Compact 4 × 4 butler matrix with non-standard phase differences for IoT applications

Adrian Bekasiewicz[1,✉] and Slawomir Koziel[1,2]

[1]Faculty of Electronics, Telecommunications and Informatics, Gdansk University of Technology, Gdansk, Poland

[2]Department of Engineering, Reykjavik University, Reykjavik, Iceland

✉Email: bekasiewicz@ru.is

Butler matrices represent a popular class of feeding networks for antenna arrays. Large dimensions and the lack of flexibility in terms of achievable output phase difference make conventional Butler structures of limited use for modern communication devices. In this work, a compact planar 4 × 4 matrix with non-standard relative phase shifts of –30, 150, –120, and 60° has been proposed. The structure is designed to operate at the centre frequency of 2.45 GHz. Small dimensions of 31.3 × 22.9 mm make it useful for Internet of Things applications. The structure operates from 2.35 to 2.55 GHz, which covers the industrial, scientific and medical bandwidth. At the centre frequency, the measured amplitude and phase imbalance are 1.65 dB and ±4.3°, respectively. The proposed circuit has been compared to the state-of-the-art structures from the literature.

Introduction: Feeding networks are the key components of antenna arrays. A type of thereof, Butler matrices (BMs), are characterised by the ability to provide different phase shifts between the outputs, depending on the excited input port. A conventional planar BM is composed of branch line couplers (BLCs), phase shifters, and crossovers [1]. In a 4 × 4 configuration, it can produce progressive phase differences of ±45 or ±135° at the outputs. Due to lack of phase-related flexibility as well as large dimensions applicability of conventional BMs to modern communication devices, including Internet of Things (IoT) systems operating within the industrial, scientific and medical band is limited.

The design of miniaturised BMs has recently gained the attention of the research community [2–7]. Popular size reduction techniques include replacement of the BM building blocks by their compact counterparts [3, 4], implementation of the structures on multi-layer substrates [5], or development of BMs without some of the components [6, 7]. Application of the mentioned techniques allows for obtaining between 60% to 85% miniaturisation. Despite a significant size reduction, compared to conventional structures [4, 6, 7], large physical dimensions of these BMs are still a limiting factor for their application in small-size devices.

BMs characterised by non-standard relative phase differences between the output ports are normally constructed using components that support arbitrary phase shifts [8, 9]. In [8], the control over the output phases is maintained using phase shifters with unequal electrical lengths. Another method, where unconventional BLCs are used to control the Butler matrix phase differences, is discussed in [9]. Both approaches proved to be useful for increasing the BM functionality over the conventional structures.

In this work, a compact planar 4 × 4 Butler matrix with non-standard relative phase differences between output ports of –30, 150, –120, and 60° is proposed. The structure comprises two pairs of hybrid BLCs with phase shifts of 75 and 60°, respectively. Dimensions and footprint of the optimised BM are 31.3 × 22.9 mm and 717 mm^2, respectively. The size reduction has been achieved using a combination of folded BLC structures and compact crossovers involving the microstrip-to-coplanar-waveguide transition. The centre frequency of the proposed circuit is 2.45 GHz, and it operates within 2.35 to 2.55 GHz bandwidth. The magnitude and phase-difference imbalances of the BM at the centre frequency are 1.65 dB and ±4.3°, respectively. Small dimensions and high performance make the proposed structure suitable for applications in small-form-factor IoT devices. The presented BM has been benchmarked against state-of-the-art circuits from the literature and validated experimentally.

