# Gaining insights into teachers' noticing and interpretations of students' responses to silent video tasks

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This work was supported by the University of Iceland under the 100 years Anniversary Grant.

**This is an 'Original Manuscript' of an article published by Taylor & Francis Group in Research in Mathematics Education © 2020 British Society for Research into Learning Mathematics on 29 May 2020, available online:**
**https://www.tandfonline.com/doi/full/10.1080/14794802.2020.1722959**

# Gaining insights into teachers' noticing and interpretations of students' responses to silent video tasks

Four upper secondary school teachers in Iceland were asked to implement an innovative task, intended to support students' understanding of mathematical concepts, with their 17-18 years old students and to give feedback to the researcher developing the task. The task was challenging for teachers because it required them to make a short transition from a transmission-oriented to a socio-constructive approach in a new role as facilitators of learning; encouraging students' discussions about mathematics and utilizing student-to-student explanations in a whole group discussion. An overarching research question asked how teachers' knowledge influenced their engagement in the innovative practices. This paper reports how teachers noticed i) mainly unclarity and what was absent in students' responses to the innovative task, and ii) some limiting factors and insecurities that constrained teachers' ability to innovate.

Keywords: task design; teacher knowledge; teacher noticing

## Introduction

In this introduction we describe the papers' aim, followed by a description of silent video tasks; including what they are as well as their potential for teaching and learning mathematics. We invite you to imagine a mathematics classroom where the teacher normally either gives lectures to introduce new topics or to revise recently worked on topics and supports students working individually on tasks in a printed textbook. It is a classroom environment where students seldom or never communicate about mathematics among themselves. Now, imagine that today the teacher assigns a new type of task. It is designed to shed light on students' thinking and understanding, it involves the use of technology, requires discussion, and it is not necessarily clear to the student what a "good response" to the task might entail. Yet, this unfamiliar task is presented in mathematics class, possibly carrying a new message about what mathematics is and what it means to do mathematics (Henningsen & Stein, 1997;

Zaslavsky 2005).

During the task implementation, teachers' beliefs, experience, knowledge and practice all influence each other, develop and grow together (Henningsen & Stein, 1997; Stahnke, Schueler, & Roesken-Winter, 2016; Swan, 2007; Watson & Geest, 2005). Research shows that change in teacher practice usually is time-consuming (e.g. Schoenfeld, 2011; Swan, 2007), at least if their beliefs do not previously support the new ways of working (Liljedahl, 2010), and because of how short the intervention is, a potential for long-term impact on the teaching practices is slim (Collopy, 2003). Many a reform carries with it innovative tasks intended to support teachers and students in getting accustomed to new practices. Tasks that seem to teachers to be cognitively demanding for students are more likely than others to get reduced in complexity (Henningsen & Stein, 1997; Stylianides & Stylianides, 2008).

It remains unclear whether the task implementation could become a spark for teachers to find and actively participate in long-term projects intended to change practice from purely supporting acquisitionist metaphors of learning to a mixture of acquisitionist and participation (Sfard, 1998). Nevertheless, it is clear that we certainly would like students to communicate with peers about mathematics and teachers to see the value of students understanding–that is the reason why we design tasks that support such social constructivist practices. Teacher noticing takes place within the context of the task implementation and we will give an example of what it is that teachers notice when listening to their students' solutions to a silent video task.

This paper reports on findings from an ongoing design-based research project (e.g. Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) where silent video tasks are currently being developed by the first author in collaboration with upper secondary school mathematics teachers in Iceland. Because it is ongoing, theories forming are not ready yet and will not be presented in this paper. In the design-cycle reported on in this paper, four teachers implemented a silent video task with 17-18 years old students in trigonometry class. Focus was set on how teachers' knowledge influenced their engagement in the innovative practices. The following two research questions were used to frame the analysis in this paper: 1) What do teachers notice when listening to student responses to the silent video task? 2) What do teachers notice about their own abilities to use innovative practices to support student understanding?

***Silent Video Tasks***

Silent videos are short (less than two minutes long) animated films without text or sound, made with the dynamic geometry software GeoGebra and a screen recording software. Each video has a clear theme and is meant to illustrate properties or characteristics of mathematical phenomena, definitions or concepts. The process of assigning a silent video task involves the screening of a silent video that focuses on one previously-worked-on mathematical concept. In turn, the teacher splits students into pairs to watch the video as often as they like on their own devices while they discuss, prepare, and record their voice-over to the video. After preparing a follow-up lesson by listening to the students' responses, the teacher initiates a whole group discussion on the basis of some example student responses.

In their voice-over responses, students are asked to discuss and describe their interpretation of the short mathematical animations. Instead of stating a problem to solve, the silent video task creates the challenge for students to observe and explain what they see, to decide: i) what to focus on, ii) in what style and iii) how much detail to describe it, and iv) whether to refer to something related but not presented in the video. The teachers' task thus becomes not only to evaluate students' responses by noticing what students paid attention to and giving them hints on possibilities to improve, but also to prepare the whole-group discussion in such a way that students' attention can be directed towards what can be learnt from different responses.

Silent video tasks are designed to encourage students to communicate verbally about the mathematical topic presented in the video, to inform teachers about their students' conceptual understanding, and to make ground for group discussion, e.g. about precision in language use and common misinterpretations connected to the mathematics shown in the video. Not every mathematical phenomenon can be used as a topic of a silent video. If the phenomenon requires text and symbols, cannot be visualized and/or is very limited in possibilities of interpretation, then it is not suitable for making a silent video. An example of a good topic for a silent video would be how triangle's base and height influence triangle area because it could be open to different responses and have the potential to assess conceptual understanding. Contrarily, the procedure of performing long division would not be suitable–it would include text and symbols and a calculation procedure that is not flexible in terms of interpretation.

