroom practice. The second section is focused on what can be learned from the active development of a PD program on creative mathematics learning. The third section looks beyond this study at how the PD program and results can be further developed. The fourth section focuses on the implications and applications of the research for teachers, schools, students, those planning PD, and policy makers.

5.1 The Influence of the PD Program

Previous research indicates that many teachers recognize the need for more creative learning in mathematics but struggle to implement it (Shriki, 2010). They may lack the experience or ability to emphasize this important component in their classroom. This can be related to antiquated ideas about how learning should happen or lack of creative experiences in their own mathematical learning. Research shows that teachers need to have experienced creative mathematics learning themselves in order to foster creativity in their classroom (Nadjafikhah et al., 2012; Sheffield, 2013; Shriki, 2010).

This study focused on a developing PD program on creative mathematics education and researched how the teachers who participated in the program reported its influence on their pedagogy and classroom practice. The PD program aimed to actively encourage and motivate participating teachers in their mathematics teaching and supported them in fostering creativity in their classroom. An effective PD is generally extensive and connected to teachers' practice, so the program was planned to be two years long (Desimone, 2015; Martin et al., 2014; Zepeda, 2019). The PD program was based on a theoretical framework that was founded in theories of creativity and creative education in general, theories of creative mathematics education, and theories of PD for mathematics teachers. The program consisted of seminars as well as teachers' work between seminars and support thereof. The seminars included instruction, reflection, active participation, and collaborative projects, all of which are connected to successful PD (Desimone, 2015; Martin et al., 2014; Zepeda, 2019).

The program focused on supporting teachers to bring their learning from seminars into their classroom, with the goal of positively impacting teaching and learning. Between the seminars, the teachers worked towards implementing creativity in their mathematics classroom. Teachers who sought support received coaching through meetings and lesson planning, as well as in the classroom itself. This coaching was based in collaborative professionalism with a focus on collective autonomy, mutual dialogue and creating a common meaning and purpose.

The research explored how the participating teachers reported the influence of the PD program on their views, pedagogy, and classroom practice. The results are based on in-depth analysis of three case studies, including five teachers, with data from all participants analyzed for a larger picture. The teachers in the case studies were chosen to get the best insight into the research topic. The method of case study was chosen to develop a deeper understanding of how the teachers experienced the influence of the PD program on their views, pedagogy, and teaching practices. The teachers explained how they experienced student learning when they focused on creative mathematics in their classroom and what barriers they encountered.

The results were gained by going through five action research cycles, based on the use of Mills' (2018) dialectical action research spiral. The first cycle was focused on developing the theoretical framework before and during the PD program. The second action research cycle analyzed the initial ideas of the teachers on creativity in mathematics education when the PD program began. The third action research cycle sought to get results and develop learning in the first PD year and the fourth cycle did the same in the second PD year. The fifth cycle was a summative cycle and happened at the end of the program, with the goal of weaving the results together and drawing conclusions that were valuable beyond the setting.

The teachers in the research explained how the PD seminars supported them in becoming clearer about their pedagogy and shared how the seminars taught them different modes of implementing creativity in their classroom and influenced their views of mathematics education. The PD seminars gave them an opportunity to communicate and collaborate with other teachers, which they felt enriched their own learning. They also noted that opportunities for hands-on learning in seminars were useful for their learning process. Some of the teachers experienced that the PD program of creative mathematics meshed well with other PD they participated in, such as a PD program on formative learning.

The teachers explained how their experience with the PD seminars influenced their views of mathematics education. They noted how the seminars provided them with a theoretical framework for creativity in mathematics that supported them to become clearer about their own views on mathematics education. The teachers explained how their beliefs about a growth mindset for mathematical learning were strengthened. They discussed how they learned the value of using open questions and conversation for creative mathematics learning through the seminars. Some teachers found that participation in the PD program supported their self-efficacy for mathematics teaching and that the seminars provided them with encouragement to implement creative mathematics in their classroom.