Design concept: A conceptual illustration of the proposed Butler matrix is shown in Figure 1. The structure is constituted by two pairs of BLCs

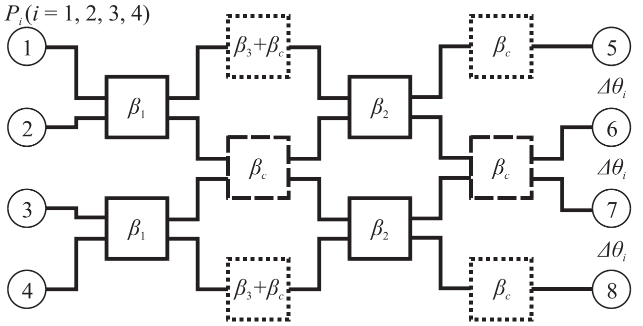

Fig 1 *Butler matrix–conceptual illustration. Solid, dashed, and dotted boxes represent branch line couplers (BLCs), crossovers, and phase shifters, respectively*

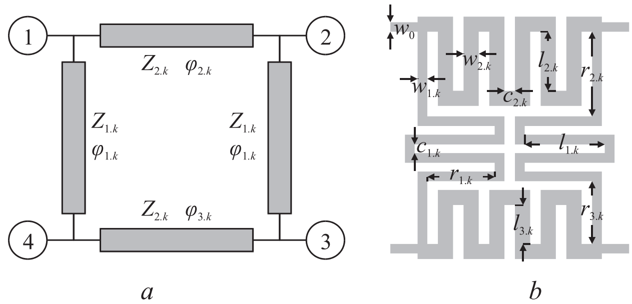

Fig 2 *BLC with an arbitrary phase difference. (a) Conceptual illustration of the structure, (b) geometry of the miniaturised circuit*

with phase differences of β_1 and β_2, respectively, two phase shifters β_3, and two crossovers introducing the phase shift of β_c.

The presented BM can provide relative phase differences of $\Delta\theta_i$ (i = 1, …, 4, corresponds to the port P_i used for structure excitation) between the outputs $P_{5\text{–}8}$ that exceed the capabilities of conventional structures. The electrical properties of the BM components (see Figure 1) required to obtain the desired $\Delta\theta_i$ can be determined from [9]

$$\beta_1 = 0.5\beta_2 - 0.25\pi \tag{1}$$

$$\beta_2 = 2\Delta\theta_1 \tag{2}$$

$$\beta_3 = -0.25\pi \tag{3}$$

Note that $\Delta\theta_2 = \Delta\theta_1 + \pi$, $\Delta\theta_3 = 0.5\beta_2 - 0.5\pi$, and $\Delta\theta_4 = \Delta\theta_3 + \pi$ [9]. Consequently, $\Delta\theta_{2\text{–}4}$ depends on $\Delta\theta_1$ from Equations (1)–(3). For demonstration, the phase differences of the BM considered here are set to $\Delta\theta_{1\text{–}4}$ = {–30, 150, –120, 60°} that are obtained using β_1 = –75, β_2 = –60, and β_3 = –45, respectively.

A conceptual illustration of the BLC structure that can realise phase differences β_1 and β_2 is shown in Figure 2(a). The kth coupler (k = 1, 2) consists of two equal-length sections with the normalised characteristic impedance of $z_{1.k}$, and the electrical length of $\varphi_{1.k}$ = –90°, as well as two sections with impedance $z_{2.k}$ and the lengths of $\varphi_{2.k}$, and $\varphi_{3.k}$, respectively. The electrical parameters of the BLCs can be found by solving [10, 11]:

$$z_{1.k} = \sqrt{\frac{z_{2.k}^2}{1 - z_{2.k}^2}} \tag{4}$$

$$z_{2.k} = \frac{\tan(\beta_k)\left(\tan^2(0.5\varphi_{2.k}) - 1\right)}{2\tan(0.5\varphi_{2.k})} \tag{5}$$

$$\varphi_{3.k} = 2\tan^{-1}\left(\sqrt{\frac{1}{\tan(0.5\varphi_{2.k})}}\right) \tag{6}$$

The values determined from Equations (4)–(6) required to obtain the selected $\Delta\theta_{1\text{–}4}$ are $Z_{1.1}$ = 44.65 Ω, $Z_{2.1}$ = 32.16 Ω, $\varphi_{2.1}$ = 100.56°,

