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ARTICLE

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# Old shocks cast long shadows over the exchange rate

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## ABSTRACT

We propose a new exchange rate model using interest rate differential (IRD) time series as the input, and we fit the new model with empirical data for calibration. We assume that exchange rate modeling cannot be based on the response to a single shock but must instead be based on the response to a series of shocks, as previous shocks could still be playing out and affecting the overall response. We extend the Dornbusch overshooting model and make adjustments to account for empirical findings. The new model is substantiated by empirical data from several currency areas and can explain the so-called exchange rate “puzzles”. Based on the model, we derive a relationship that explains when no IRD will suffice to support a stable exchange rate, which also suggests when policy-makers could be tempted to widen the IRD continually.

## ARTICLE HISTORY

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## KEYWORDS

Monetary policy; exchange rate; interest rate; IRD; UIP

## 1. Introduction

All models are wrong, but some are useful.

George E. P. Box

An exchange rate is the rate at which one currency can be exchanged for another. Demand and supply, which determine the exchange rate, are affected by many factors: the balance of payments (BoP), interest rates, inflation, investment opportunities, regulations, politics, risk sentiment, and others. An important dissemination channel for interest rate policy is derived from exchange rate responses to interest rate changes. Therefore, the ability to predict how the exchange rate will likely respond to a given interest rate policy is important for policy analysis and modeling. It is widely accepted, however, that exchange rate forecasts are no more accurate than those arising from a random walk in both short and medium term, and empirical validation is weak for long horizons. In 2017, a review of exchange rate theories in four leading economic textbooks revealed little empirical support for the theories examined, see Priewe (2017). There has been a general consensus about poor performance in comparison with benchmark random walk models for more than three decades, following seminal papers by Meese and Rogoff (1983a, 1983b). Furthermore, even though some studies, e.g., by Mark (1995) and Chinn and Meese (1995), lend some support to theory over the random walk for long horizons, more recent studies, such as by Cheung, Chinn, and

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Pascual (2005) and Clarida, Sarno, Taylor, and Valente (2003), have reinstated Meese and Rogoff's original view. Various approaches have been studied, including relationships between exchange rates and interest rate differentials using wavelets, see Hacker, Karlsson, and Månsson (2014), and time series methods such as Markov switching techniques – see, e.g., Engel (1994) – without changing the consensus on the state of exchange rate theory. Engel (2013) concluded that there was no convincing explanation for the dynamics between interest rates and the exchange rate. Therefore, an interest rate policy based on policy analysis and modeling that does not include a credible exchange rate prediction, since it does not exist, seems challenging at best.

Contemporary theory focuses primarily on the exchange rate response to a single, isolated, and unexpected interest rate shock. This can be problematic for at least two reasons. First, if every single interest rate shock can affect the exchange rate in both short term and long term, any evaluation of new shocks must also take account of previous shocks still playing out as long-term effects. If so, it is insufficient to predict a likely response to a set of consecutive interest rate shocks as independent of each other. Second, if interest rate policy-makers increasingly conduct predictable rule-based policies, are the policies still unexpected or will repeated “unexpected” shocks inevitably become expected? As interest rate policy becomes more predictable and established, the market gains greater insight into likely future policy actions and learns from experience.

In this paper, we derive and apply a new exchange rate model that extends the Dornbusch (1976) overshooting model. We assume that interest rate shocks are a series of interdependent shocks and we account for certain empirical findings. Accordingly, we use IRD time series, not a single interest rate shock, as input. Empirical data fit from various currency areas substantiate the model's predictions. Therefore, we believe that IRD time series provide more appropriate input than single shocks do. Certain exchange rate behavior, referred to herein as “perverse” or “puzzling”, can be explained by the model, which also demonstrates when interest rate decisions make carry trade feasible as an investment strategy for rational forward-looking agents.

The remainder of the paper is organized into two main parts. In the first part, we derive the new exchange rate model by combining two submodules describing short- and long-term effects into one as described in Section 2. The second part is based on applying the derived model, both for theoretical cases put forward in Section 3, and for empirical data from various currency areas put forward in Section 4.

## 2. Model

The Dornbusch (1976) overshooting model, distinguishes between instant versus long-run responses to interest rate shocks. Accordingly, the exchange rate is determined by i) purchasing power parity (PPP) holding in the long-run and ii) uncovered interest rate parity (UIP) holding in both the short-run and in the long-run. An initial overshooting of exchange rates is shown to be derived from the differential adjustment speed of markets. The asset market and exchange rates adjust fast to interest rate shocks relative to the goods market and the price of domestic output. Capital inflow will appreciate the spot rate to the point where the anticipated long-run depreciation exactly offsets the increase in domestic interest rates.

While we find the Dornbusch model to be most insightful and valuable, we believe that exchange rate determination could benefit from adjustments and extensions that are grounded in established empirical research findings. We place no restriction on the IRD duration, and use IRD time series as input instead of a single interest rate shock. Furthermore, we assume that interest rate shocks are series of interdependent shocks, not isolated or independent of each other, and that they are therefore time series. This assumption is based on the premise that if every single interest rate shock can affect the exchange rate, both in the short- and long-run, we must account for previous shocks that are potentially still playing out.

We assume a simple two-country model where  $S$  is the exchange rate between the two currencies, measured as number of foreign units per one local unit. The exchange rate development is dependent on IRD time series used as input, where IRD at time  $t$  is  $\theta(t) = i_t - i_t^*$ , where  $i$  and  $i^*$  are local and foreign interest rates, respectively. The exchange rate is a function of time and IRD, and is written as  $S(t, \theta(t))$ . The logarithmic value of  $S$  is denoted by a circumflex,  $\hat{S}$ . We assume that the exchange rate comprises three parts: a short-term response module  $\hat{S}_S$ , a long-term response module  $\hat{S}_L$ , and an error part. The error part,  $e \sim N(0, \sigma)$ , is assumed to be normally distributed with a mean value of zero and a standard deviation  $\sigma$ .

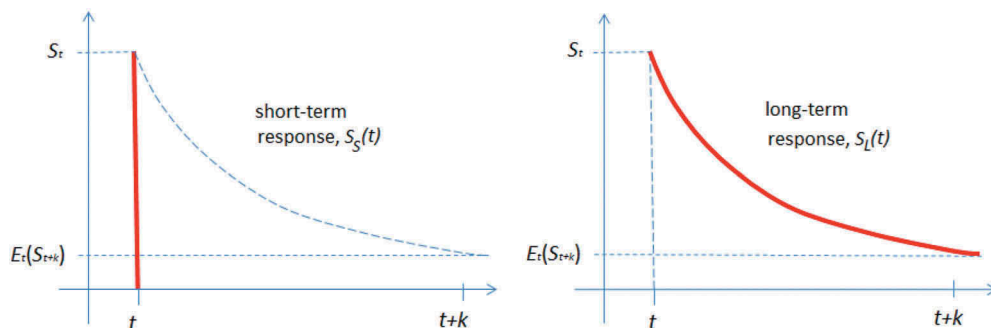
We derive the model's short-run and long-run modules separately before we combine them. The exchange rate function is therefore considered to be concurrently influenced by past and present interest rate shocks, where short-term and long-term effects are combined into a continuous time series; see [Figure 1](#).

The exchange rate function can therefore be written as

$$\hat{S}(t, \theta(t)) = \underbrace{\hat{S}_S(t, \theta(t))}_{\text{short-term}} + \underbrace{\hat{S}_L(t, \theta(t))}_{\text{long-term}} + \underbrace{e}_{\text{error}} \quad (1)$$

For a non-logarithmic form, and omitting the error part that has a zero mean, Equation (1) becomes

$$S(t, \theta(t)) = S_S(t, \theta(t)) \cdot S_L(t, \theta(t)) \quad (2)$$



**Figure 1.** The exchange rate response.

Note: The exchange rate response is assumed to be concurrently influenced by past and present interest rate shocks, since long-term responses, from past shocks, might still be playing out as new shocks hit.

Because the model's input is the IRD time series, the output can be regarded as a forecast of the exchange rate response to an interest rate policy.

### 2.1. *The short-term module*

According to Dornbusch (1976), an initial overshooting of exchange rates is shown to be derived from the differential adjustment speed of markets. The asset market and exchange rates adjust fast to interest rate shocks relative to the goods market and the price of domestic output governed by price and wage stickiness. Capital inflow will appreciate the spot rate to the point where the anticipated long-run depreciation exactly offsets the increase in domestic interest rates. Dornbusch (1976, p. 1163) has to assume perfect and consistent foresight, when he assumes the long-run exchange rate to be known, which determines the magnitude of the spot rate overshooting. For exchange rate determination this might be unrealistic. We allow for flexibility for the short-term exchange rate change due to the fact that empirical research shows it to vary in direction and magnitude. Hnatkovska, Lahiri, and Vegh (2014) demonstrated that developed and developing countries sometimes show opposite short-term exchange rate responses, which is inconsistent with the Dornbusch model.