The teachers from the case studies explained how the PD seminars influenced their views of mathematics education in a positive way, but they were clear that the coaching they received between the seminars proved instrumental in them being able to implement their learnings from seminars in their mathematics classrooms. They discussed how they were able to use conversations to foster mathematical creativity in their classroom. They shared how they helped students develop conceptual understanding through the use of manipulatives and by providing students with hands-on learning opportunities. The teachers also explained how they used games and play to make mathematics learning more enjoyable and to encourage motivation for students in their mathematics classroom.

The teachers in the case studies perceived changes in student learning throughout the course of the PD program and coaching. According to the teachers, their students were more adept at partaking in mathematical conversation and collaboration after they implemented creative mathematics in the classroom. The teachers also noticed how their students developed a deeper conceptual understanding in mathematics. Furthermore, one teacher researched students' understanding of mathematical content that she and her co-teacher had worked with in a creative mathematics workshop with the students. She detected an improvement in content learning that she contributed to the creative workshop.

These results are interesting when revisiting Desimone's (2009) writing of the ideal process for any PD. Desimone (2009) describes how a PD that teachers partake in should result in new knowledge, ability, or views, which should in turn lead to improved pedagogy and teaching. The improved teaching should then affect student learning in a positive way. The stories

from the case studies support this sentiment. All the teachers from the case studies reported on change in their own views and knowledge as a result of participating in the PD program. The teachers explained how coaching supported them to implement their learnings from seminars in their classrooms and how they believed that this resulted in improved student learning.

The teachers who received regular coaching through meetings and in the classroom described how they were able to bring their learning of creative mathematics into their practice. The teachers themselves chose to enter into a partnership and we jointly discussed their practice and how they wanted to pursue creative mathematics in their classrooms. This can be seen as partnership coaching.

Knight's (2011) writings about partnership coaching are very clear on the importance of equality, choice, voice, reflection, dialogue, praxis, and reciprocity. Throughout the coaching I supported the teachers in identifying goals, listened to the teachers actively, and asked open-ended questions. I provided the teachers with encouraging and reflective feedback on their teaching as they implemented creative mathematics in their classroom. The collaboration resulted in relationships of mutual learning and dialogue, and the teachers shared how this collaborative partnership supported their pedagogy and classroom practice.

5.2 Learning from the PD Program

The goal of any action research is learning. Teachers or other stakeholders set out to study some aspect of learning or testing a new method (Efron & Ravid, 2020; Mills, 2018). For this research the focus was on developing a collaborative PD program on creative mathematics education at Vinaskóli, where I am employed as a mathematics teacher. The school is an Icelandic compulsory school with children aged one to sixteen. The PD program in research was attended by all the teachers who taught grades one to ten, including general teachers and special subject teachers. All teachers participated in PD seminars, while teachers who were chosen for case studies received coaching between seminars. The coaching focused on collaboratively bringing creative mathematics learning to their classrooms.

The teachers reported on their learnings from participating in this study. They explained how they became stronger in their professional beliefs and shared what they found to be important for their classroom practice. The teachers became more aware of the importance of creativity for mathematics education and which ways they found beneficial for fostering

creativity in their mathematics classroom. These findings were developed into a model for creative mathematics learning which holds implications for other schools, teachers, and students. The model was derived from the data analysis and emphasizes the modalities that the teachers expressed as important when they worked with creative mathematics in their classroom. The model is a clear, concise way to present the results.

The model for creative mathematics learning depicts the features the teachers experienced as important for their mathematics teaching and that supported student learning. It also explains how creative learning can be encouraged in mathematics. While reflecting some ideas from previous literature on creative mathematics learning, the model presents a novel way to approach creativity in the mathematics classroom. Furthermore, it can serve as a framework for other teachers and those planning PD with teachers, as well as for teacher education.

The three core modalities of the model for creative mathematics learning are play and games, hands-on learning, and mathematical discussion. The teachers explained how the modalities were crucial for engaging students in creative mathematics learning and described the importance of play, games and fun for the creative mathematics learning of their students. The teachers connected fun and play to students' motivation and believed that games and playfulness provided students with rich and real learning opportunities.

The teachers in the research discussed using manipulatives and engaging their students in hands-on learning. This focus was particularly noticed for teachers who taught mathematics in grades one to four. The teachers explained how the use of manipulatives in the mathematics classroom provided students with opportunities to conceptualize their mathematical learning in a way they were not able to do when working with a procedural-based textbook.