Students recording a voice-over to silent mathematics videos is an idea that was initiated and first tested in a Nordic and Baltic collaboration project for mathematics teachers and teacher educators (Hreinsdóttir & Kristinsdóttir, 2016). After participating in that collaboration project, I (first person refers to the first author throughout the paper) became interested in developing the silent video tasks further, mainly because the experience challenged me: students' responses to silent video tasks gave me important and surprising insight into my students' conceptual understanding, and built ground for group discussion on topics such as the reason for precision in mathematical definitions. I was interested in seeing what other teachers' experiences with the silent video tasks would be and what they would notice when listening to their students' responses to such tasks.

**Icelandic Context**

After compulsory school in grade 1-10 (6-16-year-old pupils), over 90% of Icelandic youth enrol in an upper secondary school for grade 11-13 (16-19-year-old students), most of them in programs that prepare for further education rather than vocational programs. Thirty upper secondary schools offer studies intended to prepare their students for further studies in STEM (science, technology, engineering, mathematics) subjects. Each school has its own school curriculum in mathematics that is designed by mathematics teachers and in some cases also administrative school staff. Mathematics teachers in each school also design the formative and/or summative assessment that is practiced in that particular school. In most schools the final mark students receive is composed to 50-70% of the results from a final written examination, and the rest by other written assignments with basic procedural tasks to be solved by students individually (Jónsdóttir, Briem, Hreinsdóttir, Þórarinsson, Magnússon & Möller, 2014)

When designing the school curriculum, upper secondary schools in Iceland are expected to consider the Ministry of Education, Science, and Culture's (2011) publication of a general mathematics curriculum that describes goals regarding the knowledge, skills, and competences in mathematics that students are expected to reach partly or fully. Schools are expected to provide students with a foundation for understanding in mathematics, for instance, the aim is not only to become proficient in calculation methods (procedural fluency), but also to attain conceptual understanding. What we mean by conceptual understanding is specific comprehension of mathematical concepts, operations, and relations. The competence goals furthermore include some adaptive reasoning as students are expected to understand and interpret the explanations and arguments of others without prejudice, showing respect and tolerance (Ministry of Education, Science, and Culture, 2011).

Despite the general curricular objectives, interviews with teachers and observations in classrooms in nine Icelandic upper secondary schools revealed that mathematics teaching practice was mostly limited to expository methods and recitation, drill, and practice (Jónsdóttir et al., 2014). This finding was confirmed in a study by Sigurgeirsson, Eiríksdóttir, and Jóhannesson (2018), in which mathematics lessons stood out among other subjects taught in upper secondary schools for their lack of diversity in the teaching methods used. Notably, discussions were never practiced in mathematics classes. In fact, mathematics is mainly practiced in such a way that students might get an image of mathematics as a set of rules and procedures handed down to them by the teacher to be memorized and worked on individually at their desks in silence (Sigurgeirsson et al. 2018; Jónsdóttir et al., 2014). Skovsmose and Säljö (2008) describe similar practice in Scandinavia and call it an exercise paradigm. They emphasize that many alternatives to such practices exist such as inquiry-based and problem-based learning.

In connection to this, it is interesting to look into what influences possibilities for teachers in Iceland to change their teaching practice. A study by Eiríksdóttir and Jóhannesson (2016) revealed that school policy and support from colleagues had great influence in this regard. If school policy did not support changes in the teaching practice, experienced colleagues determined whether changes would be implemented or not. In mathematics–more often than other subjects–traditions defined by experienced colleagues stood in the way, limiting possibilities for change and sometimes causing frustration for teachers with innovative ideas. This aligns with findings from other international studies (e.g. Thurlings, Evers, & Vermeulen, 2015).

## Teacher Noticing

In Mason’s words “What is noticed, marked, or recorded is necessarily being attended to" (Mason, 2011). Teacher noticing shapes how teachers act–what they decide to do–and it takes place within the broader context of their knowledge and goals (Schoenfeld, 2011). Gathering information about school curriculum and classroom environment that influences teachers' goals might help us understand some part of teachers' engagement in innovative practices, but in order to answer how teachers' knowledge influenced their engagement we therefore need to pay attention mainly to what teachers notice.

A widely-referred-to approach to teacher noticing in the literature relates to teachers in a teaching situation a) identifying significant events, b) using knowledge about the context to reason about those events, and c) making connections between specific events and broader principles of teaching and learning (van Es and Sherin, 2008). Van Es (2011) further developed a framework for teachers learning to notice student mathematical thinking, describing four levels of what and how teachers notice: starting at *baseline* (level 1) or broad vision where teachers attend to whole class environment forming general impressions and providing descriptive and evaluative comments, through *mixed* (level 2) vision where teachers begin to attend to particular students' mathematical thinking and add some interpretive comments or specific events and interactions, to *focused* (level 3) or narrow vision where teachers highlight noteworthy events and attend to particular students mathematical thinking. Further she went to *extended* (level 4) vision, where teachers connect particular students' mathematical thinking to teaching strategies, making connections between events and principles of teaching and learning (van Es, 2011).