The scapegoat theory attempts to explain the short-term anomalies. The assumption is that even though agents may have a fairly accurate idea about the long-run relationship between fundamentals and exchange rates, there is substantial uncertainty about the structural parameters over the short to medium term. This implies that when currency movements over the short to medium term are inconsistent with their beliefs, agents search for scapegoats to account for these inconsistencies. Fratzscher, Rime, Sarno, and Zinna (2015) demonstrated how instability, in the relationship between exchange rates and fundamentals, can be largely explained by scapegoats; market participants have a tendency to single out different individual macro variables from one time to another.

Furthermore, Bacchetta and van Wincoop (2013) argued that the relationship is determined by expectations rather than the structural parameters themselves. Expectations vary over time, owing to perfectly rational scapegoat effects, as agents have difficulty distinguishing between unobserved fundamentals and unobserved parameters. A research on a large panel of professional forecasters who largely agreed that an interest rate increase relates to a currency appreciation, showed forecasting performance to be positively related to the performance of forecasting fundamentals and for short-term interest rates in particular, see Dick, MacDonald, and Menkhoff (2015). Expectations can also anticipate responses. As an example, Clarida and Waldman (2007) demonstrated how a worsening inflation outlook created expectations of higher interest rates: Market participants, anticipating higher interest rates as a future policy response to a worsening inflation outlook, acted promptly on those expectations and induced a currency appreciation.

In line with the above discussion, we define our short-term module using the following mechanism:

- An interest rate differential (IRD) change will cause proportional exchange rate change.

- The exchange rate change is a function of  $\chi$ , allowing for a bidirectional change of varying magnitudes over time.

The model's short-term module assumes that the exchange rate change is a response to an interest rate shock. Based on empirical evidence, such a market reaction must be flexible enough to allow for opposite responses and varying magnitude in line with the scapegoat theory. The short-term exchange rate response module is therefore defined as

$$S_S = 1 + \sum_{k=0}^t \chi(t) \cdot \Delta\theta(k) \quad (3)$$

where  $\Delta\theta(k)$  is the interest rate shock; i.e., a change in the IRD between time  $k-1$  and time  $k$ . The corresponding time-dependent reaction response magnitude is defined as  $\chi(t)$  and reflects both the magnitude and the direction. For simplification, the  $\chi$  function can also be defined as a parameter, whereupon Equation (3) becomes

$$S_S = 1 + \chi \cdot \sum_{k=0}^t \Delta\theta(k) = 1 + \chi \cdot \theta(t) \quad (4)$$

## 2.2. The long-term module

As a consequence of a short-term market response, to a wider IRD, purchasing power parity (PPP) will have changed. A change in both the interest rate- and exchange rate levels will affect PPP. We assume that long term, the pre-shock PPP equilibrium, will be restored.

Higher interest rates can discourage new investment, encourage saving, and consequently dampen output and inflation. But higher interest rates also affect funding costs, a factor of production potentially reflected in the price level. And if a wider IRD is likelier to be reflected in the price level than one that is narrower, it must also be true that although a wider IRD can dampen inflation in the short run, it is likelier to induce it in the long run. A relatively higher price level, as a result of a lasting wider IRD, is likelier to have a detrimental effect on the economy's competitiveness and exports. Declining exports induce currency depreciation, which can gradually restore export price competitiveness. But if the relatively unfavorable funding cost structure remains, the depreciation is only a temporary fix. A depreciation and a higher price level erode local purchasing power and real wages, putting pressure on wages, a factor of production. A higher interest rate level will therefore affect the economy differently, depending on the duration of the condition, and the longer the IRD lasts, the likelier such cost-push effects are to materialize.

Dornbusch (1976, p. 1163) assumes a long-run equilibrium between interest rates. Consequently, the Dornbusch model views IRD as a temporary phenomenon. We do not require long-run equilibrium between interest rates, and actually, in reality an IRD can last forever. Accordingly, cost-push effects become likelier, and the stronger the longer the IRD exists. Dornbusch (1976, p. 1171) admits that when it comes to the goods market long-term adjustment rate, there is a lack of persuasive theoretical support. He however lends support to a long-term behavior of exchange rates. For long-term behavior of

exchange rates, we do not require a 1:1 ratio between the IRD and the corresponding expected exchange rate change, in line with empirical research findings. We replace the 1:1 ratio with the  $\gamma$  function to be calibrated at later stages. In this way, the Dornbusch overshooting model becomes a special case of the new exchange rate model, derived when  $\gamma$  and  $\chi$  are positive values and if all past interest rate shocks are ignored.

Cost-push responses to higher interest rates are sometimes referred to as the Gibson paradox, a concept dating back to 1923.<sup>1</sup> The Gibson paradox is the observation that real interest rates and changes in the general price level are at times positively correlated. Tillmann described how firms' reliance on borrowing working capital caused higher interest rates to translate into a higher cost of working capital and a rise in inflation, see Tillmann (2009a, 2009b). He also posited that even if no working capital was borrowed from banks, their opportunity cost of own funds would rise with market interest rates; therefore, in this way, higher interest rates would affect the price level. He further argued that cost-push channel effects could explain inflation dynamics in forward-looking "price and wage stickiness models for the US, the UK, and the eurozone, see Tillmann (2008).

Ravenna and Walsh (2006) showed that a cost-push shock arises endogenously when a cost-push channel for monetary policy is introduced into the new Keynesian model. And the cost-push channel dynamics for inflation, for the G7 countries, demonstrated significant but varying effects for the majority of countries, see Chowdhury, Hoffmann, and Schabert (2006). With simulation, an inverse inflation response was demonstrated if the cost-push channel was sufficiently strong relative to the demand channel. In a study of five OECD countries and 21 manufacturing sectors, it was demonstrated that monetary policy decisions affected both the cost-push channel and the demand channel, see Dedola and Lippi (2005). Adolfson, Laséen, Lindé, and Villani (2005) found the cost-push channel to be dominant in the eurozone. The fact that a positive IRD can be associated with long-run depreciating of the domestic currency is therefore due to domestic inflation induced by cost-push effects.

The model's adjustment stems from the fact that empirical research has demonstrated the long-term relationship to vary. Why UIP does not hold is not the topic of this paper. Instead, we account for this fact. Empirically UIP has been rejected repeatedly, and research indicates that there are consistent deviations from unity. Bilson (1980) is the oft-cited initial study rejecting it, but many others have rejected it as well: Cumby and Obstfeld (1980), Meese and Rogoff (1981), Hansen and Hodrick (1980), Rogoff (1983), and most famously Fama (1984). Because the UIP theory is inconsistent with reality, it has been termed the UIP puzzle, and according to Engel (1996), it is one of the most studied puzzles in international economics.

The model accounts for the fact that empirical research has shown that despite being rejected, UIP holds better long term and where the 1:1 ratio is not the norm. Chinn and Meredith (2004, 2005) tested the validity of UIP and rejected it short term, but they concluded that long term, "the coefficients on interest differentials were mostly between zero and unity." Lothian (2016) conducted a 17-country panel analysis over long time

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<sup>1</sup>The term was first used by John Maynard Keynes. Shiller and Siegel (1977) confirmed such a positive correlation using British data covering 25 years. Friedman and Schwartz (1982, p. 631) found long periods of such a positive correlation but offered a variety of possible explanations. Barth and Ramey (2001) argued that the cost-push channel was the primary monetary policy transmission mechanism for some industries, but the potency was found to vary over time. Allen and Gale (2000, 2004) also concluded that the cost-push channel effect varied over time.

periods, the longest spanning 217 years, and found theory and empirical data to be “long-term largely consistent.” Lothian concluded that all long-term relationships were positive, and three-fourths of them lay in the 0.75 to 1.25 range, which could suggest a range for the  $\gamma$  ratio.

The new model’s long-term response module is an extension of Dornbusch in two ways. First, we use time series as input. Second, based on empirical findings, the long-term relationship ratio between the IRD and the corresponding expected exchange rate change is not assumed to be 1:1, but rather 1: $\gamma$ .

Accordingly, the long-term relationship is considered to be governed by

$$\frac{S_L(t)}{S_L^e(t+1)} = \frac{1+i}{1+i^*} = \frac{1+i^* + \gamma \cdot \theta}{1+i^*} \quad (5)$$

or

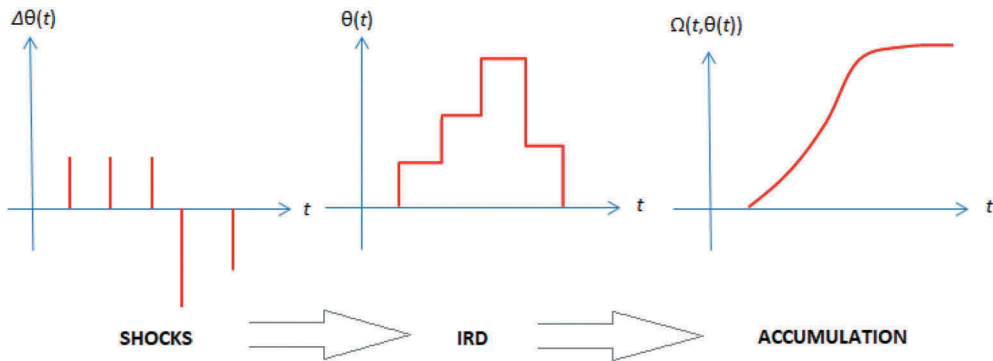
$$S_L(t+1) \simeq \frac{S_L(t)}{1 + \gamma \cdot \theta} \quad (6)$$

where  $i = i^* + \theta$  and  $S_L$  is the long-term exchange rate module. The “e” superscript indicates market expectations. With the assumption of perfect foresight,  $S_L^e(t+1)$  in Equation (5), can be replaced with  $S_L(t+1)$  in Equation (6).

