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PISA 2012 – Performance in Mathematics and School Size

Kristín Bjarnadóttir[a] – Freyja Hreinsdóttir[a*]

[a] *School of Education, University of Iceland, Sæmundargötu 2, 101 Reykjavík, Iceland*

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Abstract

A study of the results of PISA 2003 in Iceland showed that pupils in the two largest schools did was significantly better than in smaller schools. The score was particularly low in schools with 11–25 participants in PISA. A study of the PISA 2003 score in Denmark also showed better results in larger schools than in smaller ones. When the results of PISA 2012 in Iceland were published, the Educational Testing Institute of Iceland was asked to classify the results into four categories based on school size. The results in the largest schools turned out to be significantly better than in smaller schools. A questionnaire was sent to a selection of schools in each category, omitting the category of the smallest schools. In the questionnaire mathematics teachers were asked questions on their education, proportion of their work in teaching mathematics, experience in teaching mathematics in lower secondary school and material used. The results indicated that full-time work in mathematics teaching, many years of teaching mathematics and in particular continuity in teaching i.e. teachers' experience in teaching the same group and the same material for many years leads to better performance.

Keywords: Mathematics. Teacher education. Teaching experience. PISA 2012. School size.
Classification: B50, D10

Introduction

What is PISA?

PISA is an international survey into the competence and skills of 15 years olds in reading, science, mathematics and problem solving. PISA is an abbreviation of Programme for International Student Assessment. The survey is done by OECD and totally 65 nations participated in PISA 2012. The Educational Testing Institute of Iceland was responsible for the PISA survey on behalf of the Ministry of Education (Halldórsson, Ólafsson and Björnsson, 2012).

Mathematical literacy

The theoretical framework of PISA is based on the concept literacy which means the skills of students in

- drawing conclusions from what they know
- use their knowledge in new circumstances
- analyze, discuss and express their ideas when interpreting information and solving problems in different circumstances

[*]Corresponding author: freyjah@hi.is
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The concept literacy refers to the ability of students to use their knowledge and competence in key subjects and to analyze, understand and express in an efficient way solutions to various problems in many different circumstances. To acquire literacy is a lifelong process, which not only occurs in school but also through interaction with family, peers, colleagues and by participating in social activity. Mathematical literacy in PISA measures the ability of individuals to state, use and interpret mathematics in many different ways. It is the ability to reason mathematically and use mathematical concepts, methods, facts and tools to describe, explain and predict various phenomena (Halldórsson, et al., 2012).

The content of the PISA problems evolves around four main ideas on numbers, algebra and geometry. They intersect and relate in many ways to

- quantity
- shape and space
- relation and change
- uncertainty and data

It is expected that students can master calculators when solving the PISA problems. When dealing with algebraic problems the emphasis is on creating formulas and generalizing, i.e. to use mathematical symbolic language but very little emphasis is on rewriting and simplifying. A large emphasis is placed on reading graphs, interpreting and relating to given information, interpret formulas and estimate the effect of changing the value of a variable, read from maps, follow directions and calculate distance and area using appropriate scales.

Proficiency levels

The proficiency of students in answering questions in PISA is divided into 7 levels, 0–6. Students at level 6 in PISA are able to solve the most difficult problems and get more than 669 points. Students at this level have mathematical thinking and deductive abilities at a high level. They are able to draw conclusions and use information based on their research and models to solve complicated problems and use their knowledge in new contexts. They can connect information presented in different ways and adapt it to various circumstances. These students use intuition and understanding together with exceptional skills in symbolic and formal mathematical operations and relations to develop new approaches and methods to deal with new circumstances. Students at this level are able to communicate their answers precisely together with their thoughts on their discoveries, interpretations and reasoning and are able to explain why certain operations are used to bring mathematical problems from daily life to a mathematical form.

Students below level 1 get 358 points or less. They can possibly solve simple mathematical problems like reading a clearly marked value from an illustration or a table where the marks correspond to words used in the introductory text and the question posed. In this way the choice is clear and the connection between the graph and description seems obvious. These students are also able to solve mathematical exercises with integers by following direct instructions.

The average result for students in OECD countries was 494 points but the average for Iceland was 493 points. Further information on results and level distribution can be found in (Halldórsson et al., 2012, pp. 18–19, 26–28).