Noticing frameworks by van Es and Sherin and by van Es were developed for instructional settings where teachers worked in the setting of a video club; watching and discussing video clips from their classrooms (van Es and Sherin, 2008). With their frameworks in mind, in this paper we use a framework variation developed for the situation where teachers listened to students' responses to a silent video task, as they prepared a follow-up lesson, reflected on their experience of using an innovative task, and thought aloud about what they noticed. On level 1 teachers attend to student responses forming general impressions and providing descriptive and evaluative comments, on level 2 they begin to attend to students' mathematical thinking and add some interpretive comments, and on level 3 they make a connection between students' mathematical thinking and principles of teaching and learning.

## Research Design

In this section, the research project methodology and methods will be described. A part of the research project goals was to further design and develop silent video tasks and their instructional sequence. Thus, design-based research was selected as the best suitable approach.

### *Participants*

From the thirty upper secondary schools in Iceland that prepare students for studies at university, I randomly selected schools and invited them to participate in the first cycle of the research project. Originally, teachers in six schools accepted the invitation to participate, but two teachers quit participation. One of them, working in a small rural school, dropped out before data collection started because too few students signed up for the course. Another teacher, working in a large urban school, participated only in the first interview and quit participation after that; giving lack of both time and students' interest as the reason. Thus, I worked with four mathematics teachers (see Table 1) from four upper secondary schools in the urban area in and around the capital city Reykjavík in Iceland. These four teachers had all studied mathematics or engineering at university level and completed additional studies for teacher certification. Their task was to assign a silent video task to their 17-year-old students who were enrolled in a trigonometry course that is compulsory for students whose line of study prepares them for further studies in STEM subjects.

Table 1. In this table, pseudonyms for the participating teachers, the size of school that they work in, and years (rounded to five years) of teaching experience are listed.

| **Teacher pseudonym** [gender] | **School size** [number of students] | **Teaching experience** [years, rounded to 5s] |
|---|---|---|
| Gauti (m) | Medium (500-1000) | 10 |
| Lilja (f) | Large (1000-1500) | 15 |
| Magni (m) | Large (1000-1500) | 5 |
| Snorri (m) | Medium (500-1000) | 40 |

Each teacher assigned the task at a different point in time, the order reflected in the alphabetical order of their pseudonyms (see Table 1), and throughout the data gathering period, gained experiences were shared. For example, Gauti found out that students using their phones to record their voice-over did not encounter a problem created by environmental sounds that disturbed recordings from laptop microphones in screen recordings. In turn, a suggestion to advise students to use phone audio-recordings rather than laptop screen recordings was communicated to the other teachers.

### *The Silent Video Used in the Study*

The first author prepared a two-minutes-long silent video with focus on the unit circle for teachers to use in their trigonometry classes. The unit circle video was inspired by an idea from Alf Coles (2008), who got the idea from Caleb Gattegno, who restored the Nicolet films (Tahta, 1981). What follows is a text description of the video, but the reader is also advised to view it at https://ggbm.at/BfRqGSKq.

To begin with, a circle and a clearly marked orange point (represented as a dot) placed on the farthest right side of the circle are visible (see Figure 1-i). By pressing the play button, the orange point starts circulating along the circle path in positive direction. After two rounds (rotations of the dot around the circle), the circle gets divided into quarters by vertical and horizontal line segments that appear one after the other as the point moves and divide the circle into quarters (See Figure 1-ii). A coordinate system appears in the fourth round, placing the circle's centre in the coordinate system centre and making visible that the circle's radius is equal to one. After that round, a red vertical line segment connecting the moving point to the x-axis appears and after two such rounds (see Figure 1-iii), the red vertical line segment disappears, but instead a blue horizontal line segment connecting the moving point to the y-axis appears for two rounds. In the last two rounds both the red vertical and the blue horizontal line segments are visible as the orange point moves around the circle (see Figure 1-iv). In the extended definition of trigonometric functions, cosine and sine are defined respectively as the x- and y-coordinate of a point on the unit circle.

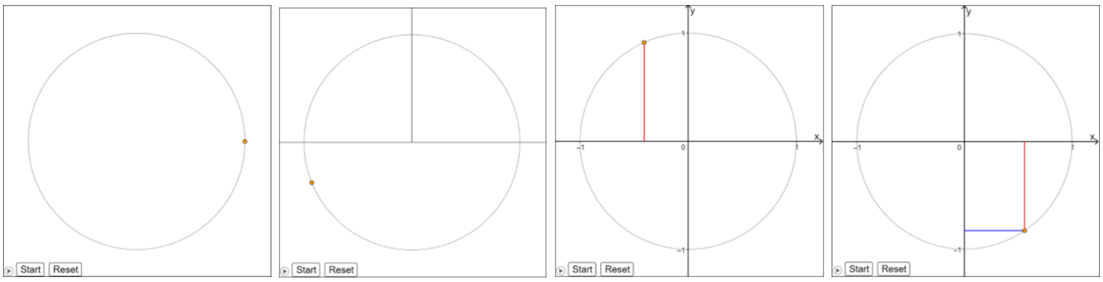


Figure 1. Stills from the silent video that was used in the study. From left to right: i) a circle and an orange point on the circle, ii) line segments that divide the circle into quarters start to appear, iii) now it is visible that the circle is the unit circle and a red vertical line segment connects the point to the x-axis, iv) a red vertical and a blue horizontal line segment connects the point to the axes of the coordinate system. View the silent video: https://ggbm.at/BfRqGSKq

### *Students' Prior Knowledge and Instructions to Teachers*

The unit circle and the extended definition of the trigonometric functions is new to students at upper secondary school level and usually taught in the second or third mathematics course that is obligatory for students on mathematics-physics, natural-science- or economics-business-oriented lines of studies. Before entering that course, students have encountered and are familiar with the limited definition of trigonometric functions in right-angled triangles. Students participated in a one to two-week learning sequence on the topic of the unit circle before participation. The learning sequence was slightly different from school to school, but all had in common presenting a definition of the unit circle, the trigonometric functions, the Pythagorean trigonometric identity, and techniques to solve first and second order trigonometric equations.