Since all interest rate shocks, past and present, must be accounted for, we define a new function,  $\Omega$ , which represents the aggregate time-weighted interest rate differential:

$$\Omega(t, \theta_c(t)) = \gamma \cdot \sum_{k=0}^t \theta_c(t) \Delta t = \gamma \cdot \sum_{k=0}^t \ln(1 + \theta(t)) \Delta t \quad (7)$$

where  $\theta_c$  is  $\theta$  continuously compounding (see, e.g., Hull (2008), where  $m = 1$ ); i.e.,  $\theta_c = \ln(1 + \theta)$  where  $\theta$  is the average time-weighted IRD, and  $\Delta t$  is the time period measured as a proportion of a single interest rate period. The  $\Omega$  function will increase over time, as long as there is not a negative interest rate differential; see Figure 2.



**Figure 2.** Accumulation of shocks.

Note: Interest rate shocks  $\Delta\theta(t)$ , past and present, are accounted for and accumulate in a time-weighted fashion into the  $\Omega$  function through the IRD function  $\theta(t)$ .

Based on Equation (6), the long-term module displays long-term exchange rate developments over  $n$  time periods, where each period is  $\Delta t$  long

$$S_L = \prod_{k=0}^n \frac{1}{(1 + \gamma \cdot \theta(k))^{\Delta t}} \quad (8)$$

and for continuous compounding IRD,  $\theta_c$

$$S_L = \prod_{k=0}^n e^{-\gamma \cdot \theta_c(k) \cdot \Delta t} = e^{-\gamma \cdot \sum_k \theta_c(k) \cdot \Delta t} \quad (9)$$

or

$$S_L = e^{-\Omega(t, \theta_c(t))} \quad (10)$$

### 2.3. Combining the two modules

We can now put forward our new model by combining the short- and long-term modules into one. The new model's input is a time series of interest rate shocks, past and present. The output is a prediction for exchange rate responses according to Equation (2); i.e.,

$$S = S_S \cdot S_L \quad (11)$$

by inserting (4) and (10) into (11), we obtain a normalized exchange rate model:

$$S(t, \theta(t)) = (1 + \chi \cdot \theta(t)) \cdot e^{-\Omega} \quad (12)$$

A special case exists when there is a one-time interest rate increase away from equilibrium by  $\theta = \Delta\theta$  basis points, lasting for time  $t$ . For this special case, Equation (12) can be simplified as follows:

$$S(t, \theta) = \frac{1 + \chi \cdot \theta}{(1 + \gamma \cdot \theta)^t} = \frac{1 + \chi \cdot \Delta\theta}{(1 + \gamma \cdot \theta)^t} \quad (13)$$

The model's output is a normalized prediction value based on the time series input of interest rate shocks. In practice, when we use the model to predict for absolute exchange rate values, we must set a reference point, determining both the exchange rate reference and the start of the time series used. The model accounts for current and past interest rate shocks based on the premise that, due to price and wage stickiness, it takes time for interest rate shocks to disseminate. Therefore, the past matters. By the same token, it also matters which reference point is chosen.

Optimally, such a reference point should be a point when the exchange rate is at "equilibrium" and under "minimal" past influences. If the time series input represents deviations from equilibrium, we should set our reference points at the beginning of an era that is aimed at influencing the exchange rate. Such a choice translates into reference points before tightening or loosening interest rate policy efforts. We should aim at choosing a reference point in a "neutral" state and continue as long as a non-neutral state exists.

Another method to consider for future development of the model would be to discount the past accumulated IRD so as to ensure an appropriate limitation to the extent to which the past is allowed to affect the present and the future. Empirical data could be used to test for the appropriate past time duration to be considered and the discount factor calibrated accordingly.

### 3. Theoretical cases

In this section, we present theoretical cases that show various responses to different IRD time series. First, we consider isometric exchange rate lines; i.e. what conditions must be satisfied in order to ensure a fixed nominal exchange rate for a given initial IRD, and how different  $\gamma$  and  $\chi$  ratios affect the outcome. Second, we run several theoretical cases for imaginary interest rate policies and display how the new model predicts the exchange rate response to each case.

#### 3.1. Isometric exchange rate paths

The new model, Equation (12), assumes that two forces determine exchange rate development, one a short-term effect and the other a long-term effect. When the  $\chi$  and  $\gamma$  ratios have the same signs (and not equal to zero) the two forces work against each other and can theoretically cancel each other out. In this case, the exchange rate would remain constant; i.e., hold “stable.” The model can be used to derive the isometric path; i.e., what IRD changes are needed over time, in order to keep the exchange rate stable.

For a stable exchange rate, the following must be true:

$$S = S_S \cdot S_L = \text{constant}, \forall t \quad (14)$$

This implies that

$$\frac{d}{dt} [(1 + \chi\theta) \cdot (1 + \gamma\theta)^{-t}] = 0, \forall t \quad (15)$$

which has the solution,

$$\dot{\theta} = \frac{(1 + \chi\theta)(\gamma\dot{\theta}t + (1 + \gamma\theta) \cdot \ln(1 + \gamma\theta))}{\chi(1 + \gamma\theta)} \quad (16)$$

A detailed derivation for Equation (16) can be seen in [Appendix A](#). Equation (16) is continuous over time and  $\dot{\theta}$  suggests the necessary change, in IRD at time  $t$ , to maintain a stable exchange rate.

At time  $t = 0$ , Equation (16) becomes,

$$\dot{\theta} = \frac{(1 + \chi\theta) \cdot \ln(1 + \gamma\theta)}{\chi} \quad (17)$$

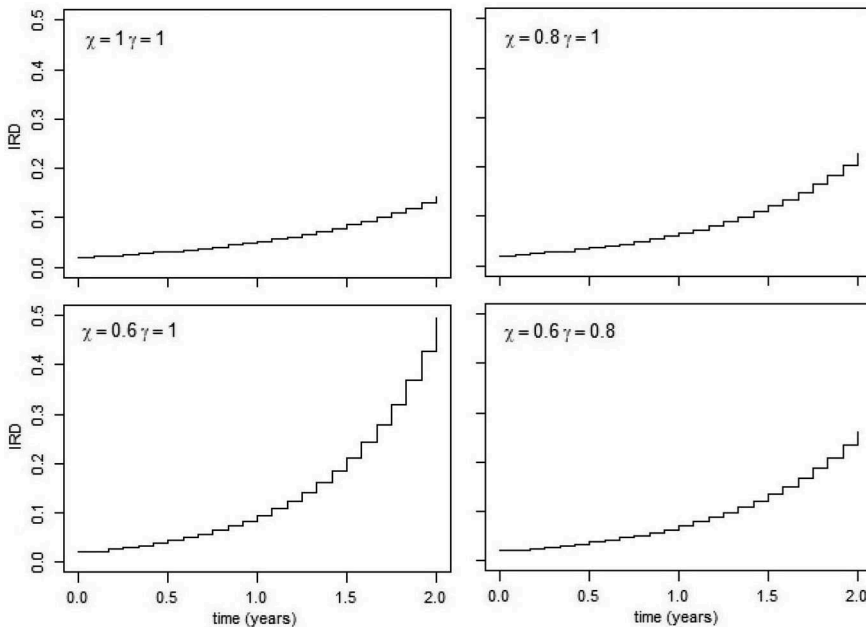
Even though Equation (17) is only valid at  $t = 0$ , we can use it as a general case to better understand the characteristics of the isometric path, and what it potentially means for policy-makers.

By observing Equation (17), we firstly see that in order to maintain a stable exchange rate, no change in interest rates is needed, unless there exists some IRD; i.e., all else equal,<sup>2</sup> change in IRD is only needed if  $\theta \neq 0$ . We see that if  $\theta = 0$  then  $\dot{\theta} = 0$ , and by the same token, if  $\theta \neq 0$  then  $\dot{\theta} \neq 0$ ; suggesting that only zero IRD is exchange rate neutral.

Second, for a positive IRD, then a positive but lower  $\gamma$  ratio, translates into a smaller change in IRD needed, to maintain a stable currency; see the two lower graphs of Figure 3. If, for whatever reason, different economies are characterized by different  $\gamma$  ratios, policy-makers should account for this.