Size categories of Schools in PISA 2003 and PISA 2012

The results of PISA 2003 in measuring mathematical literacy

The results from PISA 2003 indicated that the results were better in larger schools than smaller (Bjarnadóttir, 2008), see Figure 1.

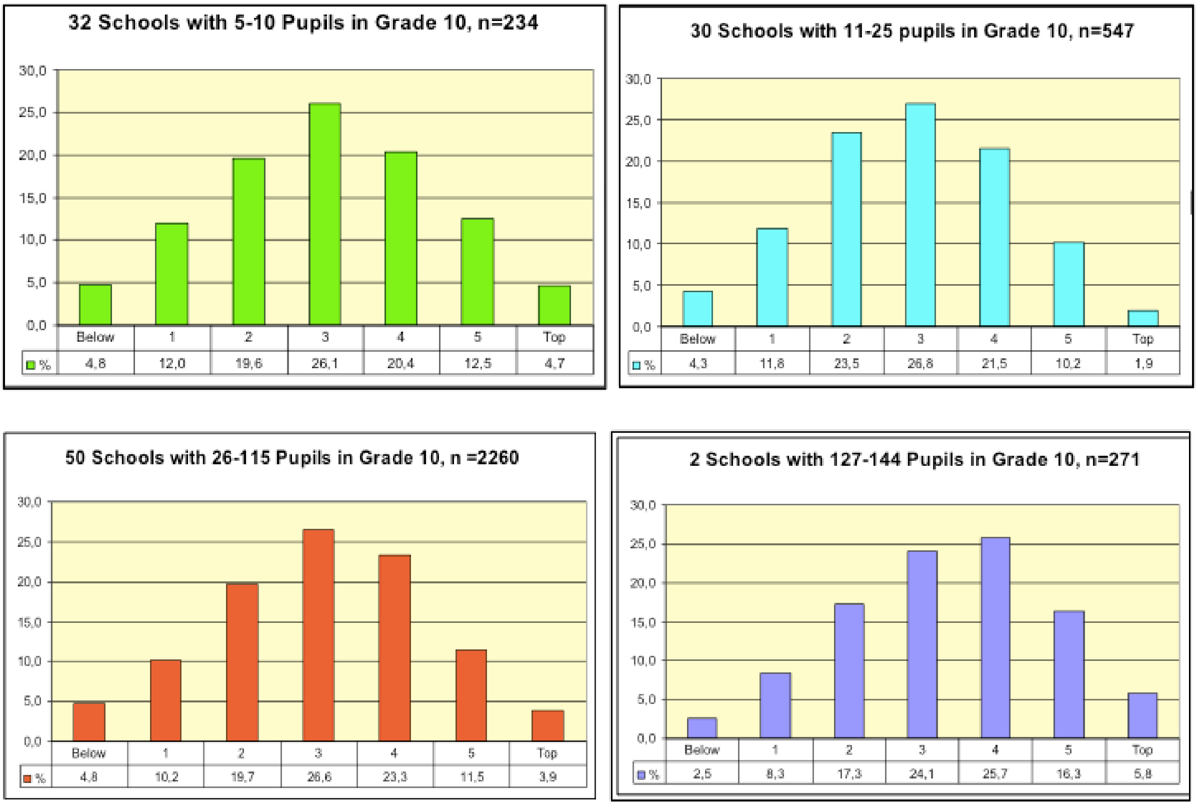

***Figure 1** Comparison of performance in mathematical literacy in PISA 2003 based on the number of participants divided into 4 categories.*

In Figure 1 it is shown that proportionally more students reached level four or higher in schools where 26–115 students participated in PISA than in schools with 11–25 participants where it can be assumed that each age group had only one class. The figure also shows that the result was proportionally better in the largest schools (only 2 schools were in that group).

Performance in PISA 2012 measuring mathematical literacy

The Educational Testing Institute of Iceland analyzed the performance in PISA 2012 in size groups in collaboration with the authors. The size groups were chosen as 1–10, 11–25, 25–40 and 41–128 participants in PISA 2012. This time it was decided to enlarge the group with the largest group from what was done for PISA 2003 (the largest group then only had 2 schools). This was done to see if the size of the school mattered and to minimize the effect of certain schools.