The teachers in the case studies, as well as a number of teachers who taught at the middle and secondary level, explained how they found mathematical conversation crucial for fostering conceptual understanding among students. They shared how mathematical discussion supports students in developing creative thinking in mathematics. The teachers connected mathematical discussions to open-ended questions, problem-solving, and the language of mathematics. Teachers relied on study buddies and whole classroom discussions to foster conversation in their mathematics classroom.

The model for creative mathematics learning also emphasizes the need for time and learning space for teachers to be able to foster creative mathematics learning. This focus sprung out of teachers' explanations of barriers to the implementation of creative mathematics in their classroom. In that regard, time was the greatest hindrance. The teachers shared how they lacked time to prepare creative mathematics lessons and how they preferred longer class periods in order to allow time for creativity.

The teachers found various aspects of learning space to be important when incorporating creativity in the classroom. They found that being able to divide the class into smaller groups was essential for creative mathematics lessons and explained how having smaller groups provided them with better opportunities to use play and games, hands-on learning, and conversation in their classroom. They commented on the importance of the physical space for group work and having learning materials such as manipulatives available for the students to use.

The model for creative mathematics education can be used as a tool for teachers who wish to focus on creativity in their mathematics classroom. The modalities in the model explain how to set up a classroom and plan lessons based on fostering creative learning in mathematics. The model for creative mathematics learning could also be used as a framework for evaluating creative learning. The conversation, hands-on work, and play and games can serve as lenses for documentation in the spirit of formative assessment.

One team of teachers explored in their action research project how they found standardized tests to be a hindrance when they wanted to focus on creativity in their mathematics classroom. This accords with literature that reports that standardized tests can be a hindrance to creative mathematics learning (Sternberg, 2017). Schools are often focused on evaluating students according to measurable goals. Since creativity is process-based, it can be more challenging to evaluate than some other aspects of learning, such as content knowledge. Therefore, the lack of creativity in education can be due to many finding it difficult to evaluate (Harris, 2014).

The teachers, however, described how they found that formative assessment worked well with the creative mathematics. Tools for formative assessment include documentation such as writing, photographs, and videos, and the assessment is focused on the learning process rather than products or outcomes. Formative learning is closely related to formative assessment and is based on methods that help students be aware of their learning while providing them with continuous feedback (Clarke, 2014).

Formative learning is based on using clear goals and criteria, discussion buddies, feedback, and varied evaluation techniques. Some of these tools were used by the participating teachers and they found it tied in well with the creative mathematics learning.

One team of teachers relied on formative assessment for an action research project they carried out. They used photographs as well as their written documentation to reflect on the learning of the students, to research it, and to present it to other teachers. Conversation can be documented through video, audio, or writings. The hands-on work can be documented with videos or photographs, and the games and play with similar methods. Writings or other documentation from students and teachers are imperative to this process.

If evaluation is a hindrance for teachers focusing on creative learning in their mathematics classroom, assisting them to find tools to evaluate creative mathematics learning must be of great importance. As described, the model for creative mathematics learning can provide a lens for viewing creative mathematics learning while using the methods of formative assessment. The model for creative mathematics learning is a useful tool for theorizing on creative mathematics learning and can be applied to further research and for supporting teachers in planning and evaluating lessons.

It is clear that the learning in this study operated on many levels. Although the teachers focused on student learning in their implementation, it was evident that they themselves learned a great deal from participating in the PD program. These learnings then affected their teaching and work with students. I also learned from the joint development of the PD program and through the cyclical action research process. As I collaborated with teachers in grades two to four, I learned about how to better approach a young student group with creative mathematics learning. By analyzing the data, I also learned through the collaboration with the teachers how important coaching is for a PD program and how valuable collaborative professionalism is when teachers wish to make changes in their classrooms. This was a result of the socio-cultural nature of the PD program, of using the design framework from Loucks-Horsley (2010), and of going through repeated cycles of analysis and adaptation throughout the research.