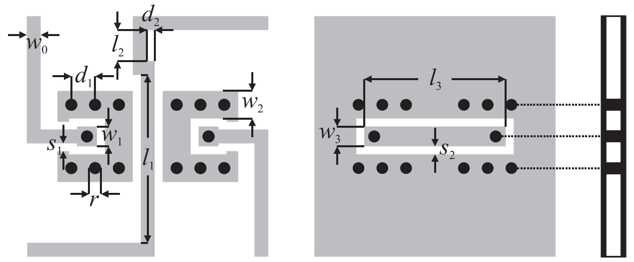

Fig 3 *Compact crossover with the microstrip-to-coplanar-waveguide transition. From left: Top, bottom, and cross-section view. Black circles represent vias*

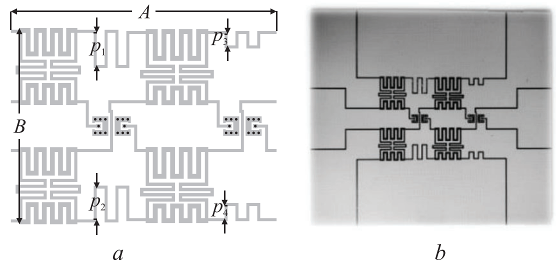

Fig 4 *Compact Butler matrix with non-standard phase differences. (a) Optimised design with a highlight on phase shifters dimensions, (b) photograph of the manufactured structure prototype*

$\varphi_{3.1}$ = 79.44°, $Z_{1.2}$ = 42.59 Ω, $z_{2.2}$ = 32.2 Ω, $\varphi_{2.2}$ = 110.71°, and $\varphi_{3.2}$ = 69.30° (note that $Z_{1\text{-}2.k} = Z_0 \cdot z_{1\text{-}2.k}$). The calculated electrical parameters are used as a starting point for the design of the BM components.

Butler matrix components: The compact BLC structure incorporated into the proposed Butler matrix is shown in Figure 2(b). The circuit and the entire BM are implemented on a dielectric substrate with $\varepsilon_r = 3.74$, $h = 0.168$ mm, and $\tan\delta = 0.0037$. The couplers are miniaturised by means of folding the conventional transmission line sections. The vector of geometry parameters that represent the kth BLC is $\boldsymbol{x}_k = [w_{1.k}\ w_{2.k}\ c_{1.k}\ c_{2.k}\ l_{1.k}\ l_{2.k}\ l_{3.k}]^T$. The relative variables are $r_{1.k} = 3w_{2.k} + 3c_{2.k} - w_{1.k}$, $r_{2.k} = l_{2.k} + w_{2.k} + c_{2.k}$, and $r_{3.k} = l_{3.k} + w_{2.k} + c_{1.k}$, whereas $w_0 = 0.35$ to ensure 50 Ω input impedance (cf. Figure 1). The unit for all design parameters is mm. The dimensions of BLC_1 and BLC_2 have been obtained through local numerical optimisation (gradient algorithm) oriented towards maintaining the reflection and isolation of each coupler below –20 dB within the 2.35 to 2.55 GHz range. Other requirements involved minimisation of the power split imbalance and maintaining the desired phase differences β_1, β_2 at the centre frequency of $f_0 = 2.45$ GHz [2, 11]. The starting points for BLCs design $\boldsymbol{x}_1^{(0)}$ = [0.34 0.55 0.3 0.3 3 2.2 1.6]^T and $\boldsymbol{x}_2^{(0)}$ = [0.46 0.6 0.3 0.3 2 2.3 1.3]^T are determined through recalculation of the electrical parameters (see the previous section) to the microstrip line dimensions. The optimised designs are $\boldsymbol{x}_1^*$ = [0.36 0.56 0.32 0.39 2.94 2.18 1.44]^T and $\boldsymbol{x}_2^*$ = [0.38 0.56 0.2 0.4 3.23 2.58 1.13]^T. The optimised BLC_1 and BLC_2 are only 8.55 × 6.77 mm = 57.85 mm^2 and 8.23 × 6.9 mm = 56.8 mm^2, respectively. The obtained circuits exhibit 80% and 79.5% miniaturisation with respect to their conventional counterparts.