Not all students in every school participated in PISA. Exemptions were approximately 5% since it is generally expected that exemptions are only made for health reasons (Halldórsson et al., 2012). The numbers however indicate that the participation was less than 95%. In 2012 there were 4500 15 year olds in Iceland (Statistics Iceland, web) but 3509 students participated in PISA 2012 or 78%.

The results can be seen in Figure 2. The figure shows that the same pattern as in PISA 2003 is repeated in PISA 2012. Proportionally more students reach level 4 or higher in the group of schools with the highest number of participants. There are also proportionally more students that only reach level 1 in the schools with the fewest participants.

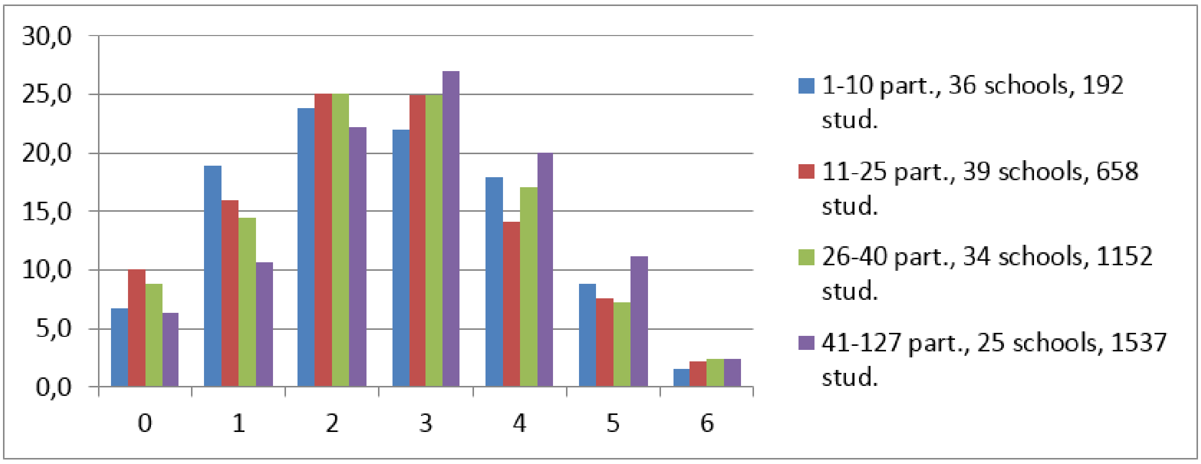

Figure 2 *Comparison of performance in Mathematical Literacy in PISA 2012 in different school size categories*

The difference is also clear when looking at the total score of the four groups:

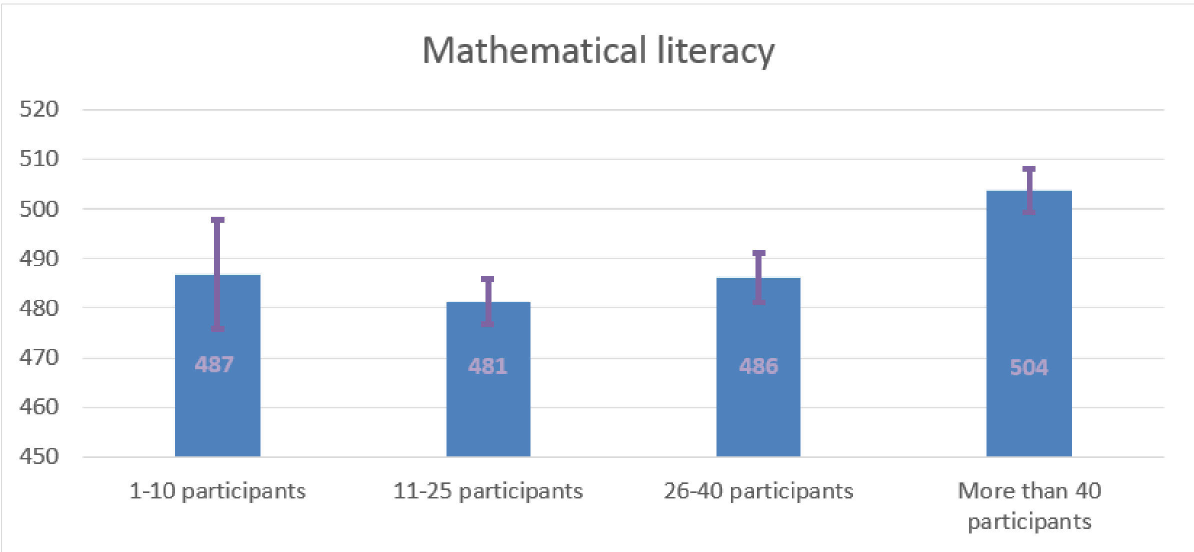

Figure 3 *Total score in Mathematical Literacy in PISA 2012 classified by the size of school (together with 90% confidence limits).*