As a summary of a previously taught topic, after a learning sequence on the topic of the unit circle and the extended definition of the trigonometric functions, teachers were asked to show the silent video to the whole class, then divide their students into pairs and give them the video link to watch the video as often as they wanted whilst working on their voice-over. In their voice-overs, teachers asked students to describe what they saw in the video and explain the mathematics as if they were narrators for educational material. Students' were given the choice of handing in an informal warm-up description along with their mathematical one, but most students only handed in their mathematical narration. Teachers were asked to refrain from handing out any list of words or concepts for students to address in their voice-overs but a suggestion for students to draft a script as a warm-up exercise could be made if they found it hard to start recording. The learning goal was to gain awareness of students' own discourse connected to the mathematical concepts in the video and to consolidate students' understanding of the concepts.

### *Data Collection*

To answer the research questions, I prepared questions for semi-structured teacher interviews, giving teachers an opportunity to engage in a professional discourse. By professional discourse I mean opportunities to talk about learning, students, subject matter and teaching (Wilson and Berne, 1999). The interviews focused on teachers' expectations and current experience with using a silent video task in their classrooms, and what they noticed when listening to student responses to the task. These interviews were conducted and recorded before and after the assignment of the silent video task, and after the follow-up lesson (see Figure 2). During the second interview, while preparing for the follow-up lesson, each teacher was invited to think aloud, and the goal was not to mark the student responses, but rather to prepare a whole-group discussion for the follow-up lesson with feedback for the students. The unit-circle-themed silent video was pilot tested with some students and interview questions were also piloted with an upper secondary school mathematics teacher. The teacher interviews were to be my main data source, however I also collected students' task responses and prepared two short questionnaires for students regarding their experiences of a) recording a voice-over to the silent video, and b) taking part in the whole-class discussion. Each questionnaire included an open comment field and five short questions to be answered on a Likert-scale. Students answered the questionnaires with response rate 86% (before the follow-up) and 70% (after the follow-up), and their results were shortly discussed in the second and third teacher interviews.

The reason for including a students' questionnaire was that teachers in Iceland do not typically design lessons in ways that give them insight into student thinking or understanding (Sigurgeirsson et al., 2018) and when it comes to evaluating new teaching methods, teachers often take students' behaviour as evidence of effectiveness (van Es, 2011). In this way student perspectives could be considered.

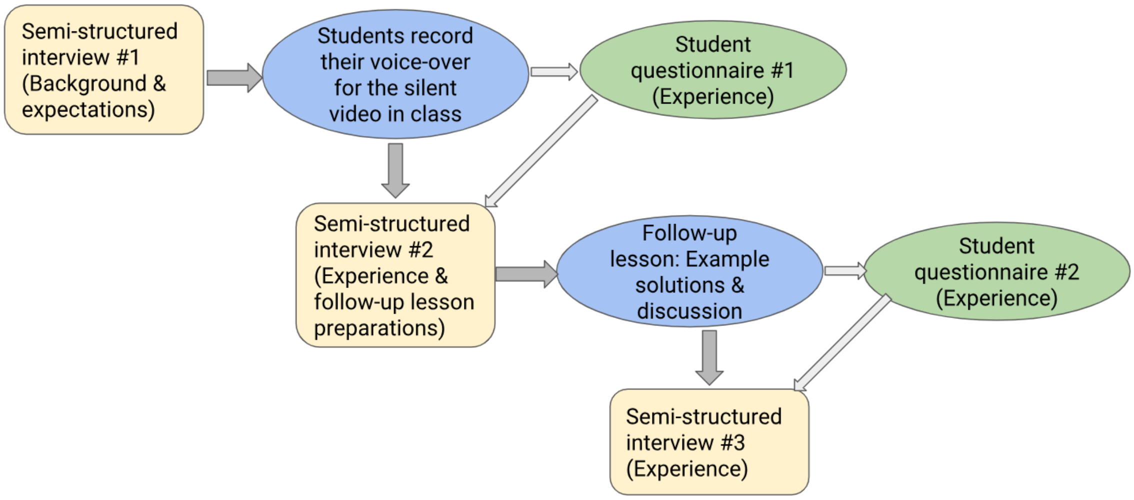


Figure 2. This flow chart shows the workflow utilized with each of the four participating teachers in the first cycle of the design-based research project. Each teacher was interviewed before and after implementing the silent video task in class, and after the follow-up lesson. The topic of each semi-structured interview is shown in parenthesis. Although not the focus of the study, the short online questionnaires for students are also shown.

### *Different Roles and Voices in the Research Process*

During the interviews, I focused on the teachers' voice and I avoided bringing my own opinions into the discourse. It was made clear both in the consent letter and orally at the beginning of our collaboration that I in no way intended to test the teachers, that I did not have a best solution yet for the process of assigning a silent video task, and that their work would contribute to reaching the goal of describing this assignment process. I took on the role of a teacher when explaining the silent video task in the first interview, but the role of a learner when learning about the interviewees' experiences in the second and third interview.