Small dimensions of crossovers are ensured by replacing the cascade-based circuits with a simple microstrip-to-coplanar-waveguide transition [12]. The geometry of the used circuit is shown in Figure 3. Its design parameters $\boldsymbol{x}_c = [w_1\ w_2\ w_3\ l_1\ l_2\ l_3\ s_1\ s_2\ d_1\ d_2\ r]^T$ have been adjusted to maximise isolation between the intersected transmission lines as well as to maintain their equal electrical length. The tuned design $\boldsymbol{x}_c^*$ = [0.5 0.7 0.5 4.22 0.78 3.55 0.2 0.2 0.6 0.2 0.3]^T provides in-band reflection and isolation both below –30 dB, as well as equal phase shift of β_c = 61.4° (cf. Figure 1). The structure size is 6.05 × 6.05 mm = 36.6 mm^2.

The proposed BM incorporates the phase shifters in the form of simple, folded transmission lines. The lengths of the meandered lines can be adjusted individually in order to provide enhanced control over the relative phase differences between P_5 to P_8 ports. The vector of phase shifters lengths is $\boldsymbol{x}_p = [p_1\ p_2\ p_3\ p_4]^T$ (cf. Figure 4(a)). Its initial values $\boldsymbol{x}_p^{(0)}$ = [4.0 4.0 1.8 1.8]^T are calculated from β_3 and β_c.

Numerical results and measurements: The geometry of the proposed Butler matrix is shown in Figure 4(a). The vector structure adjustable variables is $\boldsymbol{x}_b = [\boldsymbol{x}_1\ \boldsymbol{x}_2\ p_1\ p_2\ p_3\ p_4]^T$. The initial design is $\boldsymbol{x}_b^{(0)}$ = [0.36 0.56 0.32 0.39 2.94 2.18 1.44 0.38 0.56 0.2 0.4 3.23 2.58 1.13 4.0 4.0 1.8 1.8]^T. The final design $\boldsymbol{x}_b^*$ = [0.36 0.56 0.32 0.39 2.94 2.19 1.45 0.39 0.58 0.22 0.45 3.34 2.56 1.13 3.76 3.6 1.33 1.42]^T is obtained as a result of BM fine-tuning oriented towards reduction of the phase difference and magnitude errors. The size of the optimised structure, expressed as $A \times B$ (see Figure 4), is only 31.3 × 22.9 mm = 716.8 mm^2. The electromagnetic (EM) simulation results show that in the 2.35 to 2.55 GHz band, the BM offers reflection below –13 dB but also the amplitude and phase imbalance below 0.6 dB and slightly above ±5°, whereas at the centre frequency, the figures are below 0.55 dB and ±4.1°, respectively.