This research has shown that a PD program on creative mathematics education can influence teachers to positively engage with creative ideas and to bring those into their mathematics classrooms. The active development of the PD program shows an example of how such a program

can be used to support teachers who aim to foster creativity in their mathematics classroom. From the development of this program, it is clear that a PD program on creative mathematics education for teachers benefits from being based on a socio-cultural model with teachers actively participating in their own learning and PD development. Coaching and collaborative professionalism are important aspects, along with providing teachers with opportunities to participate in creative mathematics activities themselves and by learning and connecting to theories on how to foster creative mathematics in the classroom.

The purpose of this study was two-fold and was based on both supporting the teachers at Vinaskoli to foster creativity in their mathematics classroom and to gain a better understanding of how a PD program on creative mathematics education can work to support teachers beyond the local setting. Two aspects that developed from this research and hold the potential for transferability are the PD program itself and the model for creative mathematics learning. The PD program that has been described here can be used in schools that wish to support creative learning in mathematics. The model for creative mathematics learning can be used by teachers, teacher educators and others who plan learning endeavours for students. The next section addresses how these results can be developed in future research.

5.3 Beyond this Study

Educational action research aims to inform and improve learning (Efron & Ravid, 2020). The effects can be at a local research site or applicable to a wider setting. This research influenced the teachers who participated in it, as has been explained earlier in this thesis. However, researching the influence of the PD program on teachers' pedagogy and classroom practice has also provided valuable information that can be beneficial to other teachers, researchers, teacher educators and those planning PD programs for mathematics education.

Any research is of course limited in scope and results. The goal must always be to add to existing research. This research has provided some crucial answers while bringing to light new questions. This section focuses on how the results from this research can be developed further, as well as pointing out possible applications of the research results. Although the creative mathematics learning model is an important finding of this research, further research is needed on how the model can be applied to different situations.

The case studies that represented the most prevalent data for the creative mathematics learning model were all based on teachers of students in grades two to four. However, teachers from all levels commented on mathematical discussion as an important aspect of creative mathematics learning for their students. The teachers in the study claimed that they themselves had learned much from hands-on learning and discussion in PD seminars, and a similar view is expressed in literature on creativity in mathematics education (Pound & Lee, 2015). Whether the aspect of play transfers to older students remains unclear. Some aspect must drive internal motivation, but motivation can be driven by more abstract components with older students and adults, although play and fun could come into it as well. It would be intriguing to further investigate how the model for creative mathematics learning fits for older learners, as well as looking at the role of motivation for creative mathematics learning.

Expanding the research on the model for creative mathematics learning could provide a wider understanding of how to teach for creativity in mathematics and other subjects. The special subject teachers experienced that they benefited from the creative mathematics PD program and were able to connect their learnings from the program to their own practice. Further investigating this connection and what role a PD program on creative mathematics could play for such teachers could be of interest. It would also be of value to see if the creative mathematics learning model can be applicable to a wider setting, such as other arts and sciences.

The study did not follow the teachers in the classroom, but the coaching allowed me to partake in their practice and witness their learning and teaching throughout the PD program. Although not studying classroom practices empirically, I did experience many of the themes that the teachers discussed, such as the need for mathematical discussion, manipulatives, and playfulness for student learning. I noticed how important it was for students to develop their own solutions and how being creative in that way provided students with conceptual learning opportunities and increased joy and motivation.

Empirical research of the students' learning was outside the scope of this research. While it is possible to replicate this study for different school levels and subjects, an interesting next step would be to research how teachers' implementation of creative mathematics impacts children's learning. Such a study could apply the model for creative mathematics learning to a classroom setting and explore the influence on student

learning, their participation in mathematical conversation or their learning experiences with manipulatives, games, or play.

Another interesting research topic would be to do follow-up research with the teachers who participated in the PD program. Doing more extensive research with the teachers from this study could provide information on further development of their views of mathematics education. The research could show if the teachers would experience a longer time period with a PD program as influential for their classroom practice and pedagogy. The study might provide information on whether the teachers would continue to experience similar barriers for implementing creative mathematics in the classroom. The possible studies that have been discussed here could possibly help to develop the creative mathematics learning model further.

5.4 Researcher Reflections

Creativity is vital for the learning of mathematics. Our society faces many challenges that need individuals who can come up with new solutions and create new products and ideas. This is particularly important for scientific development, and as computers have taken over many routine operations students need opportunities to develop their own ideas, connections, and mathematical concepts. Students who are given the opportunity to create in their mathematics learning have the potential of bringing this important trait to their life and our society in the future.