During the data analysis, I kept in mind the voices to which the teacher responded (my own voice, other teachers' voices, parents' voices, students' voices…). Since I belong to the small mathematics teacher community in Iceland, I was fully aware of the fact that I am an insider, a participant in the conversation and even that my interviewees might–biased by social desirability–try to 'please' me with some answers.

Knowing roughly beforehand what kind of culture was dominant within each school that I visited made me simultaneously an insider and an outsider. I strived to set aside any pre-assumptions and approached the teachers with ears open to hear their story. In all cases, I experienced surprises and learned about the culture within each school.

## Data Analysis

All teacher interviews and students' responses to the silent video task were transcribed verbatim. In the case of special emphasis on words in speech, they were underlined in the transcript. I used thematic analysis (Braun and Clarke, 2006), using an inductive approach to code and develop themes from the data content. Examples for semantic codes (close to interviewee's language) included: "Teacher notices that sometimes only one student is actively participating", "Teacher would have liked students to use time better" and "There is too little time to cover the course curriculum". Examples for latent codes (informed by underlying concepts) included: "Teacher assumes that he/she knows what his/her students know", "Teacher gives students autonomy", and "Teacher does not expect students to think". Candidate themes were tested to see if they were present across the dataset. Data analysis was discussed with critical friends in a doctoral seminar throughout the process.

## Findings and Discussion

In order to answer the overarching question on how teachers' knowledge influences their engagement in innovative practices intended to support students' understanding of mathematical concepts, the findings will be presented and discussed according to the research questions.

***RQ1: What do teachers notice when listening to student responses to silent video tasks?***

Different responses to an open task like the silent video task are acceptable. Moreover, they create opportunities for perceiving different ways of thinking and make base for discussion. Considering the diversity of responses possible, it is interesting that teachers seemed to have a rather clear–even somewhat narrow–idea of how an ideal response might sound and furthermore teachers seemed to expect their students would readily know and display what teachers expected of them. For instance, even if nothing in the task mentions trigonometric functions, yet the teachers were surprised that students did not mention them.

The semantic theme *Teachers notice what is missing or unclear* captured that teachers identified specific words, definitions, connections and/or explanations that they had expected in students' responses and often got disappointed–either by the absence of these specific words or phrases or by responses not being explicit or clear enough for others to understand. Often the missing words were something that the teacher had "told over and over again" in class or something teachers remembered students having been able to express recently (within the preceding two weeks) when solving written assignments (mainly trigonometric equations).

Gauti: I would have thought that they would say more.

Gauti: … it is quite interesting that the cosine <u>never</u> gets larger than one or smaller than minus one and why does <u>nobody</u> say that…

Gauti: It is as if always the one who does not know what it is mixes up what is cosine and what is sine. They always precisely interchange it.

Lilja: …was somehow missing to describe cosine and sine of what. It was a bit messed up by them you see

Magni: … instead of describing the mathematical concepts they are describing what is happening something like "then the point goes two rounds and then this happens"

Snorri: They do not say anything about sine and cosine […] they do not mention the trigonometric functions.

Teachers were focused on collaboration regarding development of the task and neither were used to working with such tasks nor paying attention to students' thinking in this way in general. Their classroom environment normally included them giving instructions, students individually doing tasks following instructions, and teachers checking assigned homework or tests for errors and missing things. It is therefore not surprising that teachers continued to look for errors and missing things regardless of the new circumstances. In other words, they mostly gave descriptive and evaluative comments. This is in line with research on teacher noticing: noticing is something to be learnt and practiced in order to bring it to higher levels (van Es, 2011; Mason, 2002; Jacobs, Lamb, & Philipp, 2010; Crespo, 2002).

Despite knowing that students had limited experience with expressing themselves orally about mathematical concepts, teachers expected their students to "make good use of the time", i.e. fill nearly non-stop the two-minutes time given for their voice-over. It contradicted the fact that in the interviews, teachers reported that students were not used to talk mathematics. Only Lilja described in the interview that her students had "little, but still some" experience of taking part in mathematical group discussions. Teachers expressed their own ideas regarding what could have been said during the silent seconds and what would have needed a further explanation. For example, teachers would have liked their students to use certain words like projection or angle, or they would have liked them to add some explicit information about the properties of the sine and cosine. Gauti identified students mixing cosine and sine as a significant event and made a general statement that those who mix these two concepts are always “the one who does not know it”, which might be interpreted as the one who does not understand.

In student responses covering many of the concepts that teachers would have expected to hear, the teachers noticed that some connections or distinctions between the different concepts were missing. It is well-known in the literature that teachers, irrespective of their teaching experience, turn to a baseline level of noticing if they have not practiced and/or been trained in seeing beyond student mistakes when evaluating open tasks. They have a broad vision and form general impressions, providing descriptive and evaluative comments, but do not necessarily pay attention to individual students’ mathematical thinking, add some interpretative comments, highlight noteworthy events, or connect particular students’ mathematical thinking to teaching strategies or broader principles of teaching and learning (van Es, 2011). When connections appeared in the responses, they were seldom mentioned; the following excerpt where Magni adds an interpretative comment (level 2) being the only exception.

> Magni: The final point made by these students that as one (line segment) is getting bigger the other one is getting smaller that shows that they were looking at some… some connection there they tried to draw some conclusions.

Otherwise, Magni had expected students to think that the video “looks somewhat similar to what the teacher showed us, let’s look it up in the book” and was pleasantly surprised that only two pairs of students seemed to have done so. The fact that the silent video was an open task with no direct questions was pointed at by teachers as a possible factor causing students to forget to mention what the teachers considered important aspects.