Figure 3 shows clearly that the total score is significantly better in larger schools than in smaller ones. The average total score in the largest schools in 504 points, which is significantly better than the average score of Icelandic students, 493 points. In our research we try to examine the reasons for the better performance in the larger schools.

Theoretical background

Difference in performance in PISA 2003 in Denmark with respect to school size

Niels Egelund (2006) reached the conclusion on PISA 2003 in Denmark that the performance was better in larger schools than smaller. Egelund claims that the performance gets better with the size of schools up to 650 students (this is the total number of students in 10 age groups). (see Figure 4). He refers to international research, mainly American, but says that this research is not applicable for comparison with Danish schools. Firstly, these are not 10 year schools, secondly there are schools with very different situations from Denmark in areas of poverty and thirdly there are very few teachers in the United States that teach all age groups and many subjects while this is common in Denmark. The Icelandic school system is very similar to the school system in Denmark, and research is therefore comparable even if Denmark has very few small schools in sparsely populated areas. Egelund's theory is that large schools

- have broader competence of teachers across subjects and school levels
- have better possibilities for teachers to work as teams in each subject
- have better possibilities for teachers to teach the subjects they have specialized in
- have less effect of conflict between teachers

Figure 4 shows the relation between total number of students in Danish schools and average total score in PISA 2003.

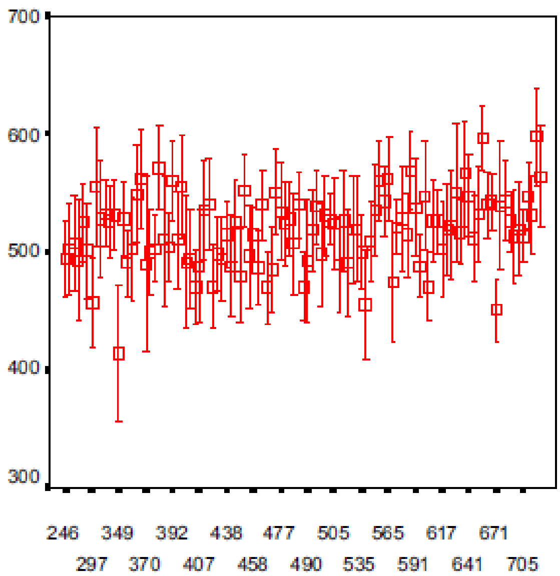

Figure 4 *Average total score in PISA 2003 (with 95% confidence interval) is the vertical axis and size of school is the horizontal axis. (Egelund, 2006, p. 311)*

Theories on subject matter knowledge and pedagogical content knowledge

Shulman (1986) defined several types of knowledge that teachers need to have. He distinguished in particular *content knowledge*, that is a deep understanding of the subject itself, and *pedagogical content knowledge*, which is to know the subject matter for teaching and to know the most useful forms of presenting it, explaining and demonstrating it. Thirdly, Shulman defined *curricular knowledge* as a necessary factor in teacher education; that is pedagogical knowledge behind the subject's curriculum and the material taught in other subjects as well as solid knowledge of the previous and future curricular content.

Shulman article is quite old and covers general subjects but not specifically mathematics. Some scholars have continued research in a similar direction. These include Krauss, Baumert, Brunner and Blum (2008) and Neubrand (2008). They claim there is a strong correlation between *content knowledge* (CK) and *pedagogical content knowledge*, PCK. Their view is that PCK is strengthened by strong CK but that CK is only one possible way to PCK and that emphasis on pedagogics in teacher education is another possible way.

More scholars have discussed the connection between subject knowledge and pedagogical knowledge, e.g. Barbara Jaworski. To capture the complexity of the many factors that need to be considered in teaching, she developed the concept *the teaching triad* (Jaworski, 1994). Jaworski based the idea of the teaching triad originally on research on the teaching of mathematics. That research showed that the teachers took into consideration three interconnected factors when organizing their teaching: *management of learning*, ML, *sensitivity to students*, SS and *mathematical challenge*, MC.