However, many have not experienced mathematics as creative in their own learning (Ambrose, 2017; Grégoire, 2016; Sternberg, 2017). This is true for many teachers who need support to foster this important trait in their classrooms (Panaoura & Panaoura, 2014; Sriraman, 2017; Sternberg, 2017). Therefore, this research has focused on developing a collaborative PD program with the goal of supporting the teachers to bring creativity into their mathematics classroom. By actively developing a PD program based on a socio-cultural model and by using Mills' (2018) dialectical action research spiral, much has been learned on how such a program can support teachers.

The PD program that has been presented here shows a way to support teachers to foster creativity in their classroom. The teachers have explained how they found PD seminars beneficial for learning about creativity and trying out creative projects. They then relied on collaborative professionalism and coaching to bring their learnings into their classrooms. The teachers explained how they experienced changes in their students'

discussions and learning. Any PD endeavour must have the ultimate goal of improved student learning (Desimone, 2009, 2015). This research has given insight into a PD program on creative mathematics and how such a program can influence teachers' views, pedagogy, and classroom practice. The goal is that this PD program can be an example of how to successfully work with schools that wish to emphasize the important factor of creativity in mathematics learning.

According to the Icelandic national curriculum guides for play schools, compulsory schools, and upper secondary schools, creativity is a guiding pillar for learning and is supposed to be fostered in all subjects and all school life (Ministry of Education, 2013). This same focus is seen in a new educational policy from The Department of School and Youth in Reykjavik, where creativity is a primary component which should be strengthened through rich opportunities of PD for teachers at all levels (Reykjavíkurborg-Skóla og frístundasvið, 2019).

This same educational policy in Reykjavík emphasizes increasing mathematics and natural sciences in the curriculum. Mathematics is allotted a proportionally large space in students' timetables in Icelandic schools and is an important focus in learning worldwide (Ministry of Education, 2013). Although jobs related to mathematics increasingly rely on creative individuals who can work collaboratively, mathematics learning rarely focuses on creativity (Leikin & Pitta-Pantazi, 2013; Sternberg, 2017) Students in schools are often expected to work on fixed problems with one correct answer and are frequently encouraged to use only one "correct" algorithm (Nadjafikhah et al., 2012).

Our society, educational policy, and the future all call for creativity to become a core factor of all learning. Students are growing up in a fast-changing world that needs creative individuals who can meet an uncertain future with an open mind and an ability to think for themselves, work cooperatively, and persevere in the face of adversity. Focusing on creativity in mathematics education can benefit students as well as our society. This study adds to the existing information on how to support teachers to incorporate creative learning in mathematics.

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Appendix A – Survey

Name:_____ Subject/Class:_____

Please answer the following questions, based on your personal experience of the professional development program Creative Mathematics for the school year 2018-2019 with the facilitator Ósk Dagsdóttir. The program includes seminars with presentations, mathematical projects, and discussion, as well as any work with the facilitator outside of the seminars.

What was I pleased with?

_____

_____

_____

_____

What could have been better?

_____

_____

_____

_____

I would have liked more of (or to add) the following:

_____

_____

_____

_____

I would have liked less of:

_____

_____

_____

_____

I have been able to use the professional development in my teaching. If yes, how? If no, why not?

_____

_____

_____

_____

I think the professional development will be useful in my teaching. If yes, how? If no, why not?

_____

_____

_____

_____

My views on creativity in mathematics are the same/have changed. Describe your views.

_____

_____

_____

_____

Other comments or ideas regarding the professional development:

_____

_____

_____

_____

Please select one option:

□ I would like these answers to be only used to improve the professional development

□ I agree that these answers can also be used for the doctoral research

Appendix B – Interview Guide

- Could you tell me about your views on mathematics teaching?
- How has the professional development project on creativity influenced your teaching (including the seminars and work outside of the seminars)?
- How would you like to continue working with creative mathematics?
- What support would help you reach your goals?
- What prevents you from meeting these goals?
- Do you see a difference in your students' learning?