Third, when policy rates change, the  $\chi$  ratio is a measure of market participants immediate response. A stronger market response, translates into a larger  $\chi$  ratio, and vice versa. A diminishing market response can stem for various factors, including diminishing policy credibility, since it can indicate that market expectations are less affected. As Equation (17) demonstrates, then a declining  $\chi$  calls for ever larger IRD change ( $\dot{\theta}$ ), ever higher interest rates. For this case, the need to raise interest rates diminishes, as policy credibility is greater.

Forth, Equation (17) suggests that it is theoretically possible to keep the currency stable forever. What is needed, is to ever-increase the IRD, since it generates an immediate, although a short-lived, fix for the exchange rate. But, the long-term adverse effects grow stronger,<sup>3</sup> for every fix. To follow the isometric path is therefore plausible,



**Figure 3.** Isometric paths (monthly adjusted).

Note: Isometric paths respond differently, depending on the  $\chi$  and  $\gamma$  ratios, as well as the initial IRD,  $\theta_0$ . In this case, the isometric paths are adjusted monthly.

<sup>2</sup>Assuming  $\gamma \neq 0$ .

<sup>3</sup>Assuming that  $\gamma$  and  $\chi$  have the same signs, and are not equal to zero.

but could turn out costly. If such exchange rate fixes are sustained long enough, it is likely to be followed by ever larger currency correction, or even a currency crash. Obviously, such a policy must end at some point. The longer such an unsustainable policy is sustained, the larger the correction will be, and the more severe the potential consequences. Postponing the inevitable change in the exchange rate can also erode credibility, translating into a lower  $\chi$ , which induces a need to accelerate IRD changes further. But, as Equation (17) suggests, no IRD change will suffice to maintain a stable currency, when the value of  $\chi \rightarrow 0$ .

### 3.2. Discrete isometric paths

In the previous section, we examined some characteristics for the continuous isometric path. In practice, however, policy interest rates are likely to be adjusted stepwise by central banks, e.g., at regular policy meetings. Consequently, since policy interest rates are likely to follow a discrete path, not a continuous one, we examine in this section, the main difference between the two for the isometric path. Similarly, just as we derived the continuous isometric path, by solving Equation (15), we derive the discrete isometric path, by solving the following equation,

$$S_i = \frac{1 + \chi(\theta_i - \theta_0)}{(1 + \gamma \frac{\sum(\theta_{i-1} \cdot \Delta t)}{t})^t} = S_0, \quad \forall i \quad (18)$$

where  $\theta_i$ ,  $\theta_0$ ,  $S_i$ , and  $S_0$  are IRDs and exchange rates, initially and at the  $i^{th}$  occurrence, where  $\Delta t$  is the time interval between occurrences. The  $\chi$  and  $\gamma$  coefficients are the same as before. We solve Equation (18), for  $\theta_i$ , by setting  $S_i = S_0 = 1$  and rearrange,

$$\theta_i = \frac{(1 + \gamma \frac{\sum(\theta_{i-1} \cdot \Delta t)}{t})^t + \chi\theta_0 - 1}{\chi} \quad (19)$$

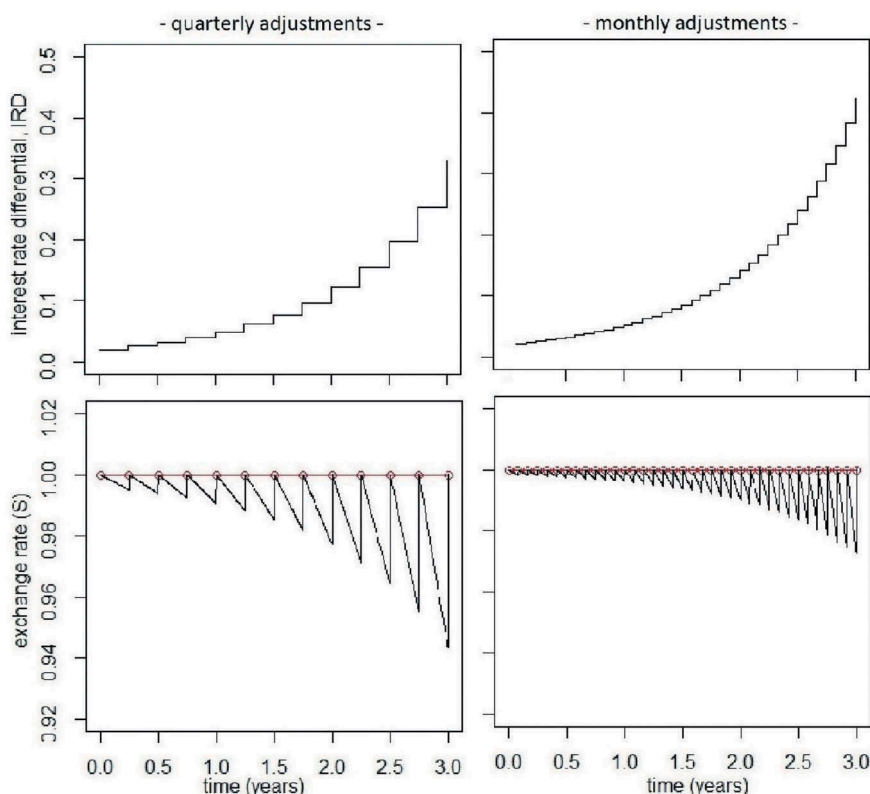
The  $\theta_i$  value of Equation (19) suggests the necessary IRD at the  $i^{th}$  occurrence, in order to maintain a stable currency.<sup>4</sup> For illustrative purposes, we compare two distinct time intervals between adjustment occurrences; one quarter, and one month, see Figure 4.

Figure 4 suggests that a higher adjustment frequency (shorter  $\Delta t$ ) comes at the cost of a higher IRD over time, see the upper part of Figure 4. But, Figure 4 also suggest that a higher adjustment frequency, will reduce exchange rate volatility and -depreciation, between adjustment occurrences, see the lower part of Figure 4. The lower part of Figure 4, also shows how the exchange rate regularly strays off the stable path, and with a single “fix” can be reverted back (red dots). However, after each “fix” it will stray again, and each time more violently.

What Figure 4 does not show, is that as soon as the IRD adjustment phase ends, the inevitable ensuing exchange rate correction will follow. The longer the path is followed, and with higher frequency, more forceful the ensuing correction will become. Whether ensuing exchange rate correction will have significant impact on domestic prices, or not, depends on several factors including pass-through. The ensuing exchange rate correction will change the relative prices of tradables and non-tradables. Imported

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<sup>4</sup>Time  $t$  is the total time that has elapsed, i.e.,  $t = i \cdot \Delta t$ .



**Figure 4.** Isometric paths.

Note: The upper part shows how higher adjustment frequency comes at the cost of a higher overall IRD. The lower part shows how higher adjustment frequency, reduces exchange rate volatility, and -depreciation between occurrences. (In both cases; monthly, and quarterly:  $\gamma = \chi = 1.00$ ).

goods become more expensive, and if pass-through is high expenditure switching from imports to domestic goods will occur and external balances will change. For some import-dependent economies there can be severe limitation to this substitution optionality, between imported and domestic goods, and then a depreciation is more likely to result in higher inflation in those cases.

### 3.3. Hypothetical interest rate policy cases

In this section, we define four cases for hypothetical interest rate policies and show how the model predicts the exchange rate response. The results are considered below as figures showing the interest rate shocks  $d\theta(t)$ , the resulting interest rate differential (IRD), the derived omega function  $d\Omega(t)$ , exchange rate developments  $S(t)$ , the carry trade profit as it develops over the period, and the net carry trade profit over the entire period as a function of  $\gamma$  and  $\chi$ .