Jaworski considers the three components as tightly woven factors in the total commitment of the teachers who need to always weigh them against each other in their teaching. She describes them by the figure below.

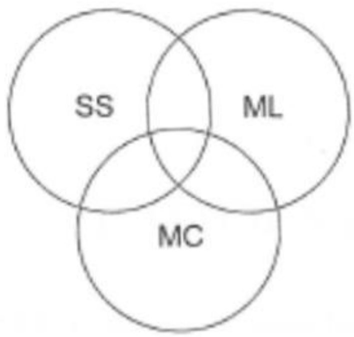

Figure 5 *The Teaching Triad (Jaworski, 1994, pp. 107–8)*

The theories of Shulman, Kreuss et. al., Neubrand and Jaworski all point in the same direction: a teacher needs to have subject knowledge in order to provoke the thought of students, he needs *pedagogical content knowledge* on the subject to be able to ask the right questions, he needs to know and take into consideration *the curriculum* and know what has already been studied and what is to be studied, and he needs to be *sensitive to his students needs*.

The investigation

Choice of schools

We contacted teachers from ten schools in each size group. Teachers in two schools didn't want to participate, but in one of those two cases (with 11 – 25 participants) we were able to find another school where teachers were willing to participate. One teacher in the size group 25–40 did not answer questions on education but all other questions.

We decided not to include the smallest school in the research. These are schools where the number of participants in PISA 2012 was between 1 and 10. This was done since we expect that the situation in these schools is different from other School Groups, several age groups may be taught together, the total number of pupils (192) is irrelevant in comparison to the total number of participants, and the teaching is adapted to the individuals in those schools.

The number of students in each group is as follows:

Table 1 *Number of students in the chosen schools*

Groups of schools	11–25 students	26–40 students	41–130 students	Total
Total number of schools	39	34	25	98
Number of chosen schools	10	9	10	29
Number of chosen schools in Reykjavík and surrounding area	5	6	6	17
Total number of students	658	1122	1537	3317
Total no. of stud. in chosen schools	174	303	721	1198
Percentage	26%	27%	47%	36%

In the year 2012 the population of Iceland was 319.575. Of those, there were 4500 15-year olds (Statistics Iceland, web) but totally 3.509 students participated in PISA 2012 or 78% as was stated earlier. In the capital region, i.e. Mosfellsbær, Reykjavík, Seltjarnarnes, Kópavogur, Garðabær, Álftanes and Hafnarfjörður 193.444 people lived or 61% of the total population, while the number of 15-year olds was 2.647 or 59% of the total number of 15-year olds. Choosing 5–6 schools in each size group from the capital region reflects therefore well on that area versus the whole country.

The goal of the investigation, research questions and methods

The aim of the research is to investigate whether the education, experience and specialization of teachers, or teaching material used, affected the results of the students. The following research questions were posed in the different School Groups:

1. Is it possible to detect a difference in the educational specialization of the teachers?
2. Is it possible to detect a difference in the experience of the teachers?
3. Is it possible to detect a difference in the specialization of the teachers in their work?
4. Is it possible to detect a difference in the use of textbooks and other teaching material?

A letter was sent to the principals of the 30 schools. It was not considered necessary to get their permission for the investigation since the results from individual schools were not published and the teachers themselves decided if they wanted to take part or not. After gathering information on who had taught the student groups considered, a questionnaire was send via email followed by a telephone interview in the next few days. The questionnaire consisted of the following items:

- the education of the teachers
- which year they had been teachers of the PISA 2012 student group
- which proportion of their work was the teaching of mathematics during the school year 2011–2012

- for how long they had been teaching mathematics in the same proportion
- what teaching material was used for the PISA 2012 group
- whether any of the students had upper secondary level mathematics as an elective course (STÆ103).

The investigators came across some hindrances in their work in locating teachers:

- schools had merged so some schools no longer existed
- teachers had moved, some of them abroad
- teachers did not remember if they had taught the group in question since two years had passed and even principals did not know which teachers to contact.

The information that was easiest to obtain was from interviewing teachers who had been working in the same place for a long time or were heads of departments. Sometimes, there was some delay in getting a hold of the teachers. The interviews took place in March, April and May in the spring of 2014.