When misunderstandings occurred in student responses, teachers in some cases reacted by pointing out previous experience of working with students, claiming that if students had been asked directly about the mathematical concepts involved, they would have answered the direct questions correctly. The latent theme *Teachers have the best intentions for their students* captured when teachers expected students to “know better” than their solutions to the silent video task showed, often drawing upon knowledge developed from working with students on other (procedural) tasks in class. In connection to disappointing student responses, teachers either blamed their own instructions or students lack of experience with talking mathematics.

Lilja: There was something that got messed up… When they were talking, I think they were thinking it correctly just they are not used to talking

Gauti: They always do it correctly on exams and then they know it

Gauti: If one would ask them questions like “What is this point on the circle called?” then they would say “It is the terminal point” like we taught it […] one knows what they know.

Gauti: …even though they know it, then they cannot explain it, that is how it is.

Magni: Could say that they did not completely realize that they are supposed to explain… It is maybe something that I should have been more explicit about you see to tell them that they should…

Magni: It is imprecise what they say it is like not completely mathematically correct what they are saying because [I think] they surely were looking at the x-axis, but they do not say it with the right words.

Snorri: …you see we have directly problems where they should not use the calculators and completed them you see and found sine and cosine of angles you see and one also did minus angles and and like this they mastered it like without a problem

Students not being used to talk mathematics was explicitly referred to by Gauti, Lilja and Snorri as a reason for why some students interchanged the x- and y-axis and/or cosine and sine. They expected their students to “mean it right” or “think it right” even though the pattern might have been interpreted as pointing at students’ lack of understanding. Gauti even went as far to say that “they always do it correctly on exams and then they know it”. In a way, it might be interpreted as an initial reaction of denial–

since instead of connecting it to possible lack of understanding, the teacher defended students referring to previous (procedural) work where students produced correct solutions. However, teachers might also be right, and it might just have been students slip of the tongue.

Gauti's words about "knowing what students know" can be interpreted in such a way that he expects his students to learn what he says one-to-one. However, he continues this discussion later in the interview underlining that he is aware of the fact that this is indeed not the case with student learning and explains that "One always thinks that more goes in than it does in fact and one somehow always thinks that 'Now they surely have got it' which most often is not the case". In doing so, Gauti makes a connection between specific events and broader principles of teaching and learning (van Es, 2011) because he relates the event of students surprisingly not knowing what a teacher expects them to know with positivist learning theories. Still, he does not transfer his knowledge to the situation. He knows that students do not "grasp things one-to-one" in mathematics class, still often expected this to be the case. It remains unclear if he, by putting this into words in the interview, got aware of the inconsistency, but it is important for me as a researcher to observe it and make others (who might have similar thoughts) aware of it. In this sense I agree with Mason (2017) that what matters to me is that people become aware of something previously overlooked or downplayed.

### *RQ2: What do teachers notice about their own abilities to use innovative practices for supporting student understanding?*

Teachers discussed a number of limiting factors and insecurities that constrained their ability to innovate. For example, teachers noticed that their expectations of students' abilities to use technology often underestimated students' actual abilities. Comparing interviews before and after the follow-up lesson, a dissonance was identified between plans and actual implementation of the innovative teaching practice. Teachers admitted stress and/or anxiety towards the possibility of losing control of the learning process, especially when leading a group discussion.

The semantic theme *Discussing student solutions with the whole group is a delicate matter* captured teachers' stress and anxiety when preparing for and/or reflecting on experience of discussing student solutions with the whole group.

> Gauti: But to get some discussion going, we do not do that much and then you often feel like you are a bit clumsy or childish at doing it, not knowing how the kids will take it because they are usually rather passive.
>
> Gauti: It is interesting to see how they cannot really say what they mean. It is interesting but they indeed are not used to talking, there is never anyone saying anything in my classes, or very little.
>
> Lilja: …I was not saying what "how was your experience" and naming any names and pointing at anyone I did not do anything like that […] I just allowed them to express themselves as they wanted you see.
>
> Lilja: I think that the teacher would profit from recording herself beforehand and check [...] then she puts herself in the kids' place [...] I tried it myself and I found it too short [...] it is good to understand this stress factor before assigning it
>
> Lilja: ...and they found it unbelievable that I interfered… that I did not approve of when they said "This is a unit circle" at the start before it was visible that it was a unit circle because they have already watched it all and then of course they see that it becomes a unit circle… and they found it strange that I should mention this and got quite heated about it […] "Why do you say that we cannot say it at the start?"
>
> Magni: […] but it is harder for me as a teacher to control that they do not start to teach each other some nonsense.
>
> Snorri: No, I did not show it… And I was probably influenced by some parties that are so stressed about covering everything (in the syllabus).

Orchestrating a whole group discussion in mathematics class using some student responses to view and discuss is a complex task that requires some training (Stein and

Smith, 2011) and no such training was provided in the intervention. Also, the procedural tasks normally used in the Icelandic classrooms do not evoke any need for discussion. According to Gauti and Snorri, their students' discussions while solving the silent video task might have been their first experience of discussing mathematics in the upper-secondary school mathematics classroom. It is therefore not surprising that teachers described how hard they expected group discussion in the follow-up lesson to be. What they feared would happen included their students possibly being exposed to wrong responses by their peers. Despite best intention, Gauti and Snorri described delivering a monologue, maybe asking some questions but receiving few or no answers and in fact refraining from leading group discussions at the last minute. Magni selected one response to show at the start of his follow-up lesson and prepared a short slide show where he described different levels of conceptual understanding, listed concepts he liked to hear in the responses, some issues regarding pacing and the use of time, and potential things to improve regarding preciseness in word use and definitions. The slide show did not spark much discussion, but he rather wanted to be in control and prevent students from being exposed to flawed responses.