We are interested not only in understanding how different interest rate policies affect exchange rate developments, but also how they affect carry trade profits. Carry trade profit is derived from the IRD and exchange rate developments. Carry trade profit, CTP, over one time period can be calculated as

$$CTP = (1 + \theta) \cdot \frac{S_{t+1}}{S_t} - 1 \quad (20)$$

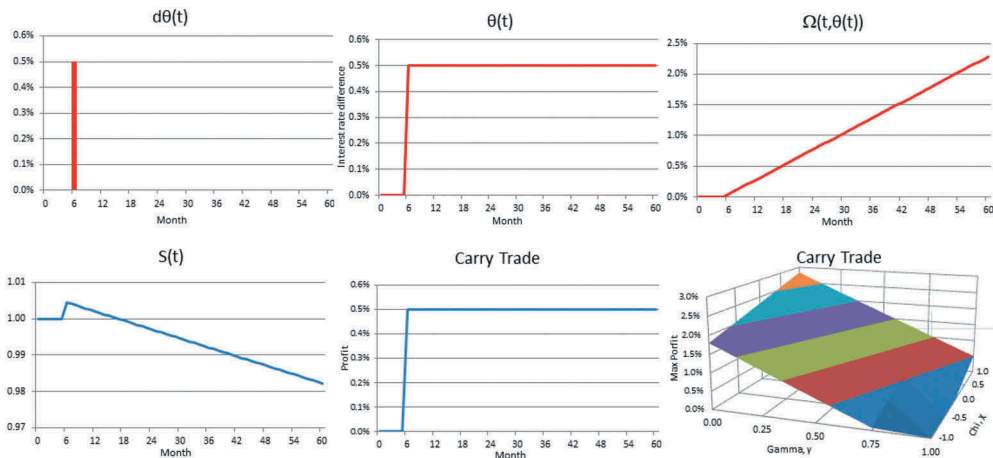
Inserting Equation (13) into Equation (20) gives the following<sup>5</sup>

$$CTP = (1 + \chi\Delta\theta) \cdot \frac{1 + \theta}{1 + \gamma \cdot \theta} - 1 \quad (21)$$

Accordingly, the conditions for profitable carry trade investments are when  $\gamma < 1.00$  and when carry trade positions are exposed to the domestic currency during a shock as long as  $\chi > 0$  and  $\gamma \leq 1.00$  and some IRD exists. This is in line with real-world experience since carry trade profitability over long periods has been substantiated by empirical research; see, e.g., Daskov and Swinkels (2015). Under UIP, carry trade should not be possible and its existence has been coined “puzzling” when viewed through the lenses of contemporary economic theory.

### 3.3.1. A lasting IRD shock (CI)

In our first hypothetical interest rate policy case (CI), we assume a single shock altering the interest rate equilibrium between two countries, see Figure 5. We assume as well that the IRD is not reversed. The model predicts that the exchange rate will respond in line with the Dornbusch overshooting model if  $\gamma = 1$  and  $\chi > 0$ . Accordingly, carry trade would yield profit as well as if  $\gamma < 1$  and  $\chi = 0$  (see graph at bottom right in Figure 5). This case (CI) demonstrates when the Dornbusch overshooting model can be viewed as a special case – i.e., when  $\gamma = 1$ ,  $\chi > 0$  – and there is no past history to complicate the exchange rate response.



**Figure 5.** A lasting IRD shock (CI).

Note: The first policy case study (CI) shows a single interest rate shock that alters the interest rate equilibrium between two countries. The value for  $\gamma$  is 1.00, and the value for  $\chi$  is 1.00.

<sup>5</sup>The  $(1 + \chi\Delta\theta)$  term is not canceled out since it happens only once when it creates the IRD, i.e., we assume that initially  $\theta = \Delta\theta$  but after that  $\Delta\theta$  is zero since IRD (i.e.,  $\theta$ ) remains the same.

### 3.3.2. A staircase and a drop (CII)

For our second hypothetical case (CII), we assume a “staircase-and-drop” interest rate policy. We assume 2 years of gradual monetary tightening, which widens the IRD before the interest rate level is brought back to equilibrium, see Figure 6. The model predicts that the exchange rate will remain stable during the gradual tightening period and then plunge as soon as it is over. If  $\gamma < 1$ , this would guarantee profitable carry trade, but expectations of an imminent end to the tightening phase would provide an incentive to close such a position before the expectations materialize.

This case demonstrates that if there is a reason to expect prolonged periods of interest rate tightening, a stable and even strong exchange rate is likely to follow. During the tightening period, both foreign funding and carry trade are feasible. However, both exogenous shocks negatively affecting the exchange rate and increased risk awareness are likely to give rise to expectations that the tightening phase might be coming to an end. Such a change in expectations would reverse the feasibility of foreign funding and carry trade, inducing closing of positions and causing the currency to depreciate. The model suggests that a depreciation will follow a tightening period, and the longer such an inevitable depreciation is postponed, the larger it will be.

### 3.3.3. The pyramid (CIII)

For our third hypothetical case (CIII), we assume a “pyramid”-like interest rate policy. We assume 2 years of gradual tightening followed by 2 years of gradual easing, see Figure 7. The model predicts that the exchange rate will remain stable during the gradual tightening period and then weakens steadily during the easing phase. The model displays milder effects for both the exchange rate and carry trade profits. However, a rational, forward-looking market participant should close any carry trade position as soon as the tightening phase is over, so as to maximize profit.

For a small open economy that is highly dependent on imported goods, the exchange rate is a determining factor for the price level. A lower exchange rate

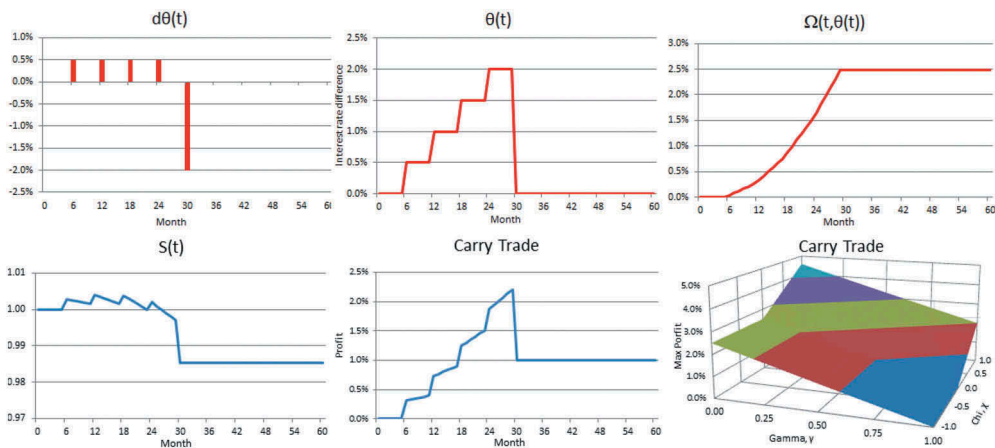
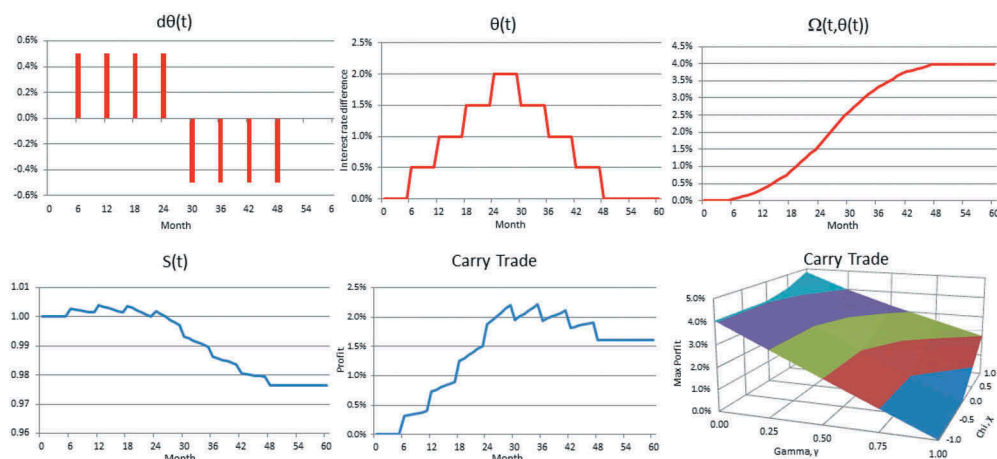


Figure 6. Staircase and drop (CII).

Note: The second policy case studied (CII) is when IRD is gradually increased, but reverted to zero by a single shock. We refer to this as a “staircase-and-the-drop” interest rate policy. The value for  $\gamma$  is 0.60, and the value for  $\chi$  is 0.60.



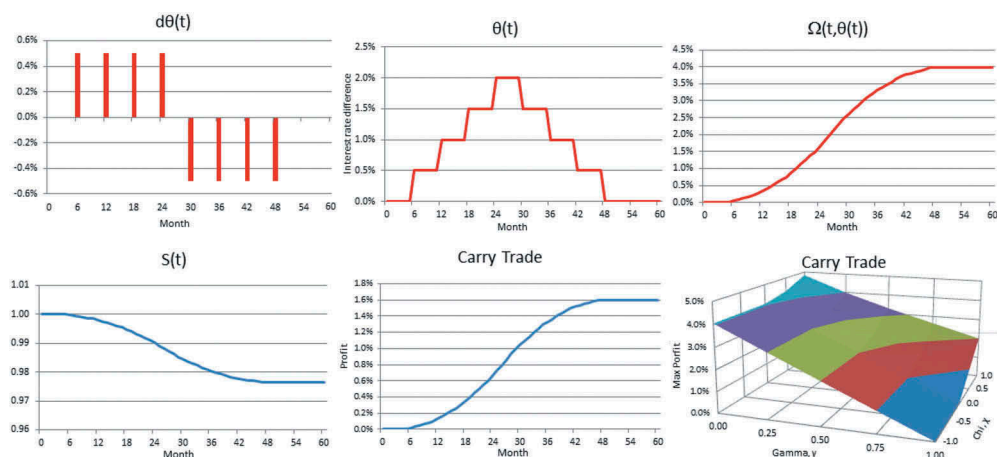
**Figure 7.** The pyramid (CIII).

Note: The third hypothetical policy case (CIII) is when IRD is gradually increased, and later gradually reverted back to zero. We refer to it as the “pyramid” interest rate policy. The value for  $\gamma$  is 0.60 and the value for  $\chi$  is 0.60.

translates into higher imported goods prices. From a policy standpoint, it might be tempting to follow an isometric line so as to ensure a stable currency and rein inflation in. Based on the model, however, such a path suggests that the  $S_S$  module will increase the exchange rate, whereas the  $S_L$  module will lower it further ahead. As is demonstrated in the above cases, and in line with Equation (19), the exchange rate will eventually fall. In order for a policy to rein inflation in following the isometric line, new capital must be attracted with offers of ever-higher interest rates. This can continue until new capital cannot be tempted, and then the policy will collapse, just as a Ponzi scheme does. A Ponzi scheme is a structure in which return on capital is paid from new capital rather than from profit earned, or by other sustainable means. Operators of Ponzi schemes usually entice new capital by offering higher returns, and because returns accumulate and grow, the need for new capital grows over time if the scheme is to work. Such growth in new capital can potentially continue until trust vanishes, whereupon the scheme will most likely collapse.

### 3.3.4. Pyramid without overshoot (CIV)

For our forth hypothetical case (CIV), we assume a “pyramid”-like interest rate policy, but unlike the third case (CIII), we assume no short-term responses to interest rate shocks, i.e.,  $\chi = 0$ , see Figure 8. This could resemble an economy where short-term speculative cross-border capital flows, are not induced by interest rate shocks, and/or fully anticipate shocks by market participants. In this case, the “carry trade” would reflect relative savings when entering into foreign-denominated versus local funding. One reason why shocks would not induce cross-border capital flows, could be due to macro-prudential policies discouraging short-term cross-border capital flows. The model shows that in comparison with the third case, volatility is reduced, but the end result is the same.



**Figure 8.** The pyramid (CIV).

Note: The fourth policy case (CIV) is a “pyramid” interest rate policy, but one where the economy is characterized by a  $\chi$  ratio equal to zero. The value for  $\gamma$  is 0.60 and the value for  $\chi$  is 0.00.

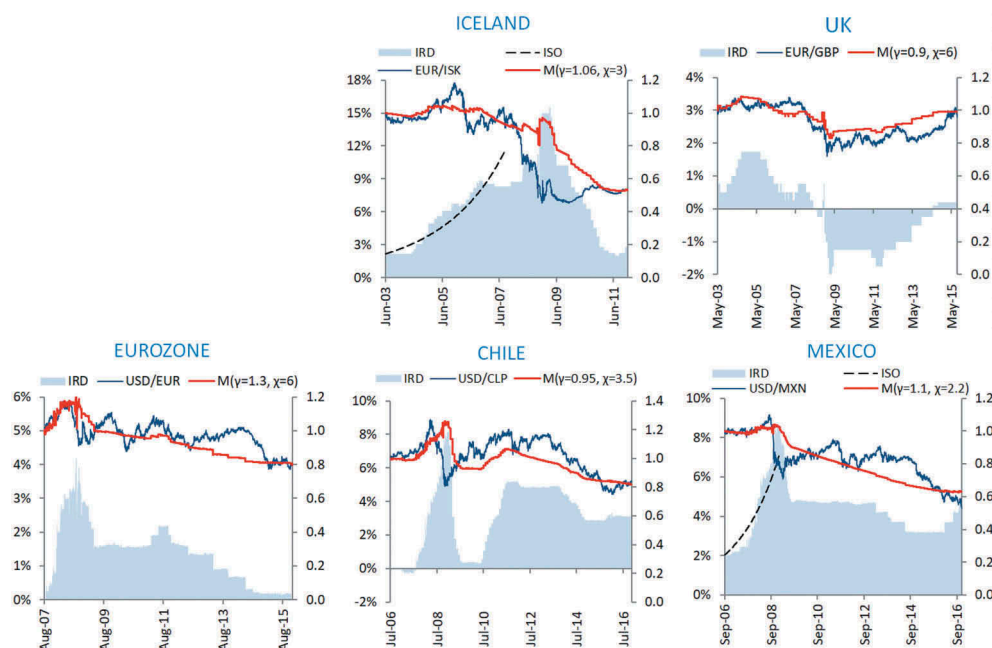
## 4. Empirical cases

In this section, we compare the model’s forecast with empirical data for five currency areas, see Figure 9. The model’s forecasts are based on the IRD time series inputs; see Figure B.1 in Appendix B. All cases have been calibrated as displayed in Table B1 in Appendix B.

### 4.1. The Eurozone, Iceland and the UK

First, we test for empirical data from Iceland, starting 5 years prior to the financial meltdown of 2008 and ending 3 years afterward. The period prior to the crash was characterized by a policy of raising interest rates; see Figures 9 and B1. During the tightening period from 2003–2007, the currency remained relatively stable, just as the model predicts. In July 2003, the nominal exchange rate was approximately the same as it was at the end of 2007, almost 5 years later. As the IRD widened, foreign credit became more popular. This willingness to borrow in foreign currency was puzzling to some economists, some of whom labeled it irrational. But, as the model shows, the isometric line and interest rate policy were closely aligned from 2003 to 2006, see Figure 9. An interest rate policy following the isometric line ensures a stable currency according to the model. A rational, forward-looking agent observing a policy that closely resembles the isometric line can therefore expect a stable currency for some time, and to profit from carry trade. An increase in foreign credit was therefore rational according to the model.

Shortly after interest rate policy departed from the isometric path, the currency started to weaken. As the depreciation became more severe, the central bank seems to have panicked and raised interest rates excessively to support the currency, but without any success. The model suggests that if an interest rate change produces a limited response for the currency, the  $\chi$  value is low or approaches zero. A lower  $\chi$  can reflect diminishing credibility since the monetary actions have limited impact. When  $\chi$  is zero, no interest rate change will suffice to support the exchange rate.



**Figure 9.** The model's forecasts.

Note: The legends abbreviations are IRD (interest rate differential) and ISO (isometric line). The  $M(\gamma, \chi)$  is the model's forecast based on the displayed calibrated values for  $\gamma$  and  $\chi$ . The normalized forecast and normalized empirical data for the exchange rate are on the right axis. The left axis shows the IRD.

Empirical data reveals strong overall consistency between reality and the model for Iceland. However, the period before and after the crash is characterized by real exchange rate developments that front-runs the model's predictions. Unlike market participants, who can act on expectations, the model is bound by realized interest rate policy actions. This front-run development suggests that market participants have accurately expected what was inevitable and have acted accordingly, before the expectations were implemented as interest rate policy actions.

The empirical data for the eurozone shows that IRD, between ECB and US policy rates, widened from 2007 to 2008, see Figure 9 and B1, followed by an appreciation of the euro, as predicted by the model. The larger IRD was caused mostly by a sharp decrease in US policy interest rates in response to the US sub-prime crisis, which surfaced in July 2007. The ECB policy rate level remained high until the problem spread to other countries. ECB policy rates remained higher than those in the US from 2007 until 2015. Based on the IRD time series input, the model predicts a general depreciation trend from 2008 to 2015, as was the case; see Figure 9.

After 2008, the eurozone experienced one financial crisis after another, followed by some rescue efforts, more often than not related to Greece. For example, in 2010 the first austerity package was passed by the Greek parliament, igniting social unrest and protests, calling into question the very existence of the euro project. The measures included a freeze in all government workers' salaries. The euro was volatile throughout this period. The most pronounced deviation from the model predictions starts in July 2012 and lasts until the end of 2014. During this period, the euro remained stronger than predicted by the

model. Notably, at the beginning of this deviation period, European Central Bank President Mario Draghi gave a speech in which he stated that "... the ECB is ready to do whatever it takes to preserve the euro ...". These remarks seemingly contributed to a stronger euro than predicted by the model for a period of two-and-a-half years.

The empirical data for the UK shows that from 2003 to 2008, interest rate policies in both the UK and the eurozone were characterized by monetary tightening, followed by monetary easing from 2009 to 2014, although at different speeds, see [Figure 9](#). The model's prediction is largely consistent with the long-term trend over a 12-year period; however, the model predicts a slightly stronger pound than actually materialized after 2008, see [Figure 9](#).