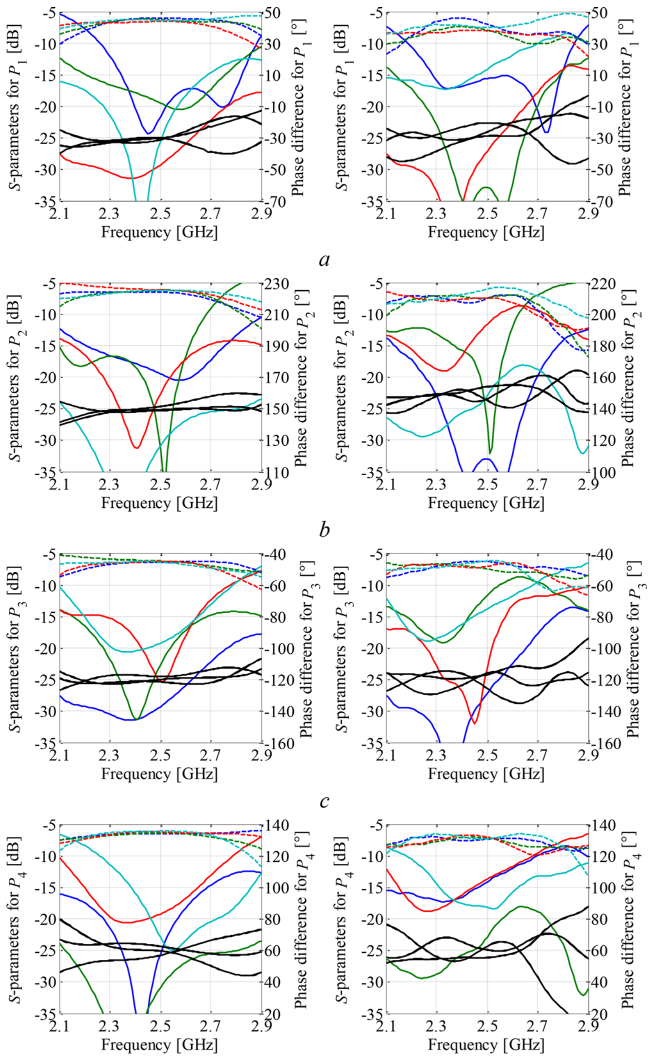

Fig 5 *Butler matrice: Comparison of simulations (left) and measurements (right) with respect to reflection/isolation (solid colour), transmission (dashed colour) and phase-shift (black) responses for the structure excited through (a) Port* P$_1$ *(phase difference at the output ports: –30°), (b) port* P$_2$ *(phase difference at the output ports: 150°), (c) port* P$_3$ *(phase difference at the output ports: –120°), (d) port* P$_4$ *(phase difference at the output ports: 60°)*

The proposed BM has been fabricated (see Figure 4(b)) and measured. A comparison of simulation and measurement results is shown in Figure 5, whereas the performance figures of the structure, that is, in-band reflection (R_{BW}) and isolation (I_{BW}), as well as magnitude (ΔM) and phase-shift (ΔP) imbalance within the frequency band and at f_0 are gathered in Table 1. The overall agreement between the simulation and the measured results is acceptable having in mind that the magnitude is

Table 1. Compact Butler matrice (BM): Comparison of simulations and measurements

	Port P_1		Port P_2		Port P_3		Port P_4	
Excitation	**Sim.**	**Meas.**	**Sim.**	**Meas.**	**Sim.**	**Meas.**	**Sim.**	**Meas.**
R_{BW} [dB]	–15.4	–15.3	–16.9	–14.4	–16.4	–16.7	–13.3	–16.6
I_{BW} [dB]	–17.2	–12.9	–17.2	–10.0	–18.2	–10.0	–18.3	–12.7
ΔM_{BW} [dB]	0.55	2.09	0.60	1.81	0.51	1.62	0.43	0.98
ΔM_{f0} [dB]	0.55	1.65	0.40	1.66	0.23	1.01	0.38	0.79
ΔP_{BW} [°]	±2.1	±7.3	±2.5	±6.6	±2.8	±8.4	±5.3	±7.5
ΔP_{f0} [°]	±1.4	±4.3	±0.9	±4.0	±1.7	±1.0	±4.1	±3.5

Table 2. Proposed BM: Benchmark against the state-of-the-art circuits

	Performance figures				**Size**	
	f_0 (GHz)	BW (%)	ΔM_{f0} (dB)	ΔP_{f0} (°)	Dimensions (mm × mm)	Dimensions ($\lambda_g \times \lambda_g$)
[9]	5.8	7.3	0.45	±6.0	71.4 × 119	1.89 × 3.15
[2]	1.0	N/A	1.20	±1.0	87.8 × 82.4	0.49 × 0.46
[6]	6.0	7.2	0.4	±0.9	58.9 × 57.9	1.96 × 1.93
[4]	1.8	5.5	2.40	±5.9	99.5 × 127	0.82 × 1.04
[5]	28	3.2	4.70	±16	16.8 × 14.9	N/A
This work	2.45	9.4	0.55	±4.1	31.3 × 22.9	0.44 × 0.32

expressed in dB. The discrepancies between the responses mostly result from the fabrication tolerances as well as the errors introduced by the manual circuit assembly and imperfections of the measurement setup.