Lilja was the only teacher who facilitated a group discussion in which more than two students took part and where students debated among themselves. She started by showing examples from two volunteering groups. Both responses had some errors in them, and she discussed with her students that every response had something that could be done better, tiny errors or imprecise word use that could be learnt from. When she mentioned that in some of her students' responses no clear distinction had been made between the concepts of a circle and the unit circle, her students entered a heated debate with her regarding at which point in time it was acceptable to say that the circle was a unit circle. Lilja managed to move towards making the unpacking of mathematics become a shared responsibility–something that Scheiner, Montes, Godino, Carrillo, and Pino-Fan (2019) have described as an anthropological-sociocultural approach and a part of what makes knowledge for teaching mathematics specialized. For Lilja, this was a starting point for further discussion about precision in mathematical definitions. Her students, for example, discussed whether to focus on the length of the line segment or to rather refer to the projected point on the corresponding axis. That discussion offered an opportunity to discuss the value of different approaches, and address some difficulties, bearing in mind that a length is always positive.

In Lilja's opinion, her own recording of a response to the silent video task composed an important step in her preparation for the task implementation, because it made her realize the complexity and cognitive load of the task. Gauti and Snorri tried recording a response to the task after assigning the task. Snorri said that this made him realize that there was "maybe more to the task than he had thought at first". In the follow-up lesson, all teachers felt the need to tell how they would have solved the exercise, i.e. to impose their own formalized version of a voice-over on the students. It is easy to get tempted to do so in an environment where mathematics is normally taught by imposition (Cobb, 1988) and where there almost always is some kind of a role-model or best solution existing.

Being confronted with an open task for the first time can make students insecure and they even might wish to hear their teachers' response, because they are used to it. Like Cobb (2000) points out, students are influenced by the classroom microculture that they participate in. It influences what it means to them to know and do mathematics, and it is hard to suddenly change their working culture. Lilja's students were especially upset about the random assignment into pairs, the short oral instructions, and the transfer of responsibility to solve possible technological issues to the students themselves, i.e. the enhancement of student autonomy. Lilja explained that after a short discussion, her students agreed that the random assignment into groups was fair, and Lilja would not have liked to rob students of the productive struggle and joy they experienced getting past some technological obstacles. She was the only teacher who received very creative solutions to the task, describing the moving point as an orange robot sending out its laser beams to the axes of the coordinate system. To some students, the open task might have felt liberating–just knowing that there is no single correct response lightens the pressure of making no mistakes. The intention with allowing students to hand in one informal warm-up response and one mathematical response was also to lower the performance pressure. In the few cases where students handed in two different responses to the task, teachers expected the mathematical response to be much better, but often it showed to be quite similar. Magni described it as "the content stays the same but the way they use the words resembles more the formal text in their textbook". This also relates to the culture of the mathematics classroom (Cobb, 2000) and the students' lack of experience with talking mathematics.

For the teachers, the intended discussion in the follow-up lesson and especially the plan to show selected students' responses, created tension. Gauti was concerned that his students were at a fragile age, and he was relieved when students reacted with a loud "No!" to his question whether he should play an example student response. He decided, instead of showing student responses, to only refer to them, point out what could be improved or made clearer, and to tell his students that they generally cannot expect the viewer to know what is going on in the video and thus must explain it precisely. Snorri was unsure of which responses to choose and not convinced anymore that showing a response would be key to get the discussion going. After the follow-up lesson he also explained that he and his students felt some time pressure. This is interesting because to a certain degree, Icelandic mathematics teachers at upper secondary school level create their own obligations described by the school curriculum or these get created by the mathematics head teacher or principal. As stated earlier, the teachers themselves prepare the final exams and these exams are to a high extent not in compliance with the curricular aims for understanding (Jónsdóttir et al., 2014). There is no standardized examination such that syllabus and assessment is each school's decision to make within the frame that the curriculum gives. Still, the teachers main reason given for not expecting to use the silent video task again, was that it did not directly prepare their students for the final exams.

Even though I had explained the intentions and purposes of the task (facilitate discussion and make discourse visible in order to further develop it), both in talking and in writing, it might have gotten lost. At least there is evidence in the data that the intentions and purposes were not clear to the teacher.  Gauti, Lilja and Snorri all gave short oral instructions when assigning the task and sent a link to the video via email or the school learning management system. Magni, however, presented a handout (~400 words, more than one A4 page) that he had prepared explaining step-by-step what to do, indicating that he had expected students to need more detailed instructions than had been suggested by me. Both Snorri and Magni noted that they would give more detailed instructions if they would assign a silent video task again. As opposed to them, Lilja explained that she would not have liked to "rob her students of the experience" of "finding out of the technological issues themselves" since despite her students' expected struggle, afterwards they had visibly "felt like they won the Olympics" and expressed that they would like to receive more challenges in the course, "not necessarily the same, but similar". Her experience is consistent with results from Liljedahl (2016) that short oral instructions are more effective than written instructions that students need to decode.

### *Development of the instructional sequence*

Even if the research project settings were only composed of a limited number of classroom settings with a particular population, the development of instructional practice and theory were expected to grow together within the research project (Cobb et al., 2003, p. 10). The intention was not merely to investigate the specific settings of the four classrooms, but on developing the silent video tasks and their pedagogy in such a way that as many teachers as possible could use it in the way that it was aimed to be used, i.e. "not just to meet local needs, but to advance theoretical agenda, to uncover, explore and confirm theoretical relationships" (Barab & Squire, 2004, p. 5).