Results

Basic information

The number of students and teacher, the average number of students per teacher and the average age of teachers in the schools that were contacted was as follows:

Table 2 *Number of students and teachers, students per teacher and average age of teachers*

	Number of students	Number of teachers	Students per teacher	Average age of teachers
School Group 4 > 40 participants	721 (47%)	25	29	48 years
School Group 3 25–40 part.	334 (30%)	21	16	50 years
School Group 2 10–25 part.	174 (26%)	17	10	42 years
Total	1229 (35%)	63		47 years

From the table we see that teachers of approximately half of students in schools with more than 40 participants were contacted. On the average each one of them had contact with 29 students so many have been teaching more than one class.

Education of the teachers

The majority of the teachers in all groups had completed a B.Ed.-degree with mathematics education as a special subject or had taken extra courses in mathematics education after finishing their first degree. Three teachers, all in School Group 2, had an old degree (from the time when teacher education was at the upper secondary level), which they had later complemented with additional education. They are classified as “other education”. Four teachers had finished their B.Ed. from the University of Akureyri, with specialization in science teaching. They are classified with the mathematics specialization group.

Table 3 *The education of the teachers*

	Number of teachers	B.Ed or M.Ed. in Mathematics Education		B.Ed. with additional courses in Mathematics Education	Other, with additional courses in Mathematics Education	B.Ed. Other spec.	Other studies
		M.Ed.	B.Ed.				
School Gr. 4 >40 part.	25	3	9	4	1	6	2
		48%		16%	4%	24%	8%
School Gr. 3 25–40 part.	20	2	12	1	0	5	0
		70%		5%	0%	25%	0%
School Gr. 2 10–25 part.	17	0	10	0	1	3	3
		59%		0%	6%	17%	17%

It seems that the education of teachers is the highest in School Group 3 when considering mathematics education and mathematics. It is hard to distinguish between the education of teachers in School Group 4 and School Group 2. No teachers had completed a B.Sc. degree in mathematics.

School years that the teachers taught the PISA 2012 group

Teachers in School Group 4 had the highest occurrence of teaching the students during all three years of lower secondary school but the teachers in School Group 2 had the lowest occurrence.

Table 4 *School years that the teachers taught the group*

	Number of teachers	2011–2012	2010–2011	2009–2010
School Gr. 4 >40 participants	25	23 (92%)	24 (96%)	19 (76%)
School Gr. 3 25–40 part.	21	19 (90%)	17 (81%)	13 (62%)
School Gr. 2 10–25 part.	17	12 (71%)	11 (65%)	10 (59%)

The proportion of mathematics teaching in the teachers' work

The majority of the teachers in School Group 4 worked full time as mathematics teachers, possibly with some administrative duties as part of their work.

Table 5 *Proportion of mathematics teaching in the teachers' work*

	Number of teachers	Full time	>50%	<50%
School Gr. 4 >40 participants	25	16 (64%)	7 (28%)	2 (8%)
School Gr. 3 25–40 part.	21	9 (43%)	11 (52%)	1 (5%)
School Gr. 2 10–25 part.	17	8 (47%)	8 (47%)	11 (6%)

The number of years the teachers had taught mathematics

Table 6 *The number of years the teachers had taught mathematics*

	Number of teachers	1 year	2 years	3 years
School Gr. 4 >40 participants	25	0	1 (4%)	24 (96%)
School Gr. 3 25–40 part.	21	0	2 (10%)	19 (90%)
School Gr. 2 10–25 part.	17	4 (25%)	2 (13%)	10 (63%)

Teachers in School Group 2 had the least experience in teaching mathematics. They also had the lowest average age, 42 years, while the average age was 48 and 50 in School Groups 4 and 3, respectively. It was clear that change in mathematics teacher for the student was most common in School Group 2. If teachers took a leave of absence or quit (due to maternity or studies or other reasons) he or she was usually replaced by an outsider where as in the larger schools it seemed to be easier for one teacher in the group to take on the teaching of an absent one.

Upper secondary level mathematics

It is common that students are offered the possibility to study the first course in upper secondary level mathematics STÆ103, as an elective course. This turned out to be more common in School Group 4 with the highest number of PISA 2012 participants, see Table 7. Generally about 20 – 30% of the students chose the course.