## 4.2. Chile and Mexico

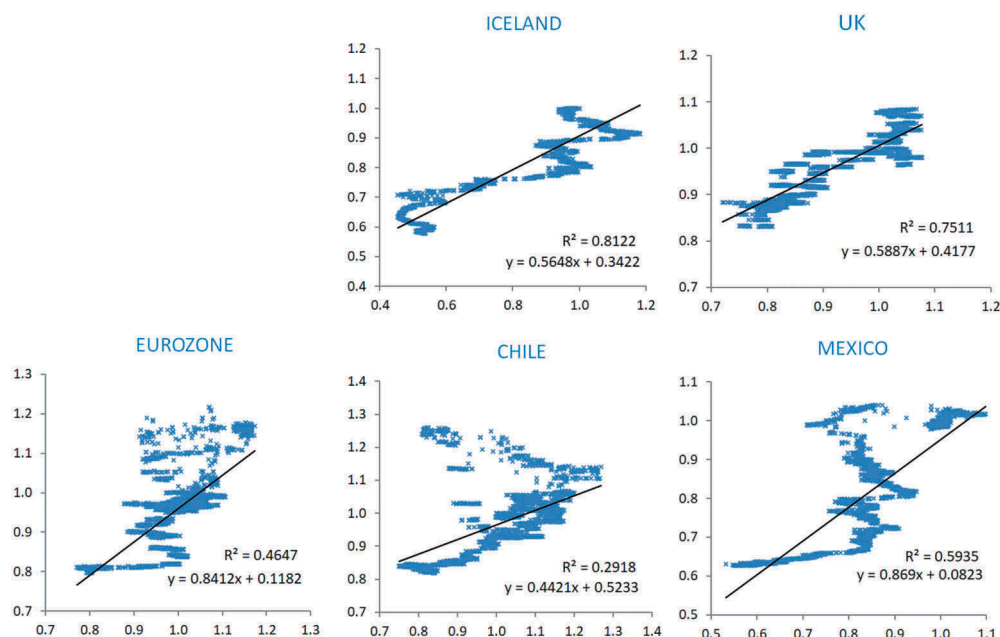
In 2008, the Chilean peso fell against the US dollar by more than 20%, despite a rapid and substantial increase in  $\theta$ , see [Figure 9](#) and [B1](#). As outsiders, we find two interesting observations for the Chilean case. First, the model "predicts" the crash, but 6 months too late! One potential explanation is that credibility (measured as  $\chi$ ) mostly evaporated during this period, and/or that market participants correctly anticipated that such a rapid, even frantic, increase in IRD, could not last, as happened half a year later. The other observation is a good fit of empirical data, with the calibrated model. The only input for the exchange rate model is; the initial value of the exchange rate ( $S_0$ ), and an IRD time series ( $t, \theta(t)$ ) over 10 years. The Chilean peso against the US dollar exchange rate development, substantiates the model.

The Mexican peso remained relatively stable against the US dollar, in the time period from 2006 to 2008, before it collapsed by more than 20% in 2008, see [Figure 9](#) and [B1](#). However, it is interesting to observe that during this period, the IRD increased in a way closely resembling the isometric path, see [Figure 9](#). But, despite resembling the isometric path, and "predicting" a substantial currency correction, it is more than half a year too late. The observed currency crashes of 2008, in Mexico, Chile, and Iceland, all happened sooner than predicted by the model. One potential explanation is that market participants simply outsmart policy-makers. Policy-makers determine the input for the exchange rate model, i.e., the IRD time series. If market participants can correctly predict what policymakers will inevitably be forced to do, e.g. cut interest rates when faced with what we now call the "great recession," they will outsmart hesitant policy makers.

Finally, we compare the model's predictions against empirical data. A comparison between the two is used to test if a significant relationship between the two exists. A significant relationship is confirmed by testing, and rejecting a null hypothesis stating that the slope between the two  $\beta = 0$ , see [Figure 10](#) and [Table B1](#) in [Appendix B](#).

## 5. Conclusion

The new model shows a good fit for all of the empirical cases observed, ranging from 8 to 12 years. We derive a formula for an isometric path that must be followed in order to maintain a stable exchange rate. The isometric path demonstrates that given some initial



**Figure 10.** Model versus empirical data.

Note: The model's normalized exchange rate forecast (vertical) and -empirical data (horizontal) plotted.

IRD, a stable exchange rate can only be maintained by widening the IRD exponentially over time. This result prompts comparison with a Ponzi scheme, in that new capital must be lured into the economy with ever-higher interest rates until it collapses.

As demonstrate both theoretically and empirically, following the isometric path by constant widening of the IRD, might seem sailing the calm sea for a while but will most likely be followed by a risky storm. The exchange rate can be kept stable for a while; however, it will be followed by a depreciation that can be postponed but not avoided. An exchange rate correction is likely to be followed by inflation. A policy aiming at reining inflation in through constant widening of the IRD will potentially be followed by higher inflation, adverse to the initial policy aim. Chile, Mexico, and Iceland all showed signs of policies resembling the isomeric path with ensuing large corrections.

The model also explains when carry trade becomes a rational investment alternative. The model further demonstrates how diminishing credibility can undermine interest rate policy and under which circumstances no IRD will suffice to support the exchange rate.

We believe that the new model can greatly advance understanding of the importance of considering interest rate policy paths rather than single interest rate shocks. According to the Dornbusch overshooting model, every single interest rate shock has both short- and long-term effects. Furthermore, every single interest rate shock is affected by past long-term shocks still playing out. Our new model accounts for this, in addition to other important extensions.

Despite encouraging results, we believe the new model can benefit from further research and development, including appropriate choice criteria for reference points, a discount factor for past events, and rigorous calibration decision criteria.

## Disclosure statement

No potential conflict of interest was reported by the author.

## Notes on contributor

**Jón Helgi Egilsson** is a former chairman of the supervisory board of the Icelandic Central Bank, former adjunct professor in financial engineering at Reykjavík University as well as a former MBA and engineering economics lecturer at the University of Iceland.

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## References

- Adolfson, M., Laséen, S., Lindé, J., & Villani, M. (2005). The role of sticky prices in an open economy dsge model: A bayesian investigation. *Journal of the European Economic Association*, 3, 444–457.
- Allen, F., & Gale, D. (2004). Comparative financial systems: A discussion. In S. Bhattacharya, A. Boot, & A. Thakor (Eds.), *Credit, intermediation, and the macroeconomy* (pp. 699–770), Oxford University Press.
- Allen, F., & Gale, D. (2000). *Comparing financial systems*. Cambridge, MA: MIT Press.
- Bacchetta, P., & van Wincoop, E. (2013). On the unstable relationship between exchange rates and macroeconomic fundamentals. *Journal of International Economics*, 91, 18–26.
- Barth, M. J., & Ramey, V. A. (2001). The cost channel of monetary transmission. In B. Bernanke & K. Rogoff (Eds.), *NBER macroeconomics annual* (Vol. 16, pp. 199–240).
- Bilson, J. (1980). The “Speculative Efficiency” Hypothesis, No 0474, NBER Working Papers, National Bureau of Economic Research, Inc, <https://EconPapers.repec.org/RePEc:nbr:nberwo:0474>
- Cheung, Y.-W., Chinn, M. D., & Pascual, A. G. (2005). Empirical exchange rate models of the nineties: Are any fit to survive? *Journal of International Money and Finance*, 24, 1150–1175.
- Chinn, M. D., & Meese, R. A. (1995). Banking on currency forecasts: How predictable is change in money? *Journal of International Economics*, 38, 161–178.
- Chinn, M. D., & Meredith, G. (2004). Monetary policy and long-horizon uncovered interest parity. *IMF Staff Papers*, 51(3), 409–430.
- Chinn, M. D., & Meredith, G. (2005). Testing uncovered interest rate parity and short and long horizons during the post-bretton woods era. (WP 11077). NBER.
- Chowdhury, I., Hoffmann, M., & Schabert, A. (2006). Inflation dynamics and the cost channel of monetary transmission. *European Economic Review*, 50, 995–1016.
- Clarida, R., & Waldman, D. (2007). Is bad news about inflation good news for the exchange rate? (working paper 13010). NBER, National Bureau of Economic Research.
- Clarida, R. H., Sarno, L., Taylor, M. P., & Valente, G. (2003). The out-of-sample success of term structure models as exchange rate predictors: A step beyond. *Journal of International Economics*, 60, 61–83.
- Cumby, R. E., & Obstfeld, M. (1980). Exchange-rate expectations and nominal interest differentials: A test of the fisher hypothesis (NBER Working Paper No. 537).
- Dedola, L., & Lippi, F. (2005). The monetary transmission mechanism: Evidence from the industries of five oecd countries. *European Economic Review*, 49, 1543–1569.
- Dick, C. D., MacDonald, R., & Menkhoff, L. (2015). Exchange rate forecasts and expected fundamentals. *Journal of International Money and Finance*, 53, 235–256.

- Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of Political Economy*, 84, 1161–1176.
- Doskov, N., & Swinkels, L. (2015). Empirical evidence on the currency carry trade, 1900–2012. *Journal of International Money and Finance*, 51, 370–389.
- Engel, C. (1994). Can the markov switching model forecast exchange rates? *Journal of International Economics*, 36, 130–141.
- Engel, C. (1996). The forward discount anomaly and the risk premium: A survey of recent evidence. *Journal of Empirical Finance*, 3, 123–192.
- Engel, C. (2013). Exchange rates and interest parity (Working Paper 19336). Nber working paper series.
- Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics*, 14, 319–338.
- Fratzscher, M., Rime, D., Sarno, L., & Zinna, G. (2015). The scapegoat theory of exchange rates: The first tests. *Journal of Monetary Economics*, 70.
- Friedman, M., & Schwartz, A. J. (1982). *Monetary trends in the united states and the united kingdom: Their relation to income, prices, and interest rates, 1867–1975*. National Bureau of Economic Research, Inc..
- Hacker, R. S., Karlsson, H. K., & Månsson, K. (2014). An investigation of the causal relations between exchange rates and interest rate differentials using wavelets. *International Review of Economics and Finance*, 29, 321–329.
- Hansen, L. P., & Hodrick, R. J. (1980). Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *Journal of Political Economy*, 88, 829–853.
- Hnatkovska, V., Lahiri, A., & Vegh, C. A. (2014). The exchange rate response puzzle. University of Toronto, Department of Economics . working paper.
- Hull, J. (2008). *Options, futures and other derivatives* (7 ed.), Prentice Hall.
- Lothian, J. R. (2016). Uncovered interest parity: The long and the short of it. *Journal of Empirical Finance*, 36, 1–7.
- Mark, N. C. (1995). Exchange rates and fundamentals: Evidence on long-horizon predictability. *The American Economic Review*, 85, 201–218.
- Meese, R., & Rogoff, K. (1983b). The out-of-sample failure of empirical exchange rate models: Sampling error or misspecification. In J. A. Frenkel (Ed.), *Exchange rate and international economics*. University of Chicago Press, 67–112.
- Meese, R., & Rogoff, K. (1981). Empirical exchange rate models of the seventies: Are any fit to survive? (International Finance Discussion Papers, No 184).
- Meese, R., & Rogoff, K. (1983a). Empirical exchange rate models of the seventies. *Journal of International Economics*, 14, 3–24.
- Priewe, J. (2017). Review of exchange-rate theories in four leading economics textbooks. *European Journal of Economics and Economic Policies: Intervention*, 14.
- Ravenna, F., & Walsh, C. E. (2006). Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 53, 199–216.
- Rogoff, K. (1983). Empirical exchange rate models of the seventies. *Journal of International Economics*, 14, 3–24.
- Shiller, R. J., & Siegel, J. J. (1977). The gibson paradox and historical movements in real interest rates. *Journal of Political Economy*, 85, 891–908.
- Tillmann, P. (2008). Do interest rates drive innovation dynamics? An analysis of the cost channel of monetary transmission. *Journal of Economic Dynamics & Control*, 32, 2723–2744.
- Tillmann, P. (2009a). Optimal monetary policy with an uncertain cost channel. *Journal of Money, Credit and Banking*, 41.
- Tillmann, P. (2009b). The time-varying cost channel of monetary transmission. *Journal of International Money and Finance*, 28, 941–953.

## Appendix A Continuous isometric path

For a stable exchange rate, the following must be true:

$$S = S_S \cdot S_L = \text{constant}, \forall t \quad (\text{A.1})$$

This implies that

$$\frac{d}{dt} [(1 + \chi\theta) \cdot (1 + \gamma\theta)^{-t}] = 0, \forall t \quad (\text{A.2})$$

and therefore,

$$\chi\dot{\theta}(1 + \gamma\theta)^{-t} + (1 + \chi\theta) \cdot \frac{d}{dt} [(1 + \gamma\theta)^{-t}] = 0 \quad (\text{A.3})$$

where

$$\frac{d}{dt} [(1 + \gamma\theta)^{-t}] = -t(1 + \gamma\theta)^{-t-1} \cdot \gamma\dot{\theta} + (1 + \gamma\theta)^{-t} \cdot \ln(1 + \gamma\theta)(-1) \quad (\text{A.4})$$

or,

$$\frac{d}{dt} [(1 + \gamma\theta)^{-t}] = -(1 + \gamma\theta)^{-t} \left[ \frac{\gamma\dot{\theta}t}{(1 + \gamma\theta)} + \ln(1 + \gamma\theta) \right] \quad (\text{A.5})$$

We use the result of Equation (A.5) in Equation (A.3) and get,

$$\chi\dot{\theta}(1 + \gamma\theta)^{-t} - (1 + \chi\theta)(1 + \gamma\theta)^{-t} \left[ \frac{\gamma\dot{\theta}t}{(1 + \gamma\theta)} + \ln(1 + \gamma\theta) \right] = 0 \quad (\text{A.6})$$

We multiply Equation (A.6) by  $(1 + \gamma\theta)^{-t}$  and get,

$$\chi\dot{\theta} - (1 + \chi\theta) \left[ \frac{\gamma\dot{\theta}t + (1 + \gamma\theta) \cdot \ln(1 + \gamma\theta)}{(1 + \gamma\theta)} \right] = 0 \quad (\text{A.7})$$

or,

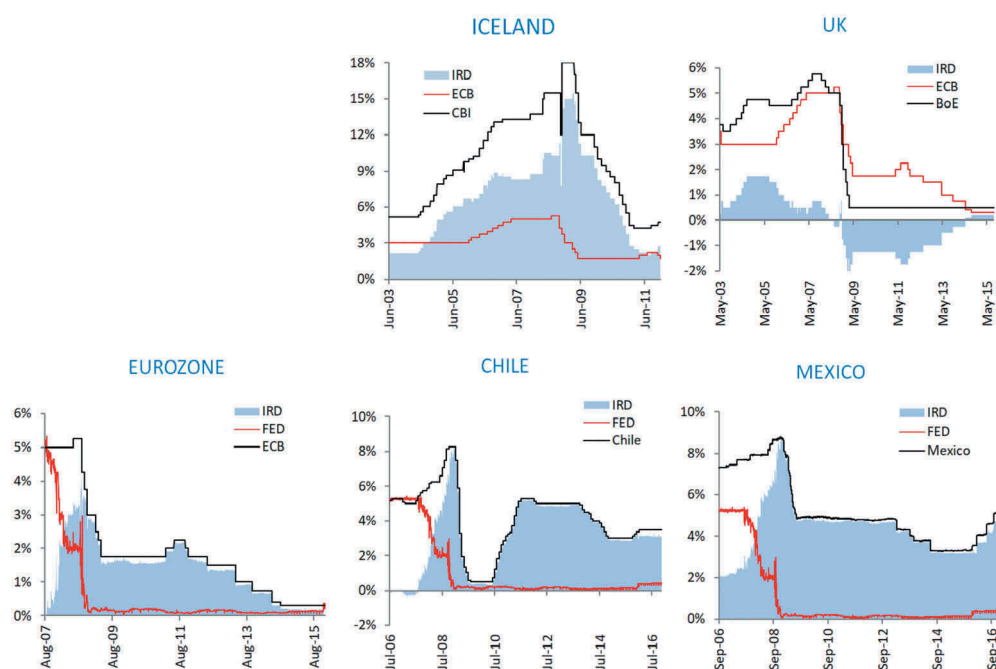
$$\dot{\theta} = \frac{(1 + \chi\theta)(\gamma\dot{\theta}t + (1 + \gamma\theta) \cdot \ln(1 + \gamma\theta))}{\chi(1 + \gamma\theta)} \quad (\text{A.8})$$

## Appendix B Empirical data overview

Table B1 provides an overview for the empirical data used, and calibrated values. Figure B1 shows the underlying nominal interest rate time series for IRD calculations.

**Table B1.** Empirical data premises and model variables.

CURRENCY		INTEREST RATE DIFFERENTIAL (IRD)		TIME PERIOD	CALIBRATION		TEST
Country	Exchange rate	Local	Foreign		Gamma (y)	Chi(X)	
Iceland	EUR/ISK	CBI collateralized lending rate	ECB-MRO, rate on marginal refinancing operations	Jan 03-Dec 11	1.06	3.0	0.81
Eurozone	USD/EUR	ECB-MRO, rate on marginal refinancing operations	US effective federal funds rate (EFFR)	Sep 07-Dec 15	1.30	6.0	0.75
The UK	EUR/GBP	BoE official rate	ECB-MRO, rate on marginal refinancing operations	May 03-Sep 15	0.90	6.0	0.46
Chile	USD/CLP	Chilean monetary policy interest rate	US effective federal funds rate (EFFR)	Jul 06-Nov 16	0.95	3.5	0.29
Mexico	USD/MXN	Banco de México	US effective federal funds rate (EFFR)	Sep 06-Nov 16	1.10	2.2	0.59



**Figure B1.** Interest rate differential (IRD).

Note: The rates used are: Chile (Chilean monetary policy interest rate), the eurozone (ECB marginal refinancing operations (MRO) rate), Iceland (Central Bank of Iceland collateralized lending rate), Mexico (Banco de México 28-day TIIE interest rate), UK (BoE official rate), and US (US effective federal funds rate (EFFR)).