In accordance with one of the research project aims, the silent video task assignment process was developed during the data collection period and continued to develop in interaction with the data with the goal to support teaching and learning, i.e. following the direction of design research where the focus is set on developing concrete tools that should be sharable and usable by different teachers (Lesh & Sriraman, 2010). Also, a more concrete description of silent videos and their characteristics was developed.

### *Limitations*

In this exploratory cycle of the design-based research project, four teachers in four different schools implemented one specific silent video task once in their classrooms. Analysis is only based on interviews with teachers and no classroom observations were collected. Teachers were equally inexperienced in describing what they noticed as students were doing their voice-over recordings. Even if the findings from this data might form a piece in the puzzle, more insights into teacher noticing when using these and other innovative tasks are needed to gain a clearer picture of how teachers' knowledge influences their engagement in innovative practices. In the final cycle of the research project, an intervention with teachers who have experience with discussions and formative assessment in their mathematics classrooms is planned. Data gathered in that cycle will include teacher interviews, student responses to three different silent video tasks implemented by their teacher in the course of one semester and classroom observations. Changes were made to the intervention to revise the manner in which teachers were asked to implement the innovation. From this data, theories about factors influencing the way teachers implement an innovative task and factors influencing the ways students act upon the innovative task as it is presented by the teacher will be formed.

## Conclusions

In our analysis we considered teachers' use of an innovative task intended to shift their attention toward students' thinking. On the basis of what was noticed about students' responses to the task, teachers made decisions regarding what to address in a follow-up lesson. To investigate how teachers' knowledge influenced their engagement in the innovative practices, we analysed four teachers' reported experiences. The teachers ranged from having short to long experience of teaching. Irrespective of their teaching experience, the teachers were not used to focus on and struggled to see students' thinking when interpreting students' task responses. Thus, the task brought teachers to novice level in terms of the task not following their normal line of practice and challenging their views on teaching and learning mathematics. I want to emphasise that by teachers in the role of novices I do not mean that they were somehow on lower level than they should have been. It is important that we as researchers are aware of the fact that new ways of working partly push people back to the start-line and I say partly because teachers keep being influenced by their knowledge and previous teaching experiences.

We focused on what teachers noticed i) while listening to students' responses to silent video tasks, and ii) about their own abilities to use innovative practices for supporting student understanding. Teachers had hoped certain learning would occur and be displayed in student responses. They were disappointed that the result of engaging learners in the silent video task did not deliver the responses they had hoped for. Knowledge is not simply transmitted to learners (acquisition). Even if students did not show what teachers expected, it does not mean that the learners were not paying attention, or that something was wrong with them. It only implies that the learners at the point of recording their voice-overs put emphasis on something else than what the teacher expected. Learners communicated in the activity (participation), even though they had little or no experience with discussions in mathematics class. Their responses reflect what they took part in. It was for most of them the first time that they had the opportunity to communicate in this way and they strived to use the language of the community, shaped by the classroom norms of what counts as being "mathematical".

In the research process, teachers were asked to work in a socio-constructivist way. They had to make a short transition from transmission-oriented teachers to a new role as organizers or facilitators of learning; encouraging students' discourse about mathematics and learning how to utilize student-to-student explanations in a whole group discussion. Teachers previously had little or no experience with encouraging students' discussions about mathematics. Preparing for examination and procedural work were prominent features of the classroom environment, influenced by examination dominated school curriculum. Interestingly, the schools' curricula were built by the schools themselves and the generic curriculum in Iceland does not force schools to set themselves tight schedules with long lists of things to cover. However, because of this emphasis, teachers' focus was prominently directed towards deficits, ambiguity, and missing things in students' responses to the silent video task. In other words, teachers noticed what they previously trained to notice. Their noticing was at the baseline level, but it might be developed in further practice (van Es, 2011; Crespo, 2000) by–for instance–attending to what there is rather than what they wish would be there in the student responses.

What makes silent video tasks valuable for teachers is first and foremost that group discussion on basis of student responses to silent video tasks can be used to identify and clarify some forms of misinterpretation, to consolidate the mathematics presented in the video and push students further towards communicating in a way precise enough for themselves and others to understand what they mean. An example for this in the data was seen with Lilja, whose students entered an enlightening debate about mathematical definitions. Nevertheless, the group discussion in the follow-up lesson created teacher tension and was considered by teachers to be the greatest challenge of implementing the task. Teachers might be better prepared for this tension, if they had access to professional development modules on the promotion of conversation in the mathematics classroom and the utilization of student mistakes to promote learning. Still, requiring teachers who already complained about time pressure at work to participate in professional development modules might have prevented their participation in the research project. Despite the challenge faced in the follow-up lesson, all teachers noted that the tasks were easily implementable. They were surprised by their students' ability to tackle new technology, but they also noticed that students needed more practice in talking about mathematics.

The analysis presented in the preceding section identified four themes answering the research questions. They were not meant to show a comprehensive list of everything noteworthy. Rather, it was intended to present some important knowledge aspects that influence how teachers engage in innovative practices and make sense of student responses (and student thinking). These results also have implications for teacher education and professional development. What the teachers noticed represents themes that may be important to target during professional development focused on teaching for understanding, supporting the development of students' discourse, and utilizing student responses in whole group discussion. Silent video task practices and theories connected thereto are currently being further developed with an upper secondary school teacher in the last cycle of the design-based research project.

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