Table 7 *Upper secondary mathematics course offered*

	Number of school that offered STÆ103 as an elective course
School Gr. 4 > 40 part.	8 of 10 schools or 80%
School Gr. 3 25–40 part.	7 of 9 schools or 77%
School Gr. 2 10–25 part.	7 of 10 schools or 70%

Textbooks

There are two possible series of textbooks to use: one of them is called *8 – Tíu (8 - Ten)* and the other one *Almenn stærðfræði (General mathematics)*, see Table 8. Most of the schools in all groups use both series; one as a main text and the other one as supplementary material.

Table 8 *Textbooks used*

	8 –Tíu		*Almenn stærðfræði*		Both series
	Main	Suppl.	Main	Suppl.	
School Gr. 4 >40 part.	**5**	2	**2**	4	**3**
School Gr. 3 25–40 part.	**6**	1	**2**	5	**1**
School Gr. 2 10–25 part.	**3**	2	**3**	2	**4**

Summary

It is of great concern that students who perform at level 6 in Iceland are fewer in PISA 2012 than in PISA 2003. When comparing the education of the mathematics teachers in the three School Groups it seems that teachers in School Group 3 have the most extensive education in mathematics or mathematics teaching. It was most common in School Group 4 that the teachers had taught the group for all three years of lower secondary school but this was least common in School Group 2. Many of the teachers had all or more than half of their teaching duties in the teaching of mathematics but this was most common in School Group 4. Teachers in School Group 4 had most experience but teachers in School Group 2 had least experience.

It was most common to offer upper secondary mathematics as an elective course in School Group 4; this was however done in the majority of all schools. The textbooks used were similar. The majority used *8-Tíu* or both textbook series. This indicates that teachers let curriculum direct their teaching more than the textbooks.

Conclusion

Figures 2 and 3 show that the performance is significantly better in large schools than in small schools. We sought explanations for this by investigating the teachers' education, experience, continuity in the teaching of the PISA 2012 group, textbooks used and students possibilities in choosing upper secondary mathematics. Our investigation did not show that difference in the teachers' education, see Table 3, could explain this and thus answer research question 1.

Investigation on the teachers' experience is two-sided; both how much they had taught the particular group who took the PISA 2012 test and how many years they had been teaching. The years of teaching the PISA 2012 group are given in Table 4. Almost every teacher in School Group 4 had been teaching the group both in their 9th and 10th year, most of them in the 8th year as well. It is also clear from the table that many of them had been teaching more than one groups which means they had to repeat their teaching. They therefore had more opportunities to contemplate the material and their teaching and thus improve it. The proportion was the lowest in School Group 2. This indicates that teachers in large schools have the most possibilities to acquire broad competence across material of consecutive years, which Egelund (2006) claims as an explanation of the better results in the largest schools. This is also in harmony with Shulman's (1986) theory on the expected curricular knowledge of these teachers. Table 6 shows that the teachers in the largest schools had also had the most experience in teaching of mathematics see research question 2, but that supports the same theories.

It can be read from Table 5 that the largest proportion of teachers in the School Group 4 were full-time math teacher, but research question 3 was on the specialization of teachers in their work. It supports the hypothesis of Egelund that large schools provide the best opportunities for teachers to teach the subject they are best prepared for. It is also Egelund's assumption that large schools have the greatest potential for teachers to build professional teams.

It is not directly possible to read from the tables a difference in the depth of pedagogical content knowledge of teachers according to the theories of Shulman and Krauss et al. (2008) It can be assumed, however, that teachers who have taught for a long time have collected useful ways to present the material and are better at presenting the material in a way that is comprehensible to others. According to Table 6, teachers in School Group 4 have the longest experience but teachers in the School Group 2 shorter experience on average than teachers in the Schools of groups 3 and 4. It can also be assumed that teachers, who have lasted long in the job, are better at taking into account the learning conditions and needs of their students, professional and non-professional, and to challenge students professionally as Jaworski (1994) believes will lead to successful teaching, although nothing will be said with certainty from the above survey.

From the above it can be concluded from the information obtained from teachers in 29 of the 98 compulsory schools in Iceland with more than 10 participants in PISA 2012, that the experience of teachers, and especially the experience which comes from teaching the respective student group long and often, weigh the most to cause that the best results are achieved with students in the largest schools. Teachers in these schools seemed to have the best opportunities to get to know students and their needs and to have a good overview of the curriculum, both what came before and what will follow.

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