Comparison with benchmark structures: The proposed circuit has been compared against BMs from the literature in terms of size and performance (EM simulation results). The considered figures include bandwidth (BW) (defined for isolation and reflection both below –15 dB), magnitude imbalance, and phase-shift error (both at the centre frequency) [2, 4–6, 9]. The dimensions of all structures are expressed in the guided wavelength λ_g calculated for a given centre frequency and the electrical parameters of the substrate used for circuit implementation. The results from Table 2 indicate that the proposed BM provides competitive performance and outperforms the benchmark circuits in terms of size.

Conclusion: In this work, a compact 4 × 4 Butler matrix for IoT applications has been presented. The structure provides non-standard relative phase shifts between output ports of –30, 150, –120, 60°, respectively. The circuit's electrical dimensions are $0.44\lambda_g \times 0.32\lambda_g$ mm, with an overall footprint of only 720 mm^2. The small size has been achieved using folded couplers and miniaturised crossovers. Simulation results indicate that the proposed BM offers reflection below –13 dB within the frequency range from 2.35 to 2.55 GHz. Furthermore, at 2.45 GHz measured magnitude and phase errors of the BM are below 1.7 dB and ±4.5°, respectively. The structure outperforms state-of-the-art circuits in terms of size while maintaining competitive performance.

Acknowledgements: This work was supported in part by National Centre for Research and Development Grant NOR/POLNOR/HAPADS/0049/2019-00, by National Science Centre of Poland Grant 2017/27/B/ST7/00563, and by Icelandic Centre for Research (RANNIS) Grant 206606051.

Received: *11 December 2020* Accepted: *25 February 2021*
doi: 10.1049/ell2.12143

References

1 Milligan, T.A.: *Modern Antenna Design*. John Wiley & Sons, New York (2005)
2 Koziel, S., Kurgan, P.: Low-cost optimization of compact branch-line couplers and its application to miniaturized Butler matrix design. In: European Microwave Conference, Rome, pp. 227–230 (2014)
3 Letavin, D.A., Shabunin, S.N.: Construction of a 4x4 Butler microstrip matrix with reduced dimensions. In: Ural Symposium on Biomedica Engineering, Radioelectronics and Information Technology, Yekaterinburg, Russia, pp. 257–260 (2020)
4 Xu, H.-X., Wang, G.-M., Wang, X.: Compact Butler matrix using composite right/left handed transmission line. *Electron. Lett.* **47**(19), 1081–1083 (2011)
5 Kim, S., et al.: A miniaturized Butler matrix based switched beamforming antenna system in a two-layer hybrid stackup substrate for 5G applications. *Electronics* **8**(11), 1–11 (2019)
6 Tian, G., Yang, J., Wu, W.: A novel compact Butler matrix without phase shifter. *IEEE Microwave Wireless Compon. Lett.* **24**(5), 306–308 (2014)
7 Bhowmik, P., Moyra, T.: Modelling and validation of a compact planar Butler matrix by removing crossover. *Wireless Pers. Commun.* **95**, 5121–5132 (2017)
8 Tajik, A., Shafiei Alavijeh, A., Fakharzadeh, M.: Asymmetrical 4 × 4 Butler matrix and its application for single layer 8 × 8 Butler matrix. *IEEE Trans. Antennas Propag.* **67**(8), 5372–5379 (2019)
9 Ren, H., et al.: A novel design of 4 × 4 Butler matrix with relatively flexible phase differences. *IEEE Antennas Wirel. Propag. Lett.* **15**, 1277–1280 (2016)
10 Wong, Y.S., Zheng, S.Y., Chan, W.S.: Quasi-arbitrary phase-difference hybrid coupler. *IEEE Trans. Microwave Theory Tech.* **60**(6), 1530–1539 (2012)
11 Nocedal, J., Wright, S.: *Numerical Optimization.* Springer, New York (2006)
12 Tajik, A., Fakharzadeh, M., Mehrany, K.: DC to 40-GHz compact single-layer crossover. *IEEE Microwave Wireless Compon. Lett.* **28**(8), 642–644 